Proceedings of the 2009 IEEE International Conference on Systems, Man, and Cybernetics
San Antonio, TX, USA - October 2009

A Haptic Teleoperation Study Using Wave Variables and Scaling Matrices

M. Alise[1], R. G. Roberts[1]
[1] Florida A&M University-Florida State University, College of Engineering, Tallahassee, FL 32310 USA
rroberts@eng.fsu.edu

D. W. Repperger[2], C. A. Moore, Jr.[1]
[2] 711 HPW AFRL/RHCV WPAFB
Ohio, 45433 USA
d.repperger@ieee.org

***Abstract*—A haptics investigation is conducted when human subjects have to deal with a bilateral teleoperation system with two levels of time delay (400 and 1,000 ms) and four types of scaling matrices that provide a passivity form of stabilization. The wave variable method was implemented to produce a controllable human-machine response. What is of interest is the resulting performance that occurs from such systems which remain passive but impacts the overall human tracking performance that can be achieved. The question addressed is how these imposed stability restrictions (via architecture constraints) may degrade human tracking performance?**

***Keywords*—haptics, teleoperation, wave variables, time delay, scaling matrices, human performance.**

I. Introduction

Time delay in a bilateral teleoperation system will generally degrade the system's performance and cause instability. Consequently, without some form of compensation for time delay, latencies in a teleoperation system would preclude the use of force feedback. Fortunately, there are approaches based on scattering theory and passivity that can compensate for time delay and allow the use of force feedback in teleoperation systems with latencies. The wave variable method discussed here is a passivity-based approach that guarantees stability for any fixed time delay. An extended family of scaling matrices was used in an experiment with human subjects and a PHANToM Omni haptic teleoperation system to study human performance in a position tracking task.

II Background Information

Since brevity must be the style herein, only the basics of the wave variable method can be presented. The reader is referred to [1-4] for background information on the topics of teleoperation, the wave variable method, and scaling matrices. For the areas of haptics, references [5-9] are instructive. First the teleoperation problem with time delay needs to be recast in terms of the wave variable method.

A. Teleoperation with Time Delay

In Figure (1) a bilateral teleoperation with time delays T_R and T_L are shown. The human operator is located at the master controller and the end effector is the slave exposed to the environment. For any small amounts of time delays, such systems easily demonstrate instability. The wave variable method [10] is based on concepts of passivity [11] and reformulates the teleoperation problem into the wave variables as displayed in Figure (1). Such a system can be guaranteed to be passive (stable) but what is unknown is the amount of performance decrement such an architecture imposes on humans who use such procedures. This paper will report on the effect of such configurations interacting with humans for a position tracking task.

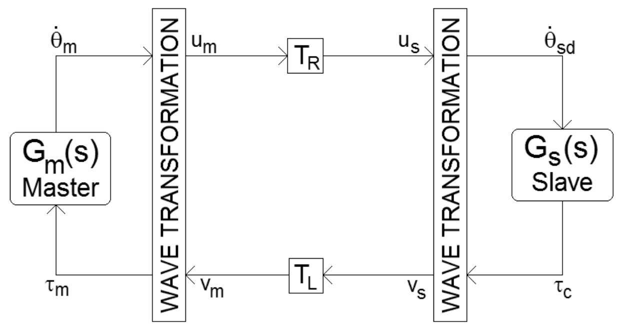

Figure (1) -A bilateral teleoperation system with wave transformations

B. Technical Details on the Wave Variable Formulation

The wave transformation relations for the single degree-of-freedom case in Figure (1) are given by:

$$u_s(t) = u_m(t-T) \tag{1}$$

$$v_m(t) = v_s(t-T). \tag{2}$$

The wave transformations for the left wave junction are:

$$u_m(t) = \frac{b\dot\theta_m(t) + \tau_m(t)}{\sqrt{2b}} \tag{3}$$

$$v_m(t) = \frac{b\dot\theta_m(t) - \tau_m(t)}{\sqrt{2b}} \tag{4}$$

and for the right wave junction are specified by:

$$u_s(t) = \frac{b\dot\theta_{sd}(t) + \tau_c(t)}{\sqrt{2b}} \tag{5}$$

$$v_s(t) = \frac{b\dot\theta_{sd}(t) - \tau_c(t)}{\sqrt{2b}} . \tag{6}$$

Although the strictly positive parameter b can be chosen arbitrarily, it defines a characteristic impedance associated with the wave variables and directly affects the system behavior [10].

C. Extension to Multiple Degrees of Freedom

Equations (3-6) are for single degree-of-freedom systems. To implement the wave variable method to a multivariable system that has more than one degree of freedom, the equations for the transforms must be generalized. Niemeyer and Slotine [10] suggest making b a positive definite matrix. Munir and Book [12, 13] have shown that in going to higher degrees of freedom, one can use a more general formulation. In particular, they introduced the following form for the wave transformation equations:

$$u_m(t) = A_w\dot{\theta}_m(t) + B_w\tau_m(t) \quad (7)$$

$$v_m(t) = C_w\dot{\theta}_m(t) - D_w\tau_m(t) \quad (8)$$

and

$$u_s(t) = A_w\dot{\theta}_{sd}(t) + B_w\tau_c(t) \quad (9)$$

$$v_s(t) = C_w\dot{\theta}_{sd}(t) - D_w\tau_c(t) \quad (10)$$

where A_w, B_w, C_w, and D_w are n x n wave variable scaling matrices and n is the number of degrees of freedom of the teleoperation system. The subscript "w" denotes the fact that the scaling matrices correspond to wave variable coefficients. These matrices cannot be chosen arbitrarily; certain relationships must hold so that the proper power relationships hold, e.g., the power flow for the master side should be:

$$\dot{\theta}_m{}^T\tau_m = \frac{1}{2}u_m{}^T u_m - \frac{1}{2}v_m{}^T v_m \quad (11)$$

$$\dot{\theta}_s{}^T\tau_c = \frac{1}{2}u_s{}^T u_s + \frac{1}{2}v_s{}^T v_s. \quad (12)$$

Furthermore, one must determine conditions for the scaling matrices to guarantee passivity [14, 15]. Substituting equations (7-10) into equation (11) or (12), expanding, and matching matrix coefficients yields the requirements:

$$C_w{}^T C_w = A_w{}^T A_w \quad (13)$$

$$D_w{}^T D_w = B_w{}^T B_w \quad (14)$$

and also

$$I = A_w{}^T B_w + C_w{}^T D_w. \quad (15)$$

D. Rules for Determining Scaling Matrices

To determine the complete set of scaling matrices, we first derive the whole family of matrices satisfying (13-15). In order to do this, we first use (13-14) to relate C_w and D_w to A_w and B_w, respectively, and then apply (15) to relate A_w and B_w. In previous work [1] it was determined that the necessary and sufficient conditions for A_w, B_w, C_w, and D_w to satisfy equations (13-15) are:

1) A_w is nonsingular.
2) $B_w = (1/2)\,(I + S_w)A_w{}^{-T}$ where S_w is any n x n skew symmetric matrix.
3) $C_w = QA_w$ where Q is any n x n orthogonal matrix.
4) $D_w = (1/2)Q(I - S_w)A_w{}^{-T}$.

Note that these four conditions guarantee that all four matrices are nonsingular. These conditions can be checked simply by substituting them back into (13-15). At this stage, it has only been shown that the conditions given in this section characterize the family of scaling matrices that result in wave variables (7-10) that satisfy the power flow equations such as equations (11,12). Characterizing the family of scaling matrices that result in passivity requires more work. Next, we derive the input-output relationship across the communication link then we use scattering theory to prove passivity.

E. The Input-Output Relationship

The input-output relationship across the communication link has the form [12]:

$$\begin{bmatrix} T_m(s) \\ s\Theta_{sd}(s) \end{bmatrix} = G_w(s)\begin{bmatrix} s\Theta_m(s) \\ -T_c(s) \end{bmatrix} \quad (16)$$

The transfer function $G_w(s)$ determines the stability of the system and is based on equations (1-2) and the wave variable relationships equations (7-10). Also, we now assume that T_R and T_L may differ. In terms of Laplace transforms, the multiple degree of-freedom version of equations (1,2) is given by:

$$\begin{bmatrix} U_s(s) \\ V_s(s) \end{bmatrix} = \begin{bmatrix} e^{-sT_R}I & 0 \\ 0 & e^{sT_L}I \end{bmatrix}\begin{bmatrix} U_m(s) \\ V_m(s) \end{bmatrix} \quad (17)$$

Substituting in the wave variable relationships (7-10) and rearranging [1] one obtains:

$$G_w(s) = \begin{bmatrix} 2A_w{}^T e^{-sT_d} & 0 \\ 0 & A_w{}^{-1} \end{bmatrix}\hat{G}_w(s)\begin{bmatrix} 2A_w e^{sT_d} & 0 \\ 0 & A_w{}^{-T} \end{bmatrix} \quad (18)$$

where

$$\hat{G}_w(s) = \frac{1}{2}\begin{bmatrix} [I - S_w\tanh(sT_a)]^{-1} & 0 \\ 0 & [I - S_w\tanh(sT_a)]^{-1} \end{bmatrix}$$
$$\times \begin{bmatrix} \tanh(sT_a)I & -\sec h(sT_a)I \\ \sec h(sT_a)I & \tanh(sT_a)(I + S_w{}^T S_w) \end{bmatrix} \quad (19)$$

and where $T_a = \frac{T_L + T_R}{2}$ and $T_d = \frac{T_L - T_R}{2}$. Note that the orthogonal matrix Q does not appear in the expression for $G_w(s)$ or $\hat{G}_w(s)$ and hence has no effect on the input-output characteristics.

F. The Family of Scaling Matrices that Result in Passivity

Now that the input-output relationship $G_w(s)$ across the communication link has been determined, it is possible to study the stability of the system. Like previous work on the wave variable method, this will be accomplished using passivity theory and the scattering operator. Applying these techniques, it will be shown that the family of scaling matrices derived earlier not only satisfy the power flow equations (11) and (12), but also result in stability for any constant time delays T_L and T_R. The scattering matrix:

$$S(j\omega) = [G_w(j\omega) - I][G_w(j\omega) + I]^{-1} \quad (20)$$

can be used to test the passivity of the system. It was shown in [11] that a system with transfer function $G_w(s)$ is passive if and only if the norm:

$$\| S \| = \sup_{\omega} \sqrt{\lambda_{\max}(S^*(j\omega)S(j\omega))} \quad (21)$$

of its scattering matrix is less than or equal to one where $\lambda_{\max}(S^*(j\omega)S(j\omega))$ denotes the largest eigenvalue of the positive definite (semi-definite) Hermitian matrix $S^*(j\omega)S(j\omega)$.

With some work [1], it can be shown that the family of scaling matrices derived previously result in passivity. Since choosing a set of scaling matrices requires the selection of an

n x n nonsingular matrix A_w, an n x n orthogonal matrix Q, and an n x n skew-symmetric matrix S_w, there are a total of $2n^2$-n degrees of freedom in choosing the scaling matrices.

It is natural to ask how this new extension of the scaling matrices affects the wave variables. To see this, we first consider the effect of Q. The v wave variables are given in equations (7-10). Substituting in the expressions for C_w and D_w yields:

$$v_m(t) = QA_w\dot{\theta}_m(t) - \frac{1}{2}Q(I - S_w)A_w^{-T}\tau_m(t) \tag{22}$$

$$= Q[A_w\dot{\theta}_m(t) - \frac{1}{2}(I - S_w)A_w^{-T}\tau_m(t)] \tag{23}$$

which clearly demonstrates that Q merely applies an orthogonal transformation to the v-variable, i.e., it will merely rotate and/or reflect the v-variable. The same holds for $v_s(t)$. This will have no effect on the power flow equations such as (11,12).

While the orthogonal matrix Q has no external effect on the behavior of the system, the matrices A_w and S_w have a significant effect. As in the scalar case, the matrix A_w affects the damping of the system. The matrix S_w, which is not present in the scalar case, influences the amount of coupling between the different degrees of freedom. S_w also allows the user to manipulate the different dimensions of the system individually. In the next section an experiment is designed to test how these matrices influence the human's ability to perform tracking tasks.

III. THE EXPERIMENT TO VALIDATE THE PROCEDURE

The wave variable method presented so far provides an elegant procedure to produce closed-loop human machine stability. The net effect of the system in Figure (1) (restructured architecture to ensure stability) on human tracking performance is not well known and will be investigated here. So far the independent variables of interest include time delay and some measure on the scaling matrices introduced so far.

A. *Objectives of the Experiment*

Human tracking performance will be studied by several means as described in the task and performance experimental sections in the sequel. The independent variables include two levels of time delay (400 and 1000ms) and four levels of a metric on the scaling matrices to be described in the experimental design section. A teleoperation scenario will be employed as the human tracks a target using the master with the slave responding in the environment.

B. *Equipment*

Figure (2) shows the three degree-of-freedom PHANToM Omni haptic device to be used by the subjects.

C. *Task and Performance Measurements*

The tracking task will be the tracing of three different shapes as depicted in Figure (3) with data displayed in Figure (4).

Figure (2) Three degree-of-freedom PHANToM Omni haptic device.

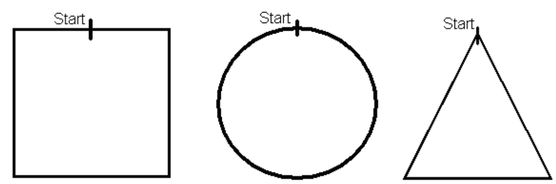

Figure (3). Three shapes to be traced by the subjects in the experiment.

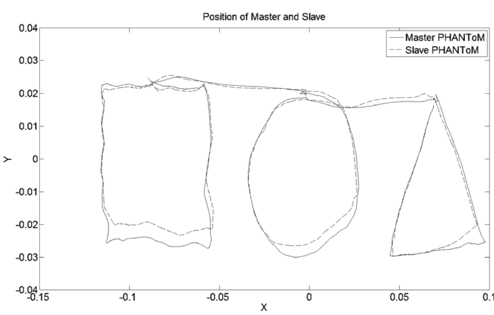

Figure (4) – A Typical Response to the Targets in Figure (3)

D. *Subjects*

A total of seven people participated in this experiment. They were chosen from both the undergraduate and graduate students at the Florida State University. Table 1 shows some of the demographics of the subjects who received no compensation for completing the experiment.

Table 1 – Demographics of Subjects Used in the Study.

Gender	Age in years	Title
Female	23	Graduate Student
Female	24	Former Student
Female	25	Former Student
Male	21	Undergraduate student
Male	25	Graduate Student
Male	26	Graduate Student
Male	26	Former Student

E. Experimental Design

The independent variables of interest were two levels of time delay (400 and 1000 ms) and four levels of scaling matrices sets described next.

E.*1 Scaling Matrix Sets Selected*

For the parameters related to scaling, the following matrices sets were selected:

$$A_{w1} = \begin{bmatrix} 2 & -0.5 & -0.5 \\ -0.5 & 2 & -0.5 \\ -0.5 & -0.5 & 2 \end{bmatrix} \quad (24)$$

$$A_{w2} = \begin{bmatrix} 4 & 1 & -1 \\ 1 & 4 & 1 \\ -1 & 1 & 4 \end{bmatrix} \quad (25)$$

$$A_{w3} = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix} \quad (26)$$

$$A_{w4} = \begin{bmatrix} 8 & 2 & 1 \\ 2 & 10 & 2 \\ 1 & 2 & 12 \end{bmatrix} \quad (27)$$

$$S_{w1} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (28)$$

$$S_{w2} = \begin{bmatrix} 0 & -0.5 & -1 \\ 0.5 & 0 & -0.5 \\ 1 & 0.5 & 0 \end{bmatrix} \quad (29)$$

$$S_{w3} = \begin{bmatrix} 0 & 0.5 & -0.25 \\ -0.5 & 0 & 0.5 \\ 0.25 & -0.5 & 0 \end{bmatrix} \quad (30)$$

E.2 Questionnaire

Once the subjects completed tracing the shapes for all of the different scaling matrix sets they were given a questionnaire to determine whether they felt they were provided enough training to complete the tasks, the quality and difficulty of the experimental tasks, and most importantly which set of scaling matrices they felt was best for the tasks.

E.3 Dependent Measures of Tracking Performance

Tracking error was of interest in this teleoperation scenario. Hence the following dependent measures were selected as representative of the position tracking task: (1) Maximum tracking position difference (between the object and the resulting trajectory at the slave site), (2) Total position tracking error difference, and (3) time to complete the task. Obviously for human machine systems, the speed-accuracy tradeoffs were manifested in the data [9] and also appeared in these data presented here.

IV. Results of the Experiment

Study 1- ANOVA

Using the JMP software from the SAS Institute, a full blown analysis of variance was conducted on the data. Again, this was a 2 x 3 x 4 experiment so the desired goal would be to have a significant difference in any of the dependent measures mentioned previously. The Effects Test results are listed below from SAS for the maximum position difference in Table V below. Similar results were obtained for the other two dependent measures.

Table 2 - The Effects Based Test From Study 1.

Source	DoF	Sum Sq.	F Ratio	Prob> F
Matrix A_w	3	0.0548	255.8679	<.0001
Matrix S_W	2	0.0051	36.0905	<.0001
Matrix A_w* Matrix S_w	6	0.2302	537.0316	<.0001
Time Delay= TD	1	0.0274	383.4554	<.0001
Matrix A_w* TD	3	0.0048	22.2444	<.0001
Matrix S_W *TD	2	0.0002	1.1513	0.3191
MatrixA_w* Matrix S_W* TD	6	0.0226	52.6868	<.0001

From Table 2 it is noted that the independent variable A_w has a significant effect on the dependent measure of maximum position difference (F Ratio= 255.8679 and p <.0001) which is a desired result. The other two independent variables (S_w and time delay) also show similar significant effects, which was desired. However, it is also seen that the cross terms Matrix A_w*Matrix S_w, Matrix A_w*time delay, and Matrix A_w*Matrix S_w*time delay also show significant effects ($p <$:0001). This is called an interaction and it is not desirable because we cannot (independently) predict the effect on performance (dependent measure) by any one variable alone. However Table 3 does show the overall performance results (differences in means) of the maximum position error variable which were consistent with the original hypothesis that increasing time delay would degrade tracking performance.

Table 3 - Maximum Position Error averaged over Delay

Level	Std. Error	Mean
1000	0.0009	0.0572
400	0.0009	0.0317

Thus it was necessary to perform Study 2 after conducting an interaction analysis on the data from Study 1. In Table 2, the significant interaction terms showed this confounding. From the interaction analysis, it was surmised that certain combinations of the A_{wi} and S_{wj} matrices were the factors that affected the overall performance [1]. In study 2 the null hypothesis that was desired to be rejected was:

H_0: There was no change in performance with the condition A_{w2} S_{w2} versus the candidate conditions $A_{w4}S_{w2}$, $A_{w3}S_{w3}$ and A_{w4} S_{w3}. The alternative hypothesis was:

H_1: There would be at least some performance change between the conditions A_{w2} S_{w2} and the other conditions listed above.

The results of Study 2 are shown in Table 4 in terms of the ANOVA performed for the dependent measure of maximum position error:

Table 4 – Results of Study 2 for Maximum Position error

Source	DoF	Sum of Squares	Mean Square	F Ratio	Prob>F
Condition	3	0.0029	0.0010	138.1011	<.0001
Error	23	0.0002	0.0000007		
C. Total	26	0.0031			

Table 5 shows the average of the human data for study 2 for all three independent measures selected:

Table 5 – Averages of the 3 Dependent Measures Selected

Matrix Set	Max. Pos. Dif.	Total Pos. Diff.	Time of Comp.
A_{w2}, S_{w2}	0.0382m	0.5284m	18.02 sec
A_{w4}, S_{w2}	0.0174m	0.2615m	19.74 sec
A_{W3}, S_{w3}	0.0130m	0.2088m	15.11 sec
A_{W4}, S_{w3}	0.0107m	0.1419m	16.40 sec

Figure (5) portrays the one way ANOVA for 400ms of time delay data and the maximum position error in Table 5.

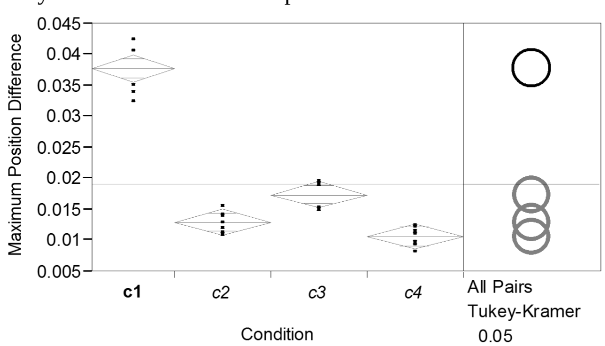

Figure (5) – One way ANOVA for the conditions in Table 5.

In reference to Figure (5) it is noted that using the conservative Tukey-Kramer test of significance at a 0.05 level that the condition “c1” which is equivalent to using the pair of scaling matrices $A_{w2}S_{w2}$ was significantly the worst performance. Thus the hypothesis H_0 could be rejected with over 95% confidence. The physical reasons why this particular pair of scaling matrices was not conducive to good tracking performance can be traced [1] to coupling between state variables induced through the A_{w2} and S_{w2} scaling matrices.

V. Summary and Conclusions

A study of teleoperation with time delays and the wave variable formulation shows that certain classes of scaling matrices are conducive to good tracking performance, yet other classes of such matrices may lead to significantly poorer results. The wave variable formulation was generalized to the matrix case through the development of scaling matrices. However these scaling matrices may impact on how humans effectively use such teleoperation systems in a performance sense.

Acknowledgment

This research was supported in part by the United States Air Force and the Consortium Research Fellows Program under contract DASW01-03-C-0010. The authors gratefully acknowledge this support.

References

[1] M. T. Alise, “Expansion and Implementation of the Wave Variable Method in Multiple Degree-of-Freedom Systems,” Ph. D. Thesis, The Florida State University, Fall, 2007.

[2] M. Alise, R. G. Roberts, D. W. Repperger, “ The Wave Variable Method for Multiple Degree of Freedom Teleoperation Systems with Time Delay,” *Proceedings of the 2006 International Conference on Robotics and Automation*, Lake Buena Vista, Florida, May 15-19, 2006, pp. 2908-2913.

[3] M. Alise, R. G. Roberts, and D. W. Repperger, “Simulating a Bilateral Teleoperation System Using Matlab and SIMULINK,” *ASEE Computers in Education Journal*, 2008.

[4] M. Alise, R. G. Roberts, D. W. Repperger, C. A. Moore, and S. Tosunoglu, “On Extending the Wave Variable Method to Multiple Degree-of-Freedom Teleoperation Systems,” *IEEE/ASME Transactions on Mechatronics*, **vol. 14**, No. 1, February, 2009, pp. 55-63

[5] D. W. Repperger, "Active Force Reflection Devices in Teleoperation," *IEEE Control Systems Magazine*, January, 1991, **11**(1),pp. 52-56.

[6] D. W. Repperger, C. A. Phillips, and T. Chelette "Study of Spatially Induced 'Virtual Force' With An Information Theoretic Investigation of Human Performance," *IEEE Transactions on Systems, Man, and Cybernetics*, **vol. 25**, No. 10, October, 1995, pp. 1392-1404.

[7] D. W. Repperger, R. H. Gilkey, R. Green, T. LaFleur, and M. W. Haas, “Effects of haptic feedback and turbulence on landing performance using an Immersive Cave Automatic Virtual Environment (CAVE),” *Perceptual and Motor Skills*, **97,** December, 2003, pp. 820-832.

[8] D. W. Repperger, “Adaptive Displays and Controllers Using Alternative Feedback,” *CyberPsychology and Behavior*, **vol 7**, No. 6, December, 2004.

[9] D. W. Repperger, C. A. Phillips, J. Berlin, A. Neidhard-Doll, M. Haas, “Human-Machine Haptic Interface Design Using Stochastic Resonance Methods, ”*IEEE Transactions on Systems, Man, and Cybernetics, Part A, Humans and Systems*, **vol 35,** No. 4, July, 2005, pp. 574-582.

[10] G. Niemeyer and J.-J. E. Slotine, “Stable adaptive teleoperation,” *IEEE J. Oceanic Engineering*, **vol. 16**, No. 1, pp 152-162, Jan. 1991.

[11] R. J. Anderson and M. W. Spong, “Bilateral control of teleoperators with time delay,” *IEEE Transactions on Automatic Control*, **vol. 34**, No. 5, May 1989, pp. 494-501

[12] S. Munir, “Internet-based teleoperation,” Ph.D. Thesis, Dept. of Mechanical Engineering, Georgia Institute of Technology, Mar. 2001.

[13] S. Munir and W. J. Book, “Internet-based teleoperation using wave variables with prediction,” *IEEE/ASME Trans. Mechatronics*, **vol.** 7, No. 2, pp. 124-133, June 2002.

[14] B. Hannaford and J.-H. Ryu, “Time domain passivity control of haptic interfaces,” *Proc. IEEE Int. Conf. Robotics Automat.*, Seoul, Korea, May 21-26, 2001, pp. 1863-1869.

[15] J.-J. E. Slotine and W. Li, *Applied Nonlinear Control*, Prentice-Hall, Inc., Englewood Cliffs, NJ, 1991.