

Uncertainty Prediction for Tool Wear Condition Using Type-2 TSK Fuzzy Approach

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Abstract – Because of the difficulty in understanding the physics of the machining process, several different intelligence methods, which employ cutting forces for estimation tool wear, have been developed in the past few years. Unfortunately, none of them can overcome the difficulty to estimate the errors of approximation during tool wear monitoring. This paper aimed at presenting a tool wear monitoring method using type-2 Takagi-Sugeno-Kang (TSK) fuzzy approach. This innovative method not only provides high reliability of the tool wear prediction over a wide range of cutting conditions, but also assesses the uncertainties associated with the modeling process and with the outcome of the model itself. The magnitude and direction of uncertainties in the machining process are described explicitly to increase the credibility of assessments.

Keywords—uncertainty estimation, type-2 TSK fuzzy logic, tool wear condition, machining, approximation

I. INTRODUCTION

The tool condition has a strong influence on the surface finish, dimensional integrity of the workpiece, and vibration levels of the machine tool. Tool wear describes the gradual failure of machine tools due to regular operation. The information obtained from tool wear monitoring can be used for several purposes that include: establishing tool change policy, economic optimization of machining operations, compensating for tool wear on-line, and avoiding to some extent catastrophic tool failures [1]. Effective monitoring of a manufacturing process is essential for ensuring product quality and reducing the production costs. The analysis, implementation and evaluation of machining processes present challenges to manufacture industry.

In a tool condition monitoring system, real-time data are first acquired from sensors located at different locations of the workpiece, tool and machine-tool, then signal processing technique is used to extract valid data, and a decision making system is used to analyze the data and classify the results in order to make a more reliable estimation of the state of the tool and consequently of the machined parts themselves [2].

Due to the difficulty in understanding the exact physics of the process, the information obtained during machining process is not complete and precise. It is difficult to establish theoretical and analytical approaches or mechanic models by using this kind information which is vague, contradictory, or

deficient in some other ways. Moreover for these models, there are many additional assumptions that reduce the reliability of the models and a large number of cutting experiments that increase the money and time consuming.

Advanced signal processing techniques and artificial intelligence play a key role in the development of modern tool condition monitoring systems [3], such as, neural network (NN), mamdani fuzzy logic (FL, the combination of NN, artificial intelligence (AI) and mamdani FL or with some automatic generate method, i.e., genetic algorithm (GA), TSK FL [4] as well. The advantages of NN are flexible structure, parallel computation and noise suppression. FL approach has high capability of representing imprecise situations. And the combination of NN, AI and FL is capable to handle large data and automatic generation of fuzzy knowledge bases. Detailed description can be found in [3]. All these methods have the similar aims – matching the estimation of average cutting tool wear and the directly measured wear value. Unfortunately, none of them can overcome the difficulty to estimate the errors of approximation during tool wear monitoring.

The aim of this paper is to present an innovative tool wear monitoring method using type-2 TSK fuzzy approach [5] based on subtractive method [6] to accomplish the integration of multi-sensor information and tool wear information. It generates fuzzy rules directly from the input-output data acquired from sensors, and provides high accuracy and high reliability of the tool wear prediction over a wide range of cutting conditions. Moreover, its interval set of output assesses the information of uncertainty in the estimation of tool wear condition, which is of great value to a decision maker and can be used to investigate the complicated tool wear condition during machining process.

In this paper, section I introduces the development of tool wear monitoring. Section II recalls the theoretical foundation of subtractive clustering based type-2 TSK fuzzy approach. Section III is a turning case studying. The experimental results show the effectiveness and advantages of type-2 TSK fuzzy approach. The magnitude and direction of uncertainties in the machining process are described explicitly to increase the credibility of assessments. Section IV is the conclusion and future research direction.

II. TYPE-2 TSK FUZZY APPROACH AND ITS IDENTIFICATION ALGORITHM

A. Type-2 TSK fuzzy approach

The fuzzy logic (FL) has been originally proposed by Zadeh in his famous paper “Fuzzy Sets” in 1965 [7]. His following proposal of linguistic approach [8, 9] was expanded into fuzzy systems modeling as a qualitative modeling. Since then Fuzzy logic has emerged as a powerful technique for controller, modeling and other expert system. TSK fuzzy rule-based system [10, 11] was proposed in an effort to develop a systematic approach to generate fuzzy rules from a given input-output data set. This model consists of rules with fuzzy antecedents and a mathematical function in the consequent part. The antecedents divide the input space into a set of fuzzy regions, while consequents describe behaviour of the system in those regions. TSK fuzzy approach is a very powerful tool for function approximation due to its capability to explain nonlinear relation using a relatively low number of simple rules.

A generalized type-1 TSK model can be described by fuzzy IF-THEN rules which represent input-output relations of the system. For a MISO first-order type-1 TSK model, its k th rule can be expressed as:

$$\begin{aligned} \text{IF } x_1 \text{ is } Q_{1k} \text{ and } x_2 \text{ is } Q_{2k} \text{ and } \dots \text{ and } x_n \text{ is } Q_{nk}, \\ \text{THEN } Z \text{ is } w^k = p_0^k + p_1^k x_1 + p_2^k x_2 + \dots + p_n^k x_n \end{aligned} \quad (1)$$

where x_1, x_2, \dots, x_n and Z are linguistic variables; $Q_{1k}, Q_{2k}, \dots,$ and Q_{nk} are the fuzzy sets on universe of discourses $U, V, \dots,$ and W , and $p_0^k, p_1^k, \dots,$ and p_n^k are regression parameters.

Because Gaussian basis functions (GBFs) have the best approximation property [12], Gaussian functions are chosen as the membership functions (MFs). A Gaussian MF can be expressed by the following formula for the v th variable:

$$Q_v^k = \exp \left[-\frac{1}{2} \left(\frac{x_v - x_v^{k*}}{\sigma} \right)^2 \right] \quad (2)$$

where x_v^{k*} is the mean of the v th input feature in the k th rule for $v \in [0, n]$. σ is the standard deviation of Gaussian MF.

Based on the extension principle [13], Mendel and his students extended previous studies and established a complete type-2 fuzzy logic theory with the handling of uncertainties [14]. For the most general type-2 fuzzy model, antecedents are type-2 fuzzy sets and consequents are type-1 fuzzy sets, then consequent parameter are assumed as convex and normal type-1 fuzzy number subsets of the real numbers, so that they are fuzzy numbers. These rules let us simultaneously account for uncertainty about antecedent MFs and consequent parameter values. The output of type-2 fuzzy system is not only a crisp output but also an interval output, which reveals the uncertainty due to antecedent or consequent parameter

uncertainties. Type-2 TSK FLS was presented in 1999 [15], and type-2 TSK FLSs have the potential to be used in control and other areas where a type-1 TSK model may be unable to perform well [14] because of its large numbers of design parameters. Type-2 TSK FLS is completely described by Prof. Mendel in his book [14] which opens a new door to estimate uncertainty using imprecise information.

A generalized k th rule in the first-order type-2 TSK fuzzy MISO system can be expressed as (4) instead of (1) in type-1 TSK fuzzy MISO system.

$$\begin{aligned} \text{IF } x_1 \text{ is } \tilde{Q}_{1k} \text{ and } x_2 \text{ is } \tilde{Q}_{2k} \text{ and } \dots \text{ and } x_n \text{ is } \tilde{Q}_{nk}, \\ \text{THEN } Z \text{ is } w = p_0 + p_1 x_1 + p_2 x_2 + \dots + p_n x_n \end{aligned} \quad (4)$$

where $\tilde{p}_0, \tilde{p}_1, \dots, \tilde{p}_n$ are consequent parameters, w output from the k th IF-THEN rule in a total of M rules FLS, $\tilde{Q}_{1k}, \tilde{Q}_{2k}, \dots, \tilde{Q}_{nk}$ are fuzzy sets on universe of discourses.

To obtain a type-2 model directly from a type-1 model, a width a_j^k of x_{jk}^* is extended to both directions of cluster center x_{jk}^* as shown in Fig. 1. By so, cluster center is expanded from a certain point to a fuzzy number:

$$\tilde{x}_{jk}^* = [x_{jk}^*(1 - a_j^k), x_{jk}^*(1 + a_j^k)] \quad (5)$$

where a_j^k is spread percentage of cluster centre x_{jk}^* as shown in Fig. 1. The cluster center x_{jk}^* becomes a constant width interval valued fuzzy set \tilde{x}_{jk}^* .

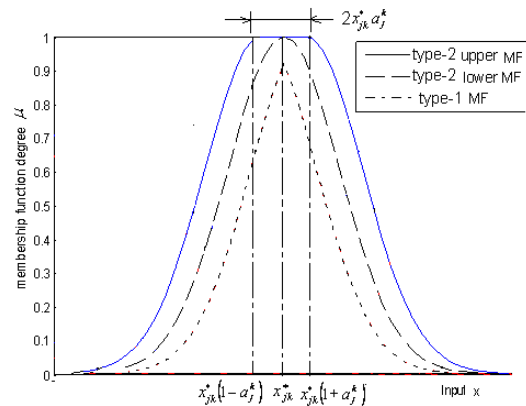


Fig. 1 Spread of cluster center

And consequent parameter \tilde{p}_j is obtained by extending the consequent parameter p_j^k from its type-1 counterpart using the following expression:

$$\tilde{p}_j^k = \begin{bmatrix} p_j^k - s_j^k & p_j^k & p_j^k & p_j^k + s_j^k \end{bmatrix}, \quad (6)$$

where $j \in [0, n]$, and s_j^k denotes the spread of fuzzy numbers p_j^k .

Hence, the premise MF is changed from type-1 fuzzy sets of (2) into type-2 fuzzy set, *i.e.*,

$$\tilde{Q}_{jk} = \exp \left[-\frac{1}{2} \left(\frac{x_j - x_{jk}^* (1 \pm a_j^k)}{\sigma_j^k} \right)^2 \right] \quad (7)$$

where σ_j^k is the standard deviation of Gaussian MF.

B. Type-2 TSK fuzzy identification algorithm

According to type-2 FLS theory [14], a type-2 TSK fuzzy FLS based on subtractive clustering method [5] was introduced. The diagram of type-2 TSK fuzzy approach algorithm is shown in Fig. 2.

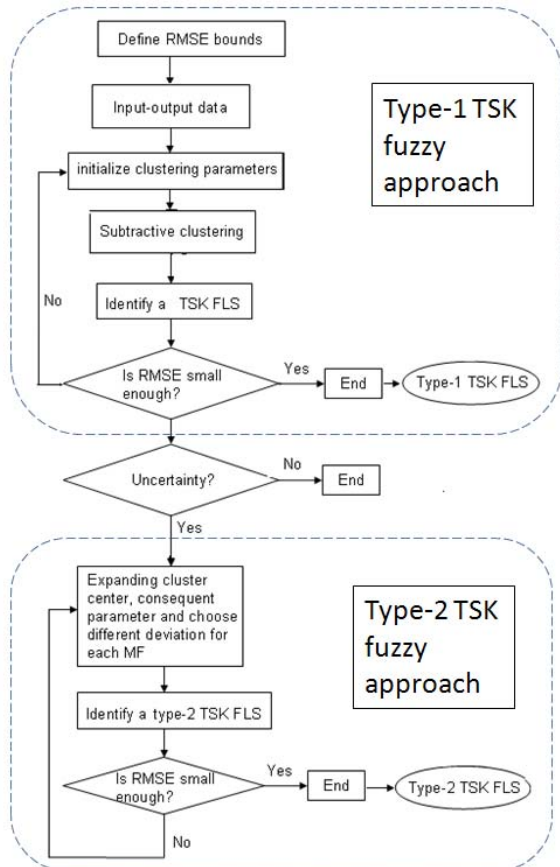


Fig. 2 Subtractive clustering based type-2 TSK fuzzy approach

Type-2 TSK fuzzy approach includes two steps. First, subtractive clustering method [6] was combined with a least-square estimation algorithm, to accomplish the integration of multi-sensor information in order to obtain a type-1 fuzzy model. The detailed description can be found in [4]. Based only on measured data without prior knowledge, there is no systematic way to obtain a TSK fuzzy model with a simple structure and sufficient accuracy. By using subtractive clustering, it is easy to obtain an efficient Type-1 TSK FLS. But this method, like other artificial intelligence methods has the difficulty to estimate the errors of approximation during tool wear monitoring. And for type-1 TSK approach itself, the usage of precise membership functions results in uncertainties in the approach results as well.

The next step is extending the type-1 fuzzy model to its type-2 counterpart. It is obtained by considering the type-1 MFs as principle MFs and assigning uncertainties to cluster centers, consequence parameters and standard deviations of Gaussian MF. Minimum error approach is obtained through enumerative search of optimum values for spreading percentage of cluster centers and consequence parameters because the model is very sensitive to them [16]. This description of the diagram of this part was given in detail in [5].

IV. CASE STUDY

A. Experimental setup

The experiments described in this paper were conducted on a conventional lathe TUD-50. A CSRPR 2525 tool holder equipped with a TiN-Al₂O₃-TiCN coated sintered carbide insert SNUN 120408 was used in the test. To simulate factory floor conditions, six sets of cutting parameters were selected and applied in sequence as presented in Fig. 3. During machining, the feed force (F_f) and the cutting force (F_c) were recorded while the tool wear was manually measured after each test.

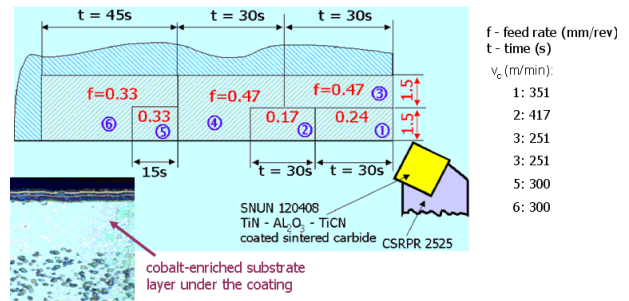


Fig. 3 Cutting parameters used in experiments

For our purposes, tool wear (VB) was estimated from three input sources: feed rate (f), F_f and F_c . The choice of input variables was based on the following two observations: F_f is independent of f , but rather depends on VB and the depth of cut, denoted a_p . Moreover, F_c depends on a_p and f ,

while being only weakly dependent on VB . So, in this paper f and the measurement F_c are used to determine a_p , and the measurement F_f is used to determine VB without requiring a_p as an input variable.

Cutting speeds were selected in such a way as to correspond to the same approximate tool life in each cut (shown on Fig. 3). VB was measured after carrying out each sequence. The value for F_f and F_c were measured by a single cut using a Kistler 9263 dynamometer during 5-s intervals while the cut was executed. Recent research has attempted to investigate the application of multiple sensors with complementing characteristics to provide a robust estimate of tool wear condition. Since the inserts used in the experiments had a soft, cobalt-enriched layer of the substrate under the coating, their tool life had a tendency to end suddenly after this coating wore through.

The experiments were carried out until a tool failure occurred. Two experiments were carried out until a tool failure occurred. In the first tests (designated W5) 10 cycles were performed until a sudden rise of the flank wear VB occurred, reaching approximately 0.5 mm. In the second test (designated W7) failure of the coating resulted in chipping of the cutting edge at the end of 9th cycle. W5 was devised for TSK fuzzy rule identification, while W7 was used to verify the performance of the different monitoring system. Figure 4 presents the cutting force components F_c and F_f versus VB obtained in the experiments. Apparently, F_c is weakly dependent of on VB . It is a function of the cutting parameters only. On the other hand, F_f is independent of f , being affected only by a_p and VB . This provides an interesting opportunity to estimate the tool wear without requiring information about a_p .

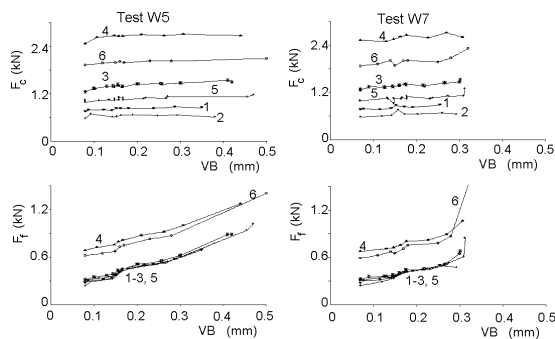


Fig. 4 Cutting force components vs. tool wear obtained in tests W5 and W7

B. TSK fuzzy approach

As mentioned, the type-2 FLS is extended from its type-1 counterpart. First, type-1 TSK fuzzy approach using the diagram shown in Fig. 2 is used to treat the information from

sensors. The results and detailed steps can be found in [4]. The 9-rule fuzzy model is shown in TABLE 1. From Fig. 5 and TABLE 2, it is observed that the type-1 TSK fuzzy approach has the lowest root-mean-square-error (RMSE) and the smallest maximum error (MAX) comparing with other artificial intelligent methods: Mamdani FL, neural network (NN), neural network based fuzzy system (NF) and TSK fuzzy approach.

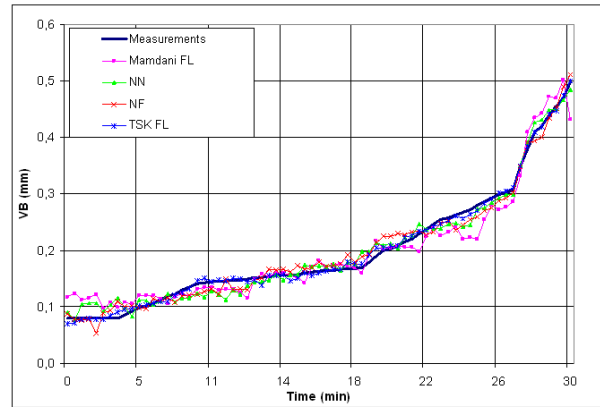


Fig. 5 Tool wear monitoring using different artificial intelligent methods: Mamdani FL, neural network (NN), neural network based fuzzy system (NF) and TSK fuzzy approach.

TABLE 2

Summary of root-mean-square-error (rmse) and maximum error (max) from the experimental results with different AI methods

AI method	Learning (W5)		Testing (W7)	
	RMSE (mm)	MAX (mm)	RMSE (mm)	MAX (mm)
Neural Network	0.015	0.036	0.029	0.081
Mamdani FL	0.024	0.068	0.034	0.056
NF	0.014	0.030	0.030	0.081
TSK FL	0.006	0.003	0.009	0.014

The next step is expanding the type-1 fuzzy filtering system to a type-2 fuzzy filtering system. The type-2 fuzzy system is extended directly from its type-1 counterpart by searching the optimal spreading percentages for cluster center and consequent parameters by comparing RMSE. The spreading percentage for clusters is confined to the range [1%; 20%] with a step size of 1%. The spreading percentage for the consequent parameters is considered in the range [0.1%; 2%] with a step size of 0.1%. A 9-rule 3-input 1-output type-2 fuzzy system with least RMSE is obtained by extending 11% to each cluster centre ($a_j^k=11\%$) and spreading the consequent parameters 1% ($b_j^k=1\%$). Figure 6 and Fig. 7 are training results and test results from the mentioned the experiments. The dotted curve and dashed curve indict the upper and lower output of the type-2 fuzzy approach. The gap between them indicts the uncertainty associated with the system.

TABLE 1 9-rule type-1 fuzzy system

rule	If X then $y = p_1x_1 + p_2x_2 + p_3x_3 + p_0$
1	If $A_f = \exp\left(-\frac{4}{0.25^2}\left(\frac{f-0.47}{0.0265}\right)^2\right)$, $A_{F_c} = \exp\left(-\frac{4}{0.25^2}\left(\frac{F_c-1397}{0.0265}\right)^2\right)$ and $A_{F_f} = \exp\left(-\frac{4}{0.25^2}\left(\frac{F_f-389}{0.0265}\right)^2\right)$, then $VB = 2.93 \times 10^5 f - 0.13 F_c + 3.45 F_f - 1.44 \times 10^5$
2	If $A_f = \exp\left(-\frac{4}{0.25^2}\left(\frac{f-0.47}{0.0265}\right)^2\right)$, $A_{F_c} = \exp\left(-\frac{4}{0.25^2}\left(\frac{F_c-1413}{0.0265}\right)^2\right)$ and $A_{F_f} = \exp\left(-\frac{4}{0.25^2}\left(\frac{F_f-431}{0.0265}\right)^2\right)$, then $VB = -6.14 \times 10^3 f - 7.05 \times 10^{-3} F_c - 0.34 F_f + 3.21 \times 10^3$
3	If $A_f = \exp\left(-\frac{4}{0.25^2}\left(\frac{f-0.33}{0.0265}\right)^2\right)$, $A_{F_c} = \exp\left(-\frac{4}{0.25^2}\left(\frac{F_c-1044}{0.0265}\right)^2\right)$ and $A_{F_f} = \exp\left(-\frac{4}{0.25^2}\left(\frac{F_f-405}{0.0265}\right)^2\right)$, then $VB = 8.32 f - 0.464 \times 10^{-4} F_c - 4.51 \times 10^{-3} F_f + 5.85$
4	If $A_f = \exp\left(-\frac{4}{0.25^2}\left(\frac{f-0.47}{0.0265}\right)^2\right)$, $A_{F_c} = \exp\left(-\frac{4}{0.25^2}\left(\frac{F_c-1395}{0.0265}\right)^2\right)$ and $A_{F_f} = \exp\left(-\frac{4}{0.25^2}\left(\frac{F_f-372}{0.0265}\right)^2\right)$, then $VB = -5.90 \times 10^5 f - 0.34 F_c + 4.57 F_f + 2.80 \times 10^5$
5	If $A_f = \exp\left(-\frac{4}{0.25^2}\left(\frac{f-0.33}{0.0265}\right)^2\right)$, $A_{F_c} = \exp\left(-\frac{4}{0.25^2}\left(\frac{F_c-1070}{0.0265}\right)^2\right)$ and $A_{F_f} = \exp\left(-\frac{4}{0.25^2}\left(\frac{F_f-365}{0.0265}\right)^2\right)$, then $VB = -10.67 f + 1.32 \times 10^{-2} F_c - 1.05 \times 10^{-3} F_f + 22.07$
6	If $A_f = \exp\left(-\frac{4}{0.25^2}\left(\frac{f-0.33}{0.0265}\right)^2\right)$, $A_{F_c} = \exp\left(-\frac{4}{0.25^2}\left(\frac{F_c-1112}{0.0265}\right)^2\right)$ and $A_{F_f} = \exp\left(-\frac{4}{0.25^2}\left(\frac{F_f-423}{0.0265}\right)^2\right)$, then $VB = 0.62 f - 1.34 \times 10^{-3} F_c + 2.61 \times 10^{-3} F_f + 4.90$
7	If $A_f = \exp\left(-\frac{4}{0.25^2}\left(\frac{f-0.47}{0.0265}\right)^2\right)$, $A_{F_c} = \exp\left(-\frac{4}{0.25^2}\left(\frac{F_c-1332}{0.0265}\right)^2\right)$ and $A_{F_f} = \exp\left(-\frac{4}{0.25^2}\left(\frac{F_f-347}{0.0265}\right)^2\right)$, then $VB = 1.14 \times 10^5 f - 1.43 \times 10^{-2} F_c - 0.18 F_f - 5.38 \times 10^5$
8	If $A_f = \exp\left(-\frac{4}{0.25^2}\left(\frac{f-0.33}{0.0265}\right)^2\right)$, $A_{F_c} = \exp\left(-\frac{4}{0.25^2}\left(\frac{F_c-1032}{0.0265}\right)^2\right)$ and $A_{F_f} = \exp\left(-\frac{4}{0.25^2}\left(\frac{F_f-334}{0.0265}\right)^2\right)$, then $VB = 2.41 f - 1.14 \times 10^{-3} F_c + 1.30 \times 10^{-2} F_f + 1.20$
9	If $A_f = \exp\left(-\frac{4}{0.25^2}\left(\frac{f-0.47}{0.0265}\right)^2\right)$, $A_{F_c} = \exp\left(-\frac{4}{0.25^2}\left(\frac{F_c-1455}{0.0265}\right)^2\right)$ and $A_{F_f} = \exp\left(-\frac{4}{0.25^2}\left(\frac{F_f-508}{0.0265}\right)^2\right)$, then $VB = 7.19 f - 9.09 \times 10^{-4} F_c - 3.28 \times 10^{-4} F_f + 2.50 \times 10^{-3}$

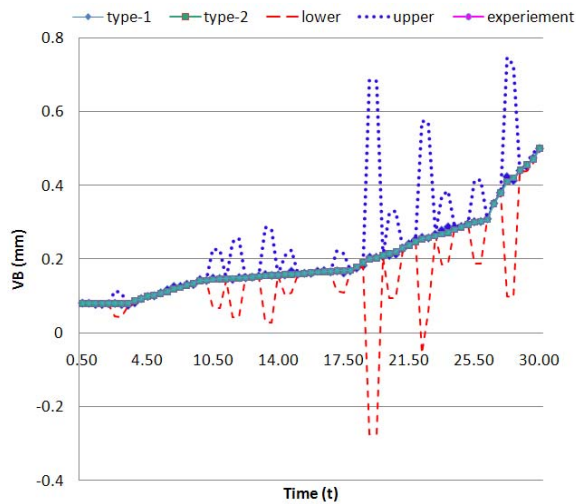


Fig. 6 Training results from type-2 fuzzy system (w5)

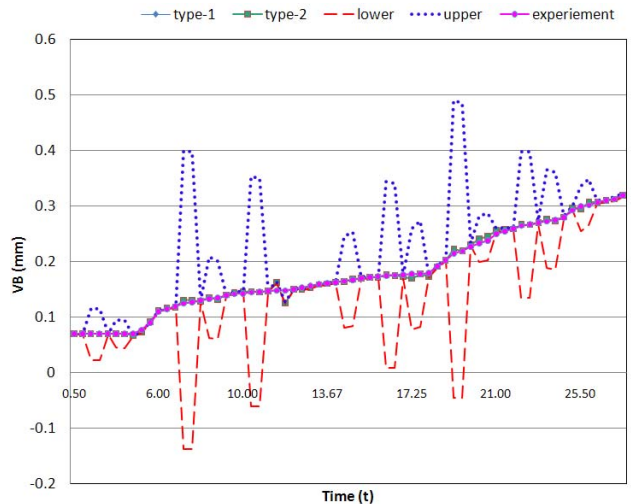


Fig. 7 testing results from type-2 fuzzy system (w7)

From Fig.6 and Fig. 7, it is observed that average output of type-2 fuzzy approach is close to the output of type-1 approach. The maximum difference between estimated tool wear values from type-2 fuzzy approach and its type-1 counterpart is 0.000156mm. It is clear that there are big uncertainty intervals corresponding to some cutting instant. The maximum uncertainty is 0.479885mm, the minimum is 0.000744mm.

The source of the uncertainties can be considered mainly from sudden changes of cutting parameters, discontinuous input-output data sets and manual measurements. In the experimentation, there were six sets of cutting parameters used from Fig. 3, 10 cycles performed for learning experiment w5 and 9 cycles performed for testing experiment w7. Instead of the cutting forces being recorded by the instrument, the tool wears were manually measured after each cycle. To do that the machine had to be stopped and restarted after each pause. Also, there are different levels of uncertainty in the result that could be caused by thermal deformation during machining, dynamic deformation of the workpiece, chatter vibration, condition of the chips [17, 18], dynamic deformation of the machine elements, structure vibration of the machine-tool [19], thermal deformation of the tool [20], sensitivity of the sensors and dimension measurement, etc. In order to reduce get better result, more accurate continuous experimental data sets are required.

From this experimentation, type-2 TSK approach has the capability to model the system matching the estimation of average cutting tool wear and the directly measured wear value. And it is clear that, comparing Fig.5 and Fig.7, type-2 fuzzy approach has an outstanding advantage that it is the only artificial intelligent method which can capture the uncertainty during the machining process and estimate the errors of approximation during tool wear monitoring

VI. CONCLUSION

This paper present an innovative tool wear monitoring method using type-2 TSK fuzzy approach based on subtractive method to accomplish the integration of multi-sensor information and tool wear information. It generates fuzzy rules directly from the input-output data acquired from sensors, and provides high accuracy and high reliability of the tool wear prediction over a wide range of cutting conditions. Also, its interval set of output assesses the information of uncertainty in the estimation of tool wear condition, which could be of a great value to a decision maker and be used to investigate the complicated tool wear condition during machining process.

The application of type-2 fuzzy logic on uncertainty prediction in machining has great meaning for continuous improvement in product quality, reliability and manufacturing efficiency in machining industry.

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