Proceedings of the 2009 IEEE International Conference on Systems, Man, and Cybernetics
San Antonio, TX, USA - October 2009

Image Segmentation Using Minimum Cross-Entropy Thresholding

Amani Al-Ajlan
Department of Information Technology, College of Computer and Information Sciences
King Saud University
Riyadh, Kingdom of Saudi Arabia
alajlan@ccis.edu.sa

Ali El-Zaart
Department of Computer Science, College of Computer and Information Sciences
King Saud University
Riyadh, Kingdom of Saudi Arabia
dr_elzaart@yahoo.com

***Abstract*— Entropy-based image thresholding is an important concept in the area of image processing. Pal (1996) proposed a minimum cross-entropy thresholding method based on Gaussian distribution. Our new method is derived from Pal method that segment images using minimum cross-entropy thresholding based on Gamma distribution and can handle bi-modal and multimodal images. Our method is tested by using Synthetic Aperture Radar (SAR) images and it gives reliable results for bi-modal and multimodal images.**

***Keywords*— Thresholding, SAR image, Gamma distribution**

I. INTRODUCTION

Image segmentation refers to the process of subdividing a digital image into multiple regions or objects [11]. The goal of segmentation is to simplify or change the representation of an image into a form that is more meaningful and easier to analyze. Image thresholding is an efficient technique for image segmentation applications and for pattern recognition [2]. The important step in thresholding is the choice of the threshold. There are two different approaches for thresholding: global and local thresholding. Global thresholding techniques segment the entire image by using a single global threshold based on gray level values [11]. On the other hand, local thresholding techniques segment the image into smaller sub-images then the thresholds will be calculated for each sub-image depending on local properties of the point or its position as well as its gray level values [11].

Recently, SAR images are widely used in many applications such as: environmental monitoring, earth-resource mapping, search-and-rescue and military systems [8]. SAR images usually contain speckles which are considered natural phenomenon generated by the coherent processing of radar echoes [1] [8]. The presence of speckle reduces the ability to resolve small details; making the segmentation a complicated process [1] [8]. Segmentation of a SAR image can be done based on grey levels or based on texture. We will deal with SAR images segmentation based on grey levels.

We propose a new method which is derived from Pal (1996) method [10], that used minimum cross-entropy thresholding method for estimating optimal threshold value based on Gamma distribution. While Pal (1996) method based on Gaussian distribution. Moreover, our new method processes bi-modal and multimodal images.

The paper is organized as follows: section II describes Gamma distribution. In section III, an introduction to thresholding is presented. Entropy-based thresholding and Pal cross entropy thresholding method are presented in section IV and section V. Section VI describes our proposed method and its algorithms. The experimental results are presented in section VII. Section VIII and IX are present the conclusion and future work.

II. GAMMA DISTRIBUTION

A gamma distribution is a general type of statistical distribution [14]. In probability theory and statistics, the Gamma distribution is a continuous probability distribution. The probability density function of the Gamma distribution in homogeneous area is given by [1] [12]:

$$f(x,\mu,N)=\frac{2q}{\mu}\frac{N^N}{\tau(N)}\left(\frac{qx}{\mu}\right)^{2N-1}e^{-N(qx/\mu)^2} \quad (1)$$

Where $q=\frac{\tau(N+0.5)}{\tau(N)\sqrt{N}}$, x is the intensity of the pixel, μ is the mean value of the distribution and N represents the parameter shape of the distribution.

Gamma distribution is better than Gaussian because Gaussian distribution works only with symmetric histograms. While Gamma distribution has ability to provide symmetric and non-symmetric histograms [12].

Gamma distribution is used due to its simplicity compared with other distribution such as: K-distribution and Beta distribution [7].

III. THRESHOLDING

Image thresholding based on the gray level histogram is an efficient and important technique for image segmentation, object detection and enhancement [6]. There are many thresholding applications such as: document image analysis, map processing and scene processing [9]. Image thresholding is the simplest technique in image segmentation and it is widely used due to its simplicity in implementation and its speed in processing [7]. It assumes that the object and the background in the image have distinct gray level distributions. Segmentation is performed by assigning the pixels with gray levels above the threshold to the object and the pixels with gray levels below the threshold to the background [2].

Several different methods for selecting a threshold have been presented in the literature. One of them is an Entropy-based method. Next a brief description of Entropy-based method is presented.

IV. ENTROPY-BASED THRESHOLDING

Entropy is "the measure of information content in probability distribution" [2]. Entropy could be used also as "a measure of separation that separates the information into two regions, above and below an intensity threshold" [12]. A number of entropy based thresholding methods are exist in the literature. These methods can be categorized into three groups: entropic thresholding, cross-entropic thresholding and fuzzy entropic thresholding. Entropic thresholding considers "the image foreground and background as two different signal sources, so that when the sum of the two class entropies reaches its maximum, the image is said to be optimally thresholded" [9]. Cross-entropic thresholding considers "the thresholding as the minimization of an information theoretic distance" [9]. Fuzzy entropic thresholding considers "the fuzzy memberships as an indication of how strongly a grey value belongs to the background or to the foreground" [9].

We will focus on cross-entropic thresholding. It was proposed by by Kullback in 1968 [7].Various algorithms have been proposed for cross-entropic thresholding. Among them, Li and Lee (1993) introduced the minimum cross entropy thresholding algorithm that selects the threshold, which minimizes the cross entropy between the segmented image and the original image [4]. Li and Lee method is sequential and based on Gaussian distribution [2] [7]. In addition, Li and Tam (1998) developed an iterative method derived from Li and Lee [4] to obtain the threshold that minimizes the cross entropy using Gaussian distribution [5]. Al-Attas and El-Zaart (2006) proposed a sequential method derived from Li and Lee [4] and Li and Tam [5] to find optimal threshold value by minimizing cross entropy using Gamma distribution [12]. Their method got good results in medical images. Al-Osaimi and El- Zaart (2008) [7] developed a new iterative algorithm derived from Li and Lee [4] and Li and Tam[5] to find optimal threshold value by minimizing cross entropy using Gamma distribution [7]. Their method got good results for SAR images.

The symmetric version of the cross entropy was proposed by Pal [10]. Pal cross entropy thresholding method is mentioned next.

V. PAL CROSS ENTROPY THRESHOLDING METHOD

Pal proposed a minimum cross entropy thresholding method where image histogram is modeled by a mixture of Poisson distributions [2]. The Pal method is as follow: Let *t* to be the threshold for segmentation. So, *t* partitions the image into two regions: the object and background. The gray level values from 0 to *t* were assumed to be object region while those from *t+1* to L were the background region where L is the possible number of gray levels [10]. Then the distribution of gray level in the object region can be defined as:

$$P_O = \{p_1^O, p_2^O, ..., p_t^O\}$$

And that for background region as:

$P_B = \{p_{t+1}^B, p_{t+2}^B, ..., p_L^B\}$, Where:

$$p_i^O = \frac{h_i}{P_{sum}} \; i = 1,2,3,......t \text{ Where } P_{sum} = \sum_{i=1}^{t} h_i \quad (2)$$

$$p_i^B = \frac{h_i}{M \times N - P_{sum}} \; i = t+1, t+2, ...L \quad (3)$$

Note that:

$$\sum_{i=1}^{t} p_i^O = 1 \text{ and } \sum_{i=t+1}^{L} p_i^B = 1$$

h_i is the frequency of gray level i, M × N is the width and the length of the image and L is the possible number of gray levels.

Let Q_O and Q_B be the probability distribution of object and background regions based on Poission model:

$$Q_O = \{q_1^O, q_2^O, ...q_t^O\}$$

$$Q_B = \{q_{t+1}^B, q_{t+2}^B, ...q_L^B\}$$

$$q_i^O = \frac{e^{-\mu_O} \mu_o^i}{i!} \; i = 1,2,........t \text{ and} \quad (4)$$

$$q_i^B = \frac{e^{-\mu_B} \mu_B^i}{i!} \; i = t+1, t+2, ...L \quad (5)$$

Where μ_O is the mean value of object region and μ_B is the mean value of background. They were estimated using Gaussian distribution:

$$\mu_O = \frac{\sum_{i=1}^{t} i h_i}{\sum_{i=1}^{t} h_i} \quad \text{and} \quad \mu_B = \frac{\sum_{i=t+1}^{L} i h_i}{\sum_{i=t+1}^{L} h_i} \quad (6)$$

h_i is the frequency of gray level i.

The cross entropy for the object region is:

$$D_O(t) = D_t(P_O, Q_O) = \sum_{i=1}^{t} p_i^O \log\left(\frac{p_i^O}{q_i^O}\right) + \sum_{i=1}^{t} q_i^O \log\left(\frac{q_i^O}{p_i^O}\right) \quad (7)$$

and the cross entropy for the background region is:

$$D_B(t) = D_t(P_B, Q_B) = \sum_{i=t+1}^{L} p_i^B \log\left(\frac{p_i^B}{q_i^B}\right) + \sum_{i=t+1}^{L} q_i^B \log\left(\frac{q_i^B}{p_i^B}\right) \quad (8)$$

The total cross entropy of segmented image can be written as:

$$D(t) = D_O(t) + D_B(t)$$

$$D(t) = \sum_{i=1}^{t} p_i^O \log\left(\frac{p_i^O}{q_i^O}\right) + \sum_{i=1}^{t} q_i^O \log\left(\frac{q_i^O}{p_i^O}\right) +$$

$$\sum_{i=t+1}^{L} p_i^B \log\left(\frac{p_i^B}{q_i^B}\right) + \sum_{i=t+1}^{L} q_i^B \log\left(\frac{q_i^B}{p_i^B}\right) \quad (9)$$

In order to segment an image, $D(t)$ is minimized with respect to *t* [10].

Our new method that derived from improving Pal cross entropy thresholding method is described in the next section.

VI. Our Proposed Method

Pal (1996) obtained threshold with Gaussian distribution while our work is based on Gamma distribution, so it can handle bi-modal and multimodal images. We were interested to use Gamma distribution due to: its capability in image modeling and its publicity over Gaussian distribution [1] [12]. In our method, we calculate mean value using Gamma distribution instead of Gaussian. The data in the image is assumed to be modeled by Gamma distribution. Thus, we can see that the image is composed by two Gamma distributions. For an object region, there is a Gamma distribution with mean μ_O and for a background region, there is a Gamma distribution with mean μ_B. The estimation of means is as follows [12]:

$$\mu_O(t) = \sqrt{\frac{\sum_{i=0}^{t} h(i) i^2 q^2}{\sum_{i=0}^{t} h(i)}} \quad \text{and} \quad (10)$$

$$\mu_B(t) = \sqrt{\frac{\sum_{i=t+1}^{L} h(i) i^2 q^2}{\sum_{i=t+1}^{L} h(i)}} \quad (11)$$

Where h(i) is the histogram defined on the gray level [0,L] and $q = \frac{\tau(N+0.5)}{\tau(N)\sqrt{N}}$

We will discuss the bi-modal and the multimodal thresholding algorithms in the next section.

A. Bi-modal Thresholding Algorithm

The bi-modal algorithm is derived from Pal method [10]. Fig.1 described the algorithm.

min = High-value

$for Start \le t \le End \;\; do$

{

$compute\ \mu_O$ using equation(10), $i = start, \ldots, t$

$compute\ \mu_B$ using equation(11), $i = t+1, .., End$

$compute\ \ p_i^o, i = start, \ldots, t$ using equation(2)

$compute\ \ p_i^B,\ i = t+1, \ldots, End$ using equation(3)

$compute\ \ q_i^o, i = start, \ldots, t$ using equation(4)

$compute\ \ q_i^B, i = t+1, \ldots, End$ using equation(5)

$compute\ \ D(t)$ using equation(9)

$if\, (\min > D(T))$

{

$\min = D(t)$

$threshold = t$

}

}

Figure 1. Bi-modal Thresholding Algorithm

Where *Start* is the initial threshold t_i, and End is the next threshold t_{i+1}. For bi-modal thresholding, we choose Start=1 and End=255.

B. Multimodal Thresholding Algorithm

Some images contain more than two classes. So, they can not threshold using bi-modal algorithm. They need some processing. We extend our application to handle multimodal thresholding.

Assume T is a set of threshold values T= $\{t_1, t_2, t_3 \ldots\ldots\ldots t_{m-1}\}$, t_0=0 and t_m=L-1

where $t_0 < t_1 < t_2 < t_3 < \ldots\ldots\ldots t_{m-1} < t_m$, m is number of classes in the image and L is the number of possible gray levels. The multimodal cross-entropy is defined as follows:

$$D(t) = \left(\sum_{i=t_0}^{t_1} p_i^O \log\left(\frac{p_i^O}{q_i^O}\right) + \sum_{i=t_0}^{t_1} q_i^O \log\left(\frac{q_i^O}{p_i^O}\right) \right) +$$

$$\left(\sum_{i=t_1+1}^{t_2} p_i^B \log\left(\frac{p_i^B}{q_i^B}\right) + \sum_{i=t_1+1}^{t_2} q_i^B \log\left(\frac{q_i^B}{p_i^B}\right) \right) + \ldots.$$

$$..+\left(\sum_{i=t_{m-2}+1}^{t_{m-1}} p_i^O \log\left(\frac{p_i^O}{q_i^O}\right) + \sum_{i=t_{m-2}+1}^{t_{m-1}} q_i^O \log\left(\frac{q_i^O}{p_i^O}\right)\right)$$
$$+\left(\sum_{i=t_{m-1}+1}^{L-1} p_i^B \log\left(\frac{p_i^B}{q_i^B}\right) + \sum_{i=t_{m-1}+1}^{L-1} q_i^B \log\left(\frac{q_i^B}{p_i^B}\right)\right) \quad (12)$$

Also, the mean value can be estimated for any mode k.

$$\mu_K(t) = \sqrt{\frac{\sum_{i=t_{k-1}}^{t_k} h(i) i^2 q^2}{\sum_{i=t_{k-1}}^{t_k} h(i)}} \qquad (13)$$

Multimodal algorithm is described in Fig. 2.

1. Compute initial values of threshold using k-mean algorithm [15]:

   $T_0 = \{t_0^1, t_0^2, t_0^3 \ldots\ldots\ldots t_0^{m-1}\}, t_0 = 0$ and $t_m = L-1$
2. Compute new threshold values T_{new} using bimodal method

   $for 1 \le i < m$ Where m is the number of classes

   Call bimodal method to get t_{new}^k

   Where Start $= t_{k-1}$, End= t_{k+1} and k=1, 2,…… m-1
3. Test whether optimal thresholds are reached

   If $|T_0 - T_{new}| < \varepsilon$ then

   Optimal threshold values

   $t_{optimal}$ $\{t_1^*, t_2^*, t_3^* \ldots\ldots\ldots t_{m-1}^*\}$ are reached

   Else

   Assign $T_0 \longleftarrow T_{new}$ and go to step 2

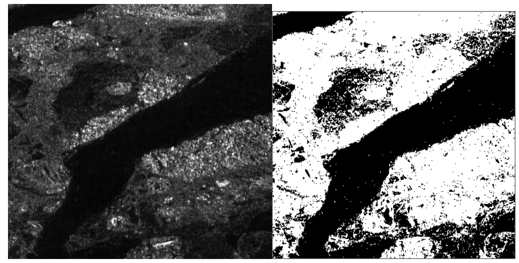
Figure 2. Multimodal Thresholding Algorithm

VII. Experimental Results

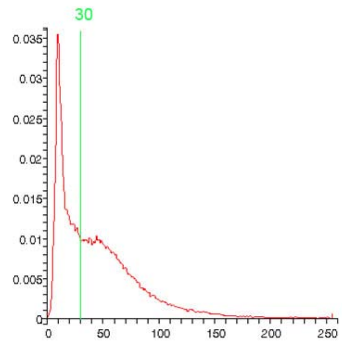
Our minimum cross entropy thresholding method is implemented and tested on many SAR images. We first apply the new method for bimodal images then on multimodal images. The result is encouraging. A brief discussion about the results will be mention in the next section.

A. Bi-modal Images Results

We applied our method that used Gamma distribution on bi-modal real SAR images. Fig. 3(b) shows segmented image with threshold 30. In Fig. 4(b) the threshold is 29, in Fig. 5(b) the threshold is 27 and in Fig. 6(b) the threshold is 26.

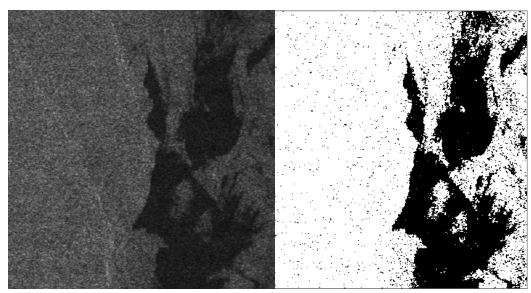


(a) (b)

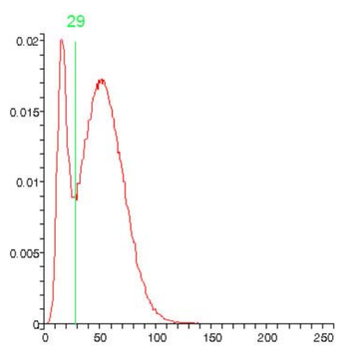


(c)

Figure 3. (a) Original SAR image. (b) Segmented image with N=10, t=30 and m=2. (c) Image histogram

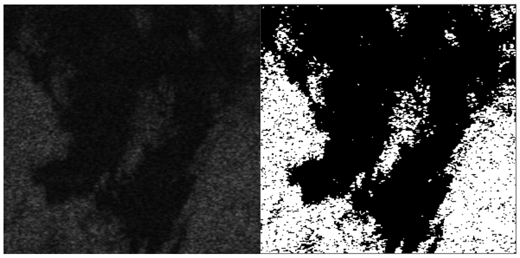


(a) (b)

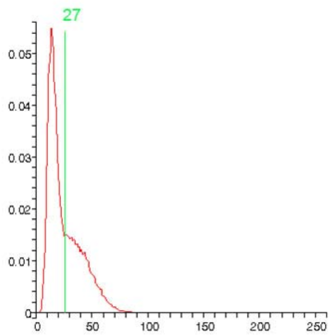


(c)

Figure 4. (a) Original SAR image. (b) Segmented image with N=8, t=29 and m=2. (c) Image histogram

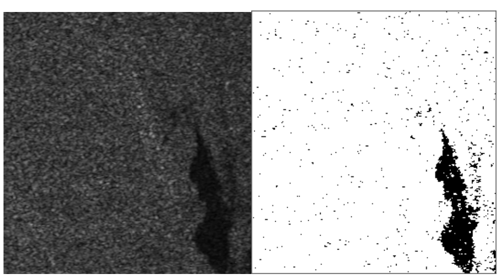


(a) (b)

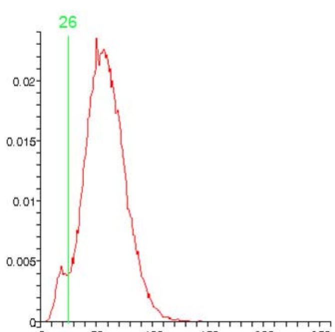


(c)

Figure 5. (a) Original SAR image. (b) Segmented image with N=12, t=27 and m=2. (c) Image histogram



(a) (b)

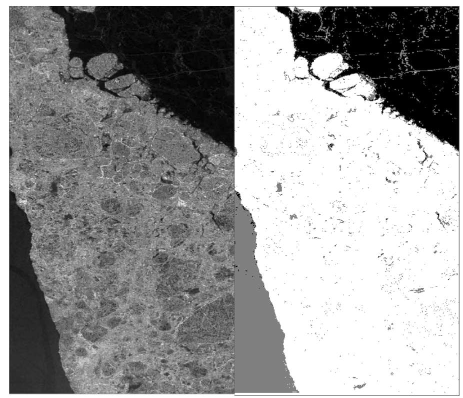


(c)

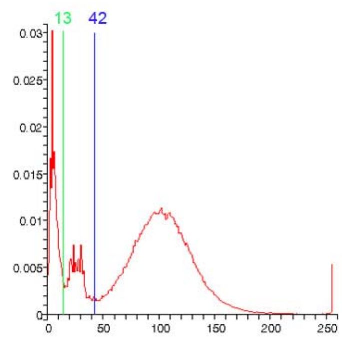
Figure 6. (a) Original SAR image. (b) Segmented image with N=2, t=26 and m=2. (c) Image histogram

B. Multimodal Images Results

We applied our multimodal algorithm on some images and the results are encouraging. Fig. 7(b) shows an example of segmented image with first threshold=13 and second threshold=42.



(a) (b)



(c)

Figure 7. (a) Original SAR image. (b) Segmented image with N=12, t1=13, t2=42 and m=3. (c) Image histogram

VIII. CONCLUSION

In this paper, we propose a new method that derived from Pal (1996) minimum cross-entropy thresholding method for estimating optimal threshold value. Our new method is based on Gamma distribution instead of Gaussian distribution and handles multimodal images that have more than two classes of pixels. Due to gamma ability of representing symmetric and non-symmetric histograms. This method is more general than Pal method. Moreover, our experimental results on SAR images showed good results in thresholding.

IX. FUTURE WORK

As future work, we will use iterative method for minimum cross-entropy thresholding which is fast compared to sequential method. Also, we will apply our method on other images such as: medical images and MRI image

REFERENCES

[1] A. El Zaart, D. Ziou, S. Wang, and O. Jiang, “Segmentation of SAR images,” Pattern Recognition Journal, vol. 35, no. 3, March 2002, pp. 713-724.

[2] A. Şengur, I. Turkoglu and M. İnce, "A Comparative Study On Entropic Thresholding Methods" Journal of Electrical & Electronics Engineering, vol.6, no. 2, 2006, pp. 183-188.

[3] B. Sankura,, and M. Sezgin, "Image Thresholding Techniques: a Survey over Categories", Pattern Recognition, 2001

[4]. C. Li, and C. Lee, “Minimum Cross Entropy Thresholding”, Pattern Recognition-Elsevier, vol. 26, 1993, pp. 617-625.

[5]. C. Li, and P. Tam, “An Iterative Algorithm Cross Entropy Thresholding,” Pattern Recognition Letter - Elsevier, vol.19, no. 8, 1998, pp. 771-776.

[6] C. Chang,Y. Du, J. Wang, S. Guo and P. Thouin, "Survey and Comparative Analysis of Entropy and Relative Entropy Thresholding Techniques", IEEE Proc.-Vis. Image Signal Process, vol. 153, no. 6, December 2006, pp. 837-850.

[7]. G. Al-Osaimi and A. El Zaart, "Minimum Cross Entropy Thresholding for SAR Images", 3rd IEEE International Conference on Information & Communication Technologies: From Theory to Applications ICTTA. Syria, April 2008.

[8] J.S. Lee, "Speckle suppression and analysis for SAR images", Optical Engineering, vol.25, no.5, 1968, pp.636-643.

[9] M. Sezgin, and B. Sankur, “Survey Over Image Thresholding Techniques And Quantitative Performance Evaluation”, Journal of Electronic Imaging, vol. 13, no. 1, January 2004, pp. 146-165.

[10] N.R. Pal, "On Minimum Cross-Entropy Thresholding", Pattern Recognition Journal-Elsevier, vol.29, no. 4 ,1996, pp. 575-580.

[11] R. Gonzalez, and R. Woods, Digital image processing. Prentice Hall, New York, 3rd Edition, 2008.

[12] R. Al-Attas, and A. El-Zaart, “Thresholding of Medical Images Using Minimum Cross Entropy”, kuala lumpur international conference on biomedical engineering, kuala lumpur, Malaysia, 2006, pp. 312-315.

[13] S. B. Mansor, H. Assilzadeh, H.M. Ibrahim and A. R. Mohamd, "Oil Spill Detection and Monitoring from Satellite Image", from http://www.gisdevelopment.net/application/miscellaneous/misc027pf.htm

[14] Mathworld http://mathworld.wolfram.com/GammaDistribution.html

[15] C. Chen, J. Luo, and K. Parker, “Image segmentation via adaptive Kmean clustering and knowledge-based morphological operations with biomedical applications,” IEEE - Transactions on Image Processing, vol. 7, issue 12, 1998,pp.1673–1683.