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Design of Observer-based Integral Adaptive Fuzzy Sliding Mode Controllers for a Class of Uncertain Nonlinear Systems

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***Abstract*— The observer-based integral adaptive fuzzy sliding mode controllers are developed for a class of uncertain nonlinear systems. By designing the state observer, the fuzzy systems, which are used to approach any unknown functions, it can be constructed using the state observer-based estimations. Based on Lyapunov stability theorem, the proposed integral adaptive fuzzy sliding mode control system can guarantee the stability of the whole closed-loop systems and obtain good tracking performance as well. The proposed methods are applied to an inverted pendulum system achieve satisfactory simulation results.**

Keywords—Integral adaptive fuzzy sliding mode control, Lyapunov stability theorem, observer-based stimation.

I. INTRODUCTION

For control applications, suppose mathematical models are not available, the fuzzy control can provide a good solution for these problems by incorporating linguistic information from experts. Control engineers are now facing more and more complex systems, and the mathematical models of these systems are more and more difficult to obtain. Thus, in control engineering, model free approaches become more important, for example, adaptive control [1], neural network control [2], fuzzy control [3-6], adaptive fuzzy control [7-9], and adaptive fuzzy neural control [10-13]. The fuzzy control has been found more extensive applications for a wide variety of industrial systems and consumer products.

In the meantime, variable structure systems VSS control [14] design technique has been developed as a popular robust strategy to treat uncertain systems with external disturbance, time-varying parameters and unmodeled dynamics. Reference [14] presented the integral variable structure control (IVSC) approach which using an integral controller followed by a variable structure controller. The variable structure control gives robust stability for system in the presence of parameter variations, uncertainties, and disturbances. Recently, adaptive fuzzy control system designs have been extensively discussed in the literature [7-9]. In an effort to improve the robustness of the adaptive fuzzy control system, many works have been published on the design of adaptive fuzzy sliding mode controller (AFSMC).

All these control methods assume that the system states are available for measurement. Suppose the total states of the nonlinear system are not available and then a state observer will be designed to estimate the nonlinear system state. Recently, some papers show how to apply observers to estimate the tracking-error vector and design suitable feedback gains to guarantee the tracking performance. By using an observer-based output feedback control law, the parameters of the adaptive fuzzy sliding mode controller can be tuned based on the Lyapunov stability theorem. Thus, this paper suggests an observer-based integral adaptive fuzzy sliding mode controller (IAFSMC). This paper is organized as follows. In Section II, system description, T-S fuzzy model and sliding function are addressed. The main results are described in Section III that includes state observer design and observer-based IAFSMC. Section IV depicts the simulation results. Finally, conclusions are given in Section VI.

II. SYSTEM DESCRIPTION, T-S FUZZY MODEL AND SLIDING FUNCTION

Consider the n th nonlinear dynamical system of the form:

$$x^{(n)} = f(x,\dot{x},...,x^{(n-1)}) + g(x,\dot{x},...,x^{(n-1)})u + d \quad (1a)$$

$$y = x \quad (1b)$$

or equivalently of the form:

$$\dot{x} = \mathrm{A}x + \mathrm{B}(f(x) + g(x)u + d) \quad (2a)$$

$$y = C^T x \quad (2b)$$

where

$$\mathrm{A}=\begin{bmatrix} 0 & 1 & 0 & ... & 0 \\ 0 & 0 & 1 & ... & 0 \\ ... & ... & ... & ... & ... \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$

$\mathrm{x} = [x,\dot{x},...,x^{(n-1)}]^T = [x_1,x_2,...,x_n]^T \in \Re^n$ is a vector of states. Not all are assumed to be available for measurement.

d : external bounded disturbance;
$u \in R$: control input;
$y \in R$: system output;
$f(x)$ and $g(x)$; unknown functions;

and $g(x)$ is, without loss of generality, chosen strictly positive. It is assumed that the existence of the solution for (1a) and (1b) are satisfied. It is required that $g(x) \neq 0$ for all x in a certain controllability region $U_x \in \Re^n$. Only the system output y is assumed to be measurable. The control objective is to force the system output y to follow a given bounded reference signal y_m .

The T–S fuzzy model has been proven to be a very good representation for a certain class of nonlinear dynamic systems. A nonlinear function of the certain system can be represented by a set of linear models interpolated by membership functions (T-S) fuzzy model. The basic configuration of fuzzy logic systems consists of some fuzzy IF-THEN rules and a fuzzy inference engine.

The fuzzy inference engine uses the fuzzy IF-THEN rules to perform a mapping from an input linguistic vector $\mathrm{x}^T = [x_1 \quad x_2 \quad \cdots \quad x_n] \in \Re^n$ to an output linguistic variable $y \in R$. The i th fuzzy IF-THEN rule is written as:

$$\mathrm{R}^{(i)}: \text{ if } x_1 \text{ is } \mathrm{A}_1^{\mathrm{i}} \text{ and ... and } x_{\mathrm{n}} \text{ is } \mathrm{A}_{\mathrm{n}}^{\mathrm{i}}$$
$$\text{then } y \text{ is } \mathrm{B}^i$$

where $\mathrm{A}_1^i, \mathrm{A}_2^i, ..., \mathrm{A}_{\mathrm{n}}^i$ and B^i are fuzzy sets. Let h be the number of the fuzzy IF-THEN rules. By using product inference, center-average and singleton fuzzifier, the output of the fuzzy logic system can be expressed as

$$\mathrm{y(x)}=\frac{\sum_{\mathrm{i}=1}^{\mathrm{h}} \bar{\mathrm{y}}^{\mathrm{i}} (\prod_{j=1}^{n} \mu A_j^i(x_j))}{\sum_{\mathrm{i}=1}^{\mathrm{h}} (\prod_{j=1}^{n} \mu A_j^i(x_j))} = \theta^T \psi(x) \tag{3}$$

where

$\mu_{A_j^i}(x_j)$: membership function value of fuzzy variable x_j
h : number of the IF-THEN rule;
$\bar{y}^i$: point at which $\mu_{B^i}(\bar{y}^i) = 1$;
$\theta^T = [\bar{y}^1 \quad \bar{y}^2 \quad \cdots \quad \bar{y}^h]$: adjustable parameter vector;
$\psi^T = \left[\psi^1 \quad \psi^2 \quad \cdots \quad \psi^h\right]$: fuzzy basis vector

Where ψ^i is defined as

$$\psi^i(x) = \frac{(\prod_{j=1}^{n} \mu A_j^i(x_j))}{\sum_{\mathrm{i}=1}^{\mathrm{h}} (\prod_{j=1}^{n} \mu A_j^i(x_j))} \tag{4}$$

The integral sliding surface s of proposed integral variable structure control system is designed as [2], $s = \lambda_0 Z + \mathbf{\Lambda}_n^T \mathbf{e} = 0$ and

$$\dot{Z} = x_1 - y_d = e_1 \tag{5}$$

where x_1 is the output signal, y_d is the desired bounded signal, λ_0 is the gain of the integral controller $\mathbf{e} = [e, \dot{e}, ..., e^{(n-1)}]^T = [e_1, e_2, ..., e_n]$ and $\mathbf{\Lambda}_n^T = [\lambda_1, \lambda_2, ..., \lambda_n]$ with $\lambda_n = 1$. If the exact system model is known and the system is free of the external disturbance d , then $\dot{s} = 0$, the equivalent control u_{eq} becomes

$$u_{eq} = \frac{1}{g(x)}\left[-f(x) + y_m^{(n)} + \mathbf{K}_c^T e\right] \tag{6}$$

Define integral sliding surface as:

$$s(t) = y(t) - \int_0^t \left[\ddot{r}(t) - k_1 \dot{e}(t) - k_2 e(t)\right] dt \tag{7}$$

$k_1 \neq 0$ and $k_2 \neq 0$. When $s(t) = \dot{s}(t) = 0$, the integral sliding surface is

$$\ddot{e}(t) + k_1 \dot{e}(t) + k_2 e(t) = 0 \tag{8}$$

III. MAIN RESULTS

Suppose not all the states of the nonlinear system are measurable, a state observer will be applied to estimate them.

A. State Observer Design

First, define the reference vector y_m , the tracking error e and estimation error vector $\hat{e}$ as

$$\mathrm{y}_{\mathrm{m}}^{\mathrm{T}} = \left[y_m \quad y_m^{'} \quad \dots \quad y_m^{(n-1)}\right] \tag{9a}$$

$$\mathrm{e} = \mathrm{y}_m - \mathrm{x} \tag{9b}$$

$$\hat{\mathrm{e}} = \mathrm{y}_m - \hat{\mathrm{x}} \tag{9c}$$

where $\hat{x}$ and $\hat{e}$ denote the estimates of x and e , respectively. Based on the certainty equivalence approach, the control law is

$$u_{eq} = \frac{1}{\hat{g}(x)}\left[-\hat{f}(x) + y_m^{(n)} + \mathbf{K}_c^T e\right]$$

(10)

where $\mathbf{K}_{\mathrm{c}}^{\mathrm{T}} = \left[k_n^c \quad k_{n-1}^c \quad ... \quad k_1^c\right]$ is the feedback gain vector, which is chosen such that the characteristic polynomial of $\mathbf{A}\text{-}\mathbf{B}\mathbf{K}_{\mathrm{c}}^{\mathrm{T}}$ is Hurwitz, and $(\mathbf{A},\mathbf{B})$ is assumed to be controllable. The functions $\hat{f}(x)$ and $\hat{g}(x)$ represent the estimates of the nonlinear functions $f(x)$ and $g(x)$, respectively. From (10), (2a), and (2b), we have

$$\dot{\mathrm{e}}=\mathrm{Ae}\text{-}\mathrm{BK}_{\mathrm{c}}^{\mathrm{T}}\hat{\mathrm{e}}+\mathrm{B}\left[\hat{f}(\mathrm{y}_m - \hat{\mathrm{e}})\text{-}f(\mathrm{y}_m - \mathrm{e})+(\hat{\mathrm{g}}(\mathrm{y}_m - \hat{\mathrm{e}})\text{-}\mathrm{g}(\mathrm{y}_m - \mathrm{e}))u\text{ - }d\right]$$

(11a)

$$c_1 = C^T \mathrm{e}$$

(11b)

where $c_1 = \mathrm{y}_m - y$ denotes the output tracking error. We have converted the tracking problem into designing a state observer for estimating the state e in order to regulate e_1 to zero.

Consider the following observer that estimates the state vector e

$$\dot{\hat{e}}=A\hat{e}-BK_c^T\hat{e}+K_o(e_1-\hat{e}_1) \tag{12a}$$

$$\hat{e}_1=C^T\hat{e} \tag{12b}$$

where $\mathbf{K}_0^T=[k_1^0 \quad k_2^0 \quad \cdots \quad k_n^0]$ is the observer gain vector, chosen such that the characteristic polynomial of $\mathbf{A}-\mathbf{K_o}\mathbf{C}^T$ is strictly Hurwitz and $(\mathbf{C},\mathbf{A})$ is assumed to be observable. Subtracting (9a), (9b) and (9c) from (1), we obtain the error dynamics

$$\mathbf{e}^{(n)}=-\mathbf{K}^T\mathbf{e}+[\hat{f}(\hat{x}|\theta_f)-f(x)]+[\hat{g}(\hat{x}|\theta_g)-g(x)]u \tag{13}$$

where

$$\mathbf{\Lambda}=\begin{bmatrix} 0 & 1 & 0 & \ldots & 0 \\ 0 & 0 & 1 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & 0 & 1 \\ -k_n & -k_{n-1} & \cdots & \cdots & -k_1 \end{bmatrix}, b=\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \tag{14}$$

the error dynamics equation

$$\dot{\mathbf{e}}=\mathbf{\Lambda e}+\mathbf{b}\{[\hat{f}(\hat{x}|\boldsymbol{\theta}_f)-f(x)]+[\hat{g}(\hat{x}|\boldsymbol{\theta}_g)]u\} \tag{15}$$

B. Observer-Based IAFSMC

In this section, our task is to use adaptive fuzzy scheme to approximate the nonlinear functions, and develop an adaptive control law to adjust the parameters of fuzzy control for the purpose of forcing the estimation error $\tilde{e}$ to converge to zero.

First, we replace the estimation functions by the fuzzy logic systems

$$\hat{f}(y_m-\hat{e}|\theta_f)=\theta_f^T\psi(y_m-\hat{e})=\theta_f^T\psi(\hat{x}) \tag{16a}$$

$$\hat{g}(y_m-\hat{e}|\theta_g)=\theta_g^T\psi(y_m-\hat{e})=\theta_g^T\psi(\hat{x}) \tag{16b}$$

In order to derive the control law, the following assumptions and lemmas must be required:

Assumption 1 [15]: Let x and $\hat{x}$ belong to compact sets U_x and $U_{\hat{x}}$, where

$$U_x=\left\{x\in R^n:\|x\|\le m_x<\infty\right\} \tag{17a}$$

$$U_{\hat{x}}=\left\{\hat{x}\in R^n:\|\hat{x}\|\le m_{\hat{x}}<\infty\right\} \tag{17b}$$

It is known a prior that the optimal parameter vectors θ_f^* and θ_g^* lie in some convex regions:

$$M_{\theta_f}=\left\{\theta_f\in R^h:\|\theta_f\|\le m_{\theta_f}\right\} \tag{18a}$$

$$M_{\theta_g}=\left\{\theta_g\in R^h:\|\theta_g\|\le m_{\theta_g}\right\} \tag{18b}$$

where the radiuses m_{θ_g} and m_{θ_f}

$$\theta_f^*=\arg\min_{\theta_f\in M_{\theta_f}}\left[\sup_{x\in U_x,\hat{x}\in U_{\hat{x}}}\left|f(x)-\hat{f}(\hat{x}|\theta_f)\right|\right] \tag{19a}$$

$$\theta_g^*=\arg\min_{\theta_g\in M_{\theta_g}}\left[\sup_{x\in U_x,\hat{x}\in U_{\hat{x}}}\left|g(x)-\hat{g}(\hat{x}|\theta_g)\right|\right] \tag{19b}$$

Assumption 2 [16]: The parameter vector θ_g is such that $\hat{g}(\hat{x}|\theta_g)$ is bounded away from zero.

Let the minimum approximation error be defined as:

$$w=\hat{f}(x|\theta_f^*)-f(x)+\left[(\hat{g}(x|\theta_g^*)-g(x)\right]u \tag{20}$$

Using adaptive fuzzy scheme to approximate the nonlinear functions $f(x)$ and $g(x)$, from (6), one can get

$$u=\frac{1}{\hat{g}(x|\boldsymbol{\theta}_g)}\left[-\hat{f}(x|\boldsymbol{\theta}_f)+y_m^{(n)}+\mathbf{K}_c^Te\right] \tag{21}$$

$$\hat{g}(x|\boldsymbol{\theta}_g)=\boldsymbol{\theta}_g^T\boldsymbol{\eta}(x) \tag{22a}$$

$$\hat{f}(x|\boldsymbol{\theta}_f)=\boldsymbol{\theta}_f^T\boldsymbol{\xi}(x) \tag{22b}$$

$$\dot{\boldsymbol{\theta}}_f=-\gamma_1\mathbf{e}^T\mathbf{Pb}\boldsymbol{\xi}(x) \tag{23}$$

$$\dot{\boldsymbol{\theta}}_g=-\gamma_2\mathbf{e}^T\mathbf{Pb}\boldsymbol{\eta}(x)u \tag{24}$$

Theorem 1: Suppose the adaptive laws are chosen as

$$\dot{\theta}_f=\begin{cases} -\gamma_1\mathbf{e}^T\mathbf{Pb}\boldsymbol{\xi}(x), \text{if } \|\theta_f\|\le m_{\theta_f} \text{ or} \\ \qquad (\|\theta_f\|=m_{\theta_f} \text{ and } \mathbf{e}^T\mathbf{Pb}\theta_f^T\boldsymbol{\xi}(x)\ge 0 \\ \Pr_f(-\gamma_1\mathbf{e}^T\mathbf{Pb}\boldsymbol{\xi}(x)), \text{if } \|\theta_f\|=m_{\theta_f} \text{ and } \mathbf{e}^T\mathbf{Pb}\theta_f^T\boldsymbol{\xi}(x)<0 \end{cases} \tag{25}$$

$$\dot{\theta}_g=\begin{cases} -\gamma_2\mathbf{e}^T\mathbf{Pb}\boldsymbol{\eta}(x)u, \quad \text{if } \|\theta_g\|\le m_{\theta_g} \text{ or} \\ \qquad (\|\theta_g\|=m_{\theta_g} \text{ and } \mathbf{e}^T\mathbf{Pb}\theta_g^T\boldsymbol{\eta}(x)u\ge 0 \\ \Pr_g(\gamma_2\mathbf{e}^T\mathbf{Pb}\boldsymbol{\eta}(x)u,), \text{if } \|\theta_g\|=m_{\theta_g} \text{ and } \mathbf{e}^T\mathbf{Pb}\boldsymbol{\eta}(x)u\theta_g^T<0 \end{cases} \tag{26}$$

where the projection operators [16] $\Pr_f(-\gamma_1\tilde{e}_1\phi(\hat{x}))$ and $\Pr_g(-\gamma_2\tilde{e}_2\phi(\hat{x}))$ are given as

$$\Pr_f(-\gamma_1\mathbf{e}^T\mathbf{Pb}\boldsymbol{\xi}(x))=-\gamma_1\mathbf{e}^T\mathbf{Pb}\boldsymbol{\xi}(x)+\gamma_1\mathbf{e}^T\mathbf{Pb}\frac{\theta_f^T\boldsymbol{\xi}(x)}{\|\theta_f\|^2}\theta_f \tag{27}$$

and

$$\Pr_g(-\gamma_2\mathbf{e}^T\mathbf{Pb}\boldsymbol{\eta}(x)u)=-\gamma_2\mathbf{e}^T\mathbf{Pb}\boldsymbol{\eta}(x)u+\gamma_2\mathbf{e}^T\mathbf{Pb}\boldsymbol{\eta}(x)u\frac{\theta_g^T\boldsymbol{\eta}(x)u}{\|\theta_g\|^2}\theta_g \tag{28}$$

From (16), one can find

$$\dot{\mathbf{e}}=\mathbf{\Lambda e}+\mathbf{b}[\hat{f}(x|\boldsymbol{\theta}_f)-\hat{f}(x|\boldsymbol{\theta}_f^*)] +[\hat{g}(\hat{x}|\boldsymbol{\theta}_g)-\hat{g}(x|\boldsymbol{\theta}_g^*)]u+w\} \tag{29}$$

Substituting (24)-(25) to (30), we can rewrite the error dynamics as:

$$\dot{\mathbf{e}}=\mathbf{\Lambda e}+\mathbf{b}[(\boldsymbol{\theta}_f-\boldsymbol{\theta}_f^*)^T\boldsymbol{\xi}(x)+(\boldsymbol{\theta}_g-\boldsymbol{\theta}_g^*)^T\boldsymbol{\eta}(x)u+w] \tag{30}$$

Proof:

Define the Lyapunov function candidate as

$$V=\frac{1}{2}e^TPe+\frac{1}{2\gamma_1}(\boldsymbol{\theta}_f-\boldsymbol{\theta}_f^*)^T(\boldsymbol{\theta}_f-\boldsymbol{\theta}_f^*) +\frac{1}{2\gamma_2}(\theta_g-\theta_g^*)^T(\theta_g-\theta_g^*) \tag{31}$$

there exists a positive definite symmetric $n\times n$ matrix $\mathbf{P}$ which satisfies the Lyapunov equation .

$$A_c^TP+PA_C=-Q \tag{32a}$$

$$\mathrm{PB_C} = C_c \tag{32b}$$

where Q is an arbitrary $n \times n$ positive-definite matrix.

$$V_1 = \frac{1}{2}\mathrm{e}^{\mathrm{T}} P \mathrm{e},$$

$$V_2 = \frac{1}{2\gamma_1}(\boldsymbol{\theta}_f - \boldsymbol{\theta}_f^*)^T(\boldsymbol{\theta}_f - \boldsymbol{\theta}_f^*),$$

$$V_3 = \frac{1}{2\gamma_2}(\boldsymbol{\theta}_g - \boldsymbol{\theta}_g^*)^T(\boldsymbol{\theta}_g - \boldsymbol{\theta}_g^*),$$

$$\mathbf{M} = \mathbf{b}[(\boldsymbol{\theta}_f - \boldsymbol{\theta}_f^*)^T \boldsymbol{\xi}(x) + (\boldsymbol{\theta}_g - \boldsymbol{\theta}_g^*)^T \boldsymbol{\eta}(x)u + w,$$

Equation (29) can be rewritten as $\dot{\mathbf{e}} = \boldsymbol{\Lambda} e + \mathbf{M}$

$$\dot{V} = \dot{V}_1 + \dot{V}_2 + \dot{V}_3$$

$$\begin{aligned}\dot{V}_1 &= \frac{1}{2}\dot{\mathbf{e}}^T \mathbf{P}\mathbf{e} + \frac{1}{2}\mathbf{e}^T \mathbf{P}\dot{\mathbf{e}} \\ &= \frac{1}{2}(\mathbf{e}^T \Lambda^T + \mathbf{M}^T)\mathbf{P}\mathbf{e} + \frac{1}{2}\mathbf{e}^T \mathbf{P}(\Lambda \mathbf{e} + \mathbf{M}) \\ &= -\frac{1}{2}\mathbf{e}^T Q \mathbf{e} + \mathbf{e}^T \mathbf{P}\mathbf{M}\end{aligned}$$

$$\dot{V}_2 = \frac{1}{\gamma_1}(\boldsymbol{\theta}_f - \boldsymbol{\theta}_f^*)^T \dot{\boldsymbol{\theta}}_f, \dot{V}_3 = \frac{1}{\gamma_2}(\boldsymbol{\theta}_g - \boldsymbol{\theta}_g^*)^T \dot{\boldsymbol{\theta}}_g$$

$$\begin{aligned}\dot{V} = &-\frac{1}{2}\mathbf{e}^T Q \mathbf{e} + \mathbf{e}^T \mathbf{P}\mathbf{b} w + \frac{1}{\gamma_1}(\boldsymbol{\theta}_f - \boldsymbol{\theta}_f^*)^T \left[\dot{\boldsymbol{\theta}}_f + \gamma_1 \mathbf{e}^T \mathbf{P}\mathbf{b}\boldsymbol{\xi}(x)\right] \\ &+ \frac{1}{\gamma_2}(\boldsymbol{\theta}_g - \boldsymbol{\theta}_g^*)^T \left[\dot{\boldsymbol{\theta}}_g + \gamma_2 \mathbf{e}^T \mathbf{P}\mathbf{b}\boldsymbol{\eta}(x)\right]\end{aligned}$$

Applying (24) and (25) to above equation

$$\dot{V} = -\frac{1}{2}\mathbf{e}^T Q \mathbf{e} + \mathbf{e}^T \mathbf{P}\mathbf{b} w \tag{33}$$

which results in $\dot{V} \le 0$.

IV. THE ILLUSTRATIVE EXAMPLE

This section presents the simulation results of the proposed observer-based integral adaptive fuzzy sliding mode controller for a class of unknown nonlinear dynamical systems to illustrate that the robust stability for the closed-loop system, and all signals involved are bounded.

Example: Consider the inverted pendulum stabilizing problem to demonstrate the robustness of the proposed control scheme. Let x_1 be the angle of the pendulum with respect to the vertical line. The dynamic equations of the inverted pendulum system [16] are

$$\begin{bmatrix}\dot{\mathrm{x}}_1 \\ \dot{\mathrm{x}}_2\end{bmatrix} = \begin{bmatrix}0 & 1 \\ 0 & 0\end{bmatrix}\begin{bmatrix}\mathrm{x}_1 \\ \mathrm{x}_2\end{bmatrix} + \begin{bmatrix}0 \\ 1\end{bmatrix}\left(f + gu + d\right) \tag{34}$$

where

$$f = \frac{ml\mathrm{x}_2 \sin \mathrm{x}_1 \cos \mathrm{x}_1 - (M+m)g \sin \mathrm{x}_1}{ml\cos^2 \mathrm{x}_1 - \frac{4}{3}l(M+m)} \tag{35a}$$

$$g = \frac{-\cos \mathrm{x}_1}{ml\cos^2 \mathrm{x}_1 - \frac{4}{3}l(M+m)} \tag{35b}$$

M is the mass of the cart, m is the mass of the rod, $g = 9.8\ \frac{\mathrm{m}}{\sec^2}$, is the acceleration due to gravity, l is the half length of the rod, and u is the control input. In this example, $M = 1.0$ kg, $m = 0.1$ kg, $l = 0.5$ m. Also, it is assumed that the external disturbance $d(t)$ is a square wave with the amplitude ± 5 and the period 2π. Our control objective is to control the state x_1 of the system to track the reference trajectory $y_m = 0.1\sin(\pi t)$ when only the system y is measurable. The feedback and observer gain vectors are given as $K_C^T = [145 \quad 45]$ and $K_o^T = [60 \quad 900]$. The following membership functions are

$$\mu_{\mathrm{NM}}(x_i) = \exp[-((x_i + \pi/6)/(\pi/24))^2,$$
$$\mu_{\mathrm{NS}}(x_i) = \exp[-((x_i + \pi/12)/(\pi/24))^2,$$
$$\mu_{\mathrm{Z}}(x_i) = \exp[-((x_i)/(\pi/24))^2,$$
$$\mu_{\mathrm{PS}}(x_i) = \exp[((x_i - \pi/12)/(\pi/24))^2,$$
$$\mu_{\mathrm{PM}}(x_i) = \exp[-((x_i - \pi/6)/(\pi/24))^2$$

Because the estimation states are two, the total fuzzy rules are twenty-five. Fig. 1 shows membership functions. Suppose the initial state is $[\pi/60, 0]$, parameters are set as $\boldsymbol{\theta}_f = \boldsymbol{\theta}_g = 0.1$, $Q = \begin{bmatrix}10 & 0 \\ 0 & 10\end{bmatrix}$, $\gamma_1 = 50$, and $\gamma_2 = 1$, then the simulation results

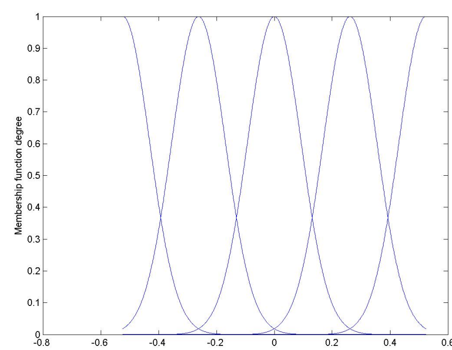

Fig. 1. Membership functions

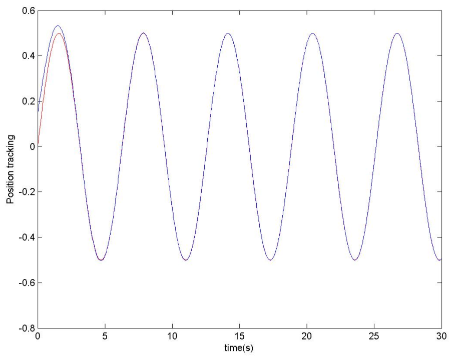

Fig. 2. Time responses of the states.

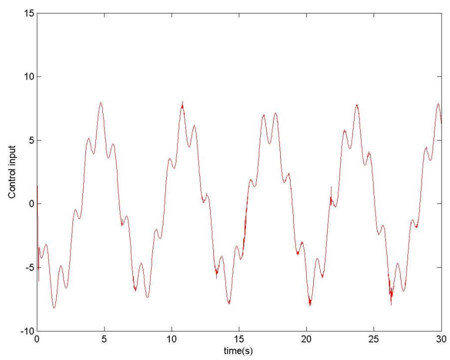

Fig. 3. Time response of control input.

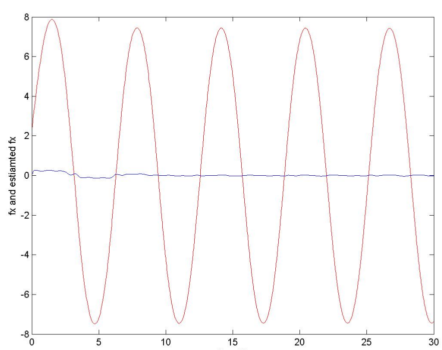

Fig. 4. Time responses of real $f(x)$ and $f(x) - \hat{f}(x)$.

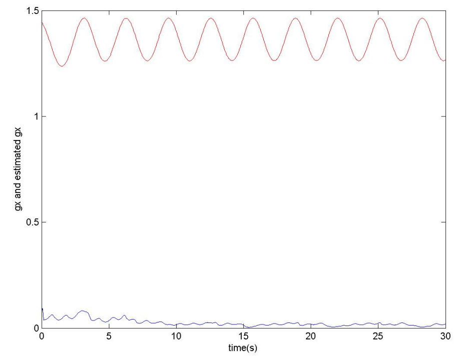

Fig. 5. Time responses of real $g(x)$ and $g(x) - \hat{g}(x)$.

are described in Figures 2 to 4. Figure 2 depicts that the state x_1 can indeed track the reference trajectory. Figure 3 shows that the magnitude of control input is bounded within ± 10 V. One can find from Figures 4 and 5 that both $f(x) - \hat{f}(x)$ and $g(x) - \hat{g}(x)$ are bounded and close to zero.

V. CONCLUSION

In this paper, the observer-based integral adaptive fuzzy sliding mode controller for a class of unknown nonlinear dynamical systems is developed. The observer-based adaptive output feedback control law and update law to tune on-line the adaptive factors of the adaptive fuzzy sliding mode controller are derived. The system nonlinearities approximated by adaptive fuzzy sliding mode are not assumed to be functions of the system states. The overall adaptive scheme provides robust stability for the closed-loop system and guarantees that all signals involved are bounded. Finally, this method has been applied to control the inverted pendulum to track a reference trajectory. The computer simulation results show that the adaptive controller can perform successful control and achieve desired performance.

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