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Positive and Negative Predictive Power as an Aid in Robust/Dynamic Decision Making

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***Abstract*—The diagnostic capability of using positive and negative predictive power in dynamic and robust decision making paradigms is explored. Several mathematical formulations are discussed to employ this concept to assist in decision making when human-machine systems may have characteristics that change with time.**

***Keywords*—signal detection theory, dynamic decision making.**

I. Introduction

Signal detection theory (SDT) has been studied for over 50 years with many of its principals for decision making being widely used and highly accepted [1,2]. In medical diagnosis, drug testing, and other areas, SDT provides the current standard for testing statistical hypotheses. However, numerous modern day systems now have properties that may vary with time. For example, the traditional signal to noise ratio measure d' and bias variable c from SDT may adapt with time and static models may be less effective. Robust and dynamic decision making needs to have measures and procedures that can adjust with time [3]. Positive and negative predictive power are explored in this paper as a possible paradigm to assist in robust and dynamic decision making.

II. Background

A. The Basics of Signal Detection Theory

Since brevity must be the style herein, Figure (1) portrays a truth table for a binary decision task. H denotes the number of hits, CR is correct rejects, FA denotes false alarms and M represents the number of misses. The standard definitions that will be used here are described in equations (1-4):

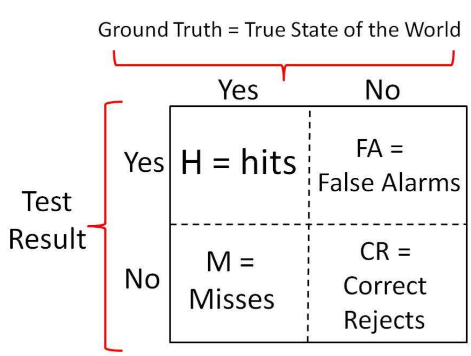

Figure (1) – Truth Table for Statistical Test

$$\text{Sensitivity} = Sn = \frac{H}{H+M} \tag{1}$$

$$\text{Specificity} = Sp = \frac{CR}{CR+FA} \tag{2}$$

$$\text{Positive Predictive Power} = \text{PPP} = \frac{H}{H+FA} \tag{3}$$

$$\text{Negative Predictive Power} = \text{NPP} = \frac{CR}{CR+M} \tag{4}$$

B. The Use of PPP and NPP to Assist in Decision Making

Figure (2) shows the typical statistical test and the appropriate parameters to discern objects with the signal to noise ratio d'and bias parameter c as defined. In the medical diagnostics area physicians now highly embrace the concepts of PPP and NPP rather than just using d' and c for several important reasons: (1) The data properties may change with time with d' and c adapting [3,4], and (2) d' and c are variables calculated with a static analysis, however, the underlying system may experience a dynamic change requiring modification of the decision making process. It is desired to improve upon the decision rule as the fundamental model characterizing the physical process may vary with time. We consider some ways to generalize signal detection theory to handle dynamic decision making yet still maintain the optimality properties and other advantages that classical signal detection theory brings to the study of decision processes.

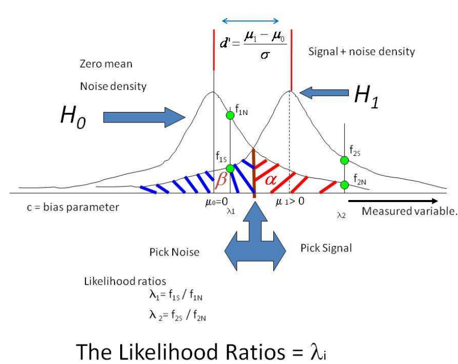

Figure (2) – Classical Binary Decision Making with Optimality

III. Some Key classical relationships

To relate the terms in equations (1-4) to some of the well known quantities in statistical testing for binary hypotheses, the elements of figure (2) can be described via:

$$\alpha = \frac{FA}{CR + FA} = 1 - specificity \qquad (5)$$

or

$$specificity = 1 - \alpha \qquad (6)$$

and

$$\beta = \frac{M}{M + H} = 1 - sensitivity \quad = \text{power of the test} \qquad (7)$$

with

$$sensitivity = 1 - \beta \qquad (8)$$

Note, also in Figure (2):

$$d' = \frac{\mu_1 - \mu_0}{\sigma} \qquad (9)$$

IV. Optimality Measures for decision making

Three types of optimality in decision making are discussed herein. The first two types of decision making can be developed using the d' and c values in Figure (2).

Optimality Test -1 – Neyman-Pearson (fix α, minimize β)

From Figure (2) and the classical maximum likelihood ratio test, it is well known [5] that using a decision rule based on the likelihood ratio is optimal in the Neyman-Pearson sense. This means for a fixed α (type 1 error), the minimum type 2 error (β in Figure (2)) is realized. The decision rule employs the ratio of the probability density functions in Figure (2). The decision rule is to select choice H_1 over choice H_0 if

$$\lambda = (f_1/f_0) > \text{Threshold} \qquad (10)$$

The second, and well accepted test of optimality involves the area under the ROC (receiver operator characteristic) curve.

Optimality Test 2 – Area under an ROC curve

In the fields of medical testing, drug evaluation and diagnostisity in general, a concept called "discriminability" is highly touted in the field. Herein a brief description of the ROC curve is provided to be instructive. Figure (3) shows the ROC curve as a plot of sensitivity versus 1-specificity. The area under the ROC curve (discriminability) accounts for the general ability of a statistical test to include the two types of errors (misses and false alarms). It represents an overall measure of the efficacy of the test. This is precisely what PPP and NPP can bring to the decision making process.

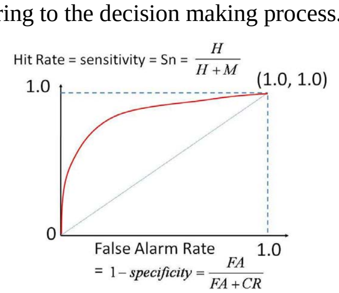

Figure (3) The ROC (Receiver Operator Characteristic) Curve

From Figure (3), the efficacy of the test (discriminability) is the total area under the ROC curve which is desired to be maximized. It should be pointed out that ROC curves are not arbitrary functions and have certain restrictive properties. Figure (4), adapted from [6], shows some of the constraints on points that fall on the ROC curve. Thus certain ROC curves may be "proper" or not. In addition, the placing of the points on the ROC curve needs to be corrected for certain biases as pointed out in [6].

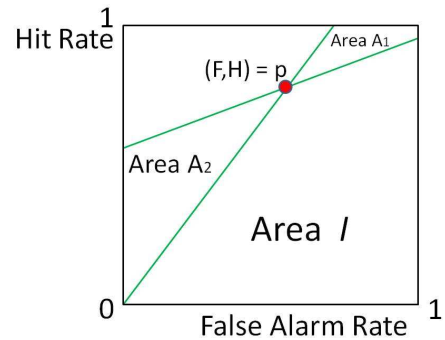

Figure (4) – Adapted from [6] on Proper ROC curves

So far the optimality tests for decision making have considered minimizing certain types of error or maximizing overall discriminability. The next optimality test will generalize signal detection theory to dynamic systems.

Optimality Test 3 - The Wald Sequential Test

The classical Wald sequential algorithm test involves two types of optimality and provides an entry into dynamic decision making [7]. Figure (5) portrays a dynamic decision making situation in which the ROC curve may be changing with time.

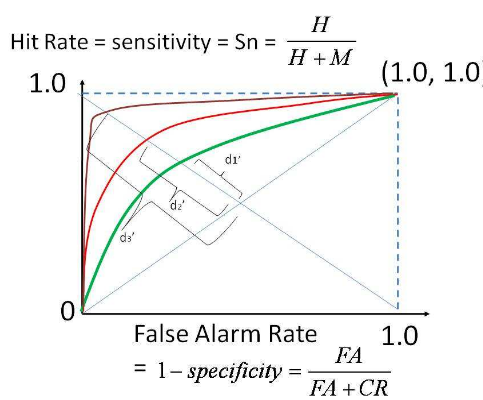

Figure (5) – Adapting ROC curves as System Parameters Vary

In Figure (5) the d' and c parameters vary with time and the goal is to adjust the decision rule, accordingly. To complete this background information, the optimal Wald sequential decision test can be briefly described as follows [7]:

A statistical measure $\gamma(t)$ (likelihood ratio) is computed in real time (Wald test), where

$$A(t) = \log\frac{\beta}{1-\alpha} \leq \gamma(t) \leq \log\frac{1-\beta}{\alpha} = B(t) \quad (11)$$

The decision rule is: If the determined $\gamma(t) > B(t)$, then select hypothesis H_1 in Figure (2). However, if $\gamma(t) < A(t)$, then choose H_0. If neither these conditions are true, then collect more data.

The beauty of the Wald test is that if α or β vary with time, then the statistical parameters A(t) and B(t) adjust accordingly. The decision making process then modifies. It is shown [7] that the Wald test still maintains two types of optimality: (1) It is still Neyman-Pearson optimal in the sense that for a fixed α (type 1 error) then β (type 2 error) is minimized, as well as (2) the algorithm will converge in minimum time to a decision. What this means is that a decision is reached in about 50% of the time [7] as compared to a static maximum likelihood decision making process with the same values of α and β. Thus dynamic decision making with time varying variables has significant advantages over static methodologies.

Finally, the concepts of PPP and NPP can now be generalized into dynamic decision making employing the background materials developed so far.

V. Adapting Decision Making with PPP and NPP

In [8] and Appendix A additional facts on the derivation and relationships of PPP and NPP to dynamical statistical hypothesis testing are detailed. A brief description of the pertinent steps will be presented here. Equations (12-19) show key relationships between the classical quantities CR, M, H, and FA from Figure (1) to the PPP and NPP parameters as well as sensitivity (*Sn*) and specificity (*Sp*).

$$CR = M\left(\frac{NPP}{1-NPP}\right) \quad (12)$$

$$M = CR\left(\frac{1-NPP}{NPP}\right) \quad (13)$$

$$H = (FA)\left(\frac{PPP}{1-PPP}\right) \quad (14)$$

$$FA = (H)\left(\frac{1-PPP}{PPP}\right) \quad (15)$$

$$CR = (FA)\frac{Sp}{1-Sp} \quad (16)$$

$$FA = (CR)\frac{1-Sp}{Sp} \quad (17)$$

$$H = (M)\frac{Sn}{1-Sn} \quad (18)$$

$$M = (H)\frac{1-Sn}{Sn} \quad (19)$$

The following key result has its details contained in Appendix A and can be described via Theorem 1:

Theorem 1:

Sensitivity (*Sn)* and specificity (*Sp*) can be related to PPP and NPP through the following formula:

$$\left(\frac{Sn}{1-Sn}\right)\left(\frac{Sp}{1-Sp}\right) = \left(\frac{PPP}{1-PPP}\right)\left(\frac{NPP}{1-NPP}\right) \quad (20)$$

Proof: Appendix A and reference [8] derive the result in equation (20).

The second major result is to extend the Wald sequential test from equation (11) into a form for the use of PPP and NPP. With this dynamic decision making criterion the Neyman-Pearson property of optimality is still preserved and the minimum time for a decision to be made is still left intact. Theorem 2 presents this main result:

Theorem 2:

The equivalent to the Wald sequential test of equation (11) can be written in terms of PPP and NPP via the following method:

The optimal Wald sequential algorithm can be modified to include the PPP and NPP terms. For the test statistic $\gamma(t)$ (likelihood ratio) in equation (11), the new regions for testing now include:

$$\log\left[\left(\frac{1-\beta}{\alpha}\right)\left(\frac{1-PPP}{PPP}\right)\left(\frac{1-NPP}{NPP}\right)\right] \leq \gamma(t) \leq \log\left[\left(\frac{\beta}{1-\alpha}\right)\left(\frac{PPP}{1-PPP}\right)\left(\frac{NPP}{1-NPP}\right)\right] \quad (21)$$

Proof: Appendix B and [8] outline the derivation of equation (21). Equation (21) is equivalent to equation (11) and still preserves the two types of optimality as enjoyed by the Wald sequential test.

Remarks: The formulation (21) has new advantages over prior methods. It is known that the parameters d' and c of Figure (2) may vary with time. From equation (21) it is seen that the decision parameters α and β are known *apriori*. Also the experimental parameters PPP and NPP may vary with time since the d' and c values are known to change but are easily measured in real time. Thus by using equation (21) rather than (11) provides a dynamic decision making procedure which can adapt with time having an emphasis on the time varying PPP and NPP variables. The result in equation (21) still enjoys the same optimality properties as in the Wald sequential algorithm in equation (11) but is cast within the framework of the PPP and NPP variables. Again, it is emphasized that the advantages afforded by the representation in equation (21) over the formulation in (11) is that the PPP and NPP methods can be used which are now highly embraced in the medical field due to their utility in obtaining real time, on line, measurements.

VI. SUMMARY AND CONCLUSIONS

Using a PPP and NPP formulation for optimal decision making, it is shown that a dynamic Wald-type sequential algorithm can be developed depending only on the PPP, NPP, α and β values. The new dynamic decision making algorithm preserves the two types of statistical optimality provided by the Wald method but uses more modern procedures such as PPP and NPP.

ACKNOWLEDGMENT

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Appendix A – Derivation of Equation (20)

Starting with the equation (4), multiplying through by *(CR+M)* yields:

$$(CR+M)\ \text{NPP} = CR \tag{A.1}$$

or

$$CR\ (1\text{-NPP}) = M\ (\text{NPP}) \tag{A.2}$$

which gives rise to the following two relationships:

$$CR = M\left(\frac{NPP}{1-NPP}\right) \tag{A.3}$$

$$M = CR\left(\frac{1-NPP}{NPP}\right) \tag{A.4}$$

Starting with equation (3), multiply through by (*H+FA*) to yield:

$$(H+FA)\ \text{PPP} = H \tag{A.5}$$

or

$$H\ (1\text{-PPP}) = FA\ (\text{PPP}) \tag{A.6}$$

which arrives at the following two relationships:

$$H = (FA)\left(\frac{PPP}{1-PPP}\right) \tag{A.7}$$

$$FA = (H)\left(\frac{1-PPP}{PPP}\right) \tag{A.8}$$

To continue, start with equation (2) and multiply through by (*CR+FA*) to yield:

$$Sp\ (CR + FA) = CR \tag{A.9}$$

or:

$$CR\ (1 - Sp) = Sp\ FA \tag{A.10}$$

which results in the following relationships:

$$CR = (FA)\frac{Sp}{1-Sp} \tag{A.11}$$

$$FA = (CR)\frac{1-Sp}{Sp} \tag{A.12}$$

Finally, starting with equation (1), multiplying through by (*H+M*) yields:

$$Sn\ (H+M) = H \tag{A.13}$$

or

$$H\ (1\text{-}Sn) = Sn\ (M) \tag{A.14}$$

which gives rise to the following relationships:

$$H = (M)\frac{Sn}{1-Sn} \tag{A.15}$$

$$M = (H)\frac{1-Sn}{Sn} \tag{A.16}$$

To now develop the dependency between PPP, NPP, *Sn* and *Sp*, set CR=CR using equations (A.3) and (A.11) yielding:

$$M\left(\frac{NPP}{1-NPP}\right)=FA\left(\frac{Sp}{1-Sp}\right) \quad \text{(A.17)}$$

Now set M=M in equations (A.4) and (A.16) which results in:

$$CR\left(\frac{1-NPP}{NPP}\right)=H\left(\frac{1-Sn}{Sn}\right) \quad \text{(A.18)}$$

Following the same procedure by setting H=H in equations (A.7) and (A.15) which yields:

$$FA\left(\frac{PPP}{1-PPP}\right)=M\left(\frac{Sn}{1-Sn}\right) \quad \text{(A.19)}$$

Finally setting FA=FA in equations (A.8) and (A.12) results in the following relationship:

$$H\left(\frac{1-PPP}{PPP}\right)=CR\left(\frac{1-Sp}{Sp}\right) \quad \text{(A.20)}$$

The relationships (A.17)-(A.20) can be further reduced. Computing the relationship (CR)/H from both equations (A.18) and (A.20) yields:

$$\frac{CR}{H}=\frac{\left(\frac{1-Sn}{Sn}\right)}{\left(\frac{1-NPP}{NPP}\right)}=\frac{\left(\frac{1-PPP}{PPP}\right)}{\left(\frac{1-Sp}{Sp}\right)} \quad \text{(A.21)}$$

Cross multiplying implies the following classical dependence between PPP, NPP, *Sn* and *Sp:*

$$\left(\frac{1-Sn}{Sn}\right)\left(\frac{1-Sp}{Sp}\right)=\left(\frac{1-PPP}{PPP}\right)\left(\frac{1-NPP}{NPP}\right) \quad \text{(A.22)}$$

To check why this is true, similar calculations can be made using both equations (A.17) and (A.19) for (FA)/M:

$$\frac{FA}{M}=\frac{\left(\frac{Sn}{1-Sn}\right)}{\left(\frac{PPP}{1-PPP}\right)}=\frac{\left(\frac{NPP}{1-NPP}\right)}{\left(\frac{Sp}{1-Sp}\right)} \quad \text{(A.23)}$$

Cross multiplying implies the reciprocal of equation (A.22) is true which further validates this approach because if things are equal, their reciprocals should also equate:

$$\left(\frac{Sn}{1-Sn}\right)\left(\frac{Sp}{1-Sp}\right)=\left(\frac{PPP}{1-PPP}\right)\left(\frac{NPP}{1-NPP}\right) \quad \text{(A.24)}$$

Appendix B – The Wald Algorithm for PPP and NPP

Starting with the Wald test of equation (11), where α and β are known:

$$\log\frac{\beta}{1-\alpha} \le \gamma(t) \le \log\frac{1-\beta}{\alpha} \quad \text{(B.1)}$$

where, from equations (6) and (8)

$$\alpha = 1 - Sp \quad \text{(B.2)}$$

and

$$\beta = 1 - Sn \quad \text{(B.3)}$$

Thus the Wald optimality criterion (B.1) can be written:

$$\log\frac{1-Sn}{Sp} \le \gamma(t) \le \log\frac{Sn}{1-Sp} \quad \text{(B.4)}$$

But it is known that:

$$\left(\frac{Sn}{1-Sn}\right)\left(\frac{Sp}{1-Sp}\right)=\left(\frac{PPP}{1-PPP}\right)\left(\frac{NPP}{1-NPP}\right) \quad \text{(B.5)}$$

Thus
$$\left(\frac{1-Sn}{Sp}\right)=\left(\frac{Sn}{1-Sp}\right)\left(\frac{1-PPP}{PPP}\right)\left(\frac{1-NPP}{NPP}\right) \quad \text{(B.6)}$$

Hence
$$\left(\frac{1-Sn}{Sp}\right)=\left(\frac{1-\beta}{\alpha}\right)\left(\frac{1-PPP}{PPP}\right)\left(\frac{1-NPP}{NPP}\right) \quad \text{(B.7)}$$

Also starting with equation (B.5) yields:

$$\left(\frac{Sn}{1-Sp}\right)=\left(\frac{1-Sn}{Sp}\right)\left(\frac{PPP}{1-PPP}\right)\left(\frac{NPP}{1-NPP}\right) \quad \text{(B.8)}$$

Which can be written as:

$$\left(\frac{Sn}{1-Sp}\right)=\left(\frac{\beta}{1-\alpha}\right)\left(\frac{PPP}{1-PPP}\right)\left(\frac{NPP}{1-NPP}\right) \quad \text{(B.9)}$$

Hence the optimal Wald sequential algorithm can be modified to include the PPP and NPP terms as follows: For the test statistic $\gamma(t)$ assuming constant α and β values with NPP and PPP possibly changing with time, the new regions for testing now become:

$$\log\left[\left(\frac{1-\beta}{\alpha}\right)\left(\frac{1-PPP}{PPP}\right)\left(\frac{1-NPP}{NPP}\right)\right] \le \gamma(t) \le \log\left[\left(\frac{\beta}{1-\alpha}\right)\left(\frac{PPP}{1-PPP}\right)\left(\frac{NPP}{1-NPP}\right)\right] \quad \text{(B.10)}$$