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Study on 2-tuple Linguistic Assessment Method based on Grey Cluster with Incomplete Attribute Weight Information

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***Abstract*—Grey cluster decision-making is an important part of grey system theory. And 2-tuple linguistic assessment resolves the calculation problems of the result of expert qualitative linguistic assessment. But both of them not mention how to calculate the weight of index. Based on the definitions of entropy, a method of getting weight is proposed, and considers the character of grey cluster decision-making and 2-tuple linguistic assessment, this paper proposed the method of 2-tuple linguistic assessment based on grey cluster. It can combine qualitative analysis and quantitative analysis well.**

***Keywords*—entropy weight, grey cluster, linguistic assessment, 2-tuple linguistic**

I. INTRODUCTION

Grey cluster decision which is applied to analyze the objects in the "few data" and "deficient information" system takes an important role in grey system theory. Gray cluster analysis theory cluster quantitative rating data by using whitenization weight to analyze and assess. When the criteria for clustering have different meanings, dimensions, and sizes of observational data, applying variable weight clustering may lead to the problem that some criteria participate in the clustering process very weakly.

In a complex and uncertain environment, with the complexity of the object and the ambiguity of human thought, it is often difficult for the evaluators to give accurate and quantitative assessment results. And description based on natural language can express the wishes of evaluation experts more flexibly and accurately. In recent years, research on group comprehensive assessment theory based on the information of linguistic assessment has become a hot spot of assessment method research [1] – [6]. One way to solve this problem is to define a weight for each individual criterion before clustering processing.

Assessment based on linguistic information has resolved the calculation problems of the result of expert qualitative linguistic assessment, however, which without quantitative evaluation. In reality, the assessment indicators often include both qualitative and quantitative indicators. In ref [7], Combined with the characteristics of grey cluster analysis method and 2-tuple linguistic assessment method, but the weight of index is not studied. In this paper, using entropy weight theory, 2-tuple linguistic assessment method based on grey cluster analysis is proposed.

The remain of this paper is organized as follows: section 2 is a brief description of the traditional 2-tuple linguistic assessment method; section 3 introduces gray cluster analysis method; and in the 4th section, describes the entropy weight theory; section 5, 2-tuple linguistic assessment method based on grey cluster analysis method is proposed to tackle with problems with qualitative and quantitative assessment information; in the end, in 6th section, concludes the paper.

II. TRADITIONAL 2-TUPLE ASSESSMENT

Assume that assessment indicators set is $P=\{P_1,P_2,\cdots,P_n\}\ \ (n\geq 2)$, and P_j is j^{th} indicator; the decision group is $D=\{D_1,D_2,\cdots,D_q\}\ \ (q\geq 2)$,and D_k is k^{th} decision-maker. Assume that weight of n assessment indicators is $R=(r_1,r_2,\cdots,r_n)^T$. Assessment vector with linguistic form given by the decision-maker D_k is $B^k=(b_j^k)$, and b_j^k is indicator value of P_j selected by decision-maker D_k from pre-defined linguistic assessment set S . Here S is an ordered set that pre-defined and has odd elements. S has attributions as follows:

1) Ordering

When $i\geq j$, it has $S_i"\geq"S_j$, where $"\geq"$ is the meaning of "better than or equal to".

2) Reversibility

There is a reversible operator called $"neg"$, when $j=T-1-i$, it has $neg(S_i)=S_j$, here T Represent the elements number in set S .

3）Maximizing Operation

When $S_i"\geq"S_j$, $\max\{S_i,S_j\}=S_i$.

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4）Minimizing Operation

When S_i"≥"S_j, $\min\{S_i,S_j\}=S_j$.

Because elements in the result set concluded from traditional linguistic assessment method is often inconsistent with that in pre-defined linguistic assessment set. And the result set can only be approximately expressed by linguistic assessment set, which led to loss of information and the inaccuracy of the results. In 2000, Spain Professor Herrera put forward the 2-tuple semantic analysis method about gathering the linguistic information, which overcomes deficiencies of the past research method [8].

2-tuple semantic is a method based on symbolic translation to express linguistic assessment information by using (S_i,α_i). S_i represent language phrase in pre-defined linguistic assessment information set, and α_i is the deviation of most close language phrase in linguistic assessment information gained from calculation and the initial linguistic assessment set.

Definition 1: Assume that $S_i\in S$ and it is language phrase. The corresponding 2-tuple semantic can be gained from below transfer function θ:

$$\theta : S \rightarrow S\times[-0.5,0,5)$$
$$\theta(S_i)=(S_i,0),\ S_i\in S \tag{1}$$

Definition 2: Assume that linguistic assessment set $S=\{S_0,S_1,\cdots,S_T\}$, $\beta\in[0,T-1]$ is a real number. It represents the result of linguistic symbol gathering operation. And then the corresponding dyadic semantic with β may be gained from below function Δ:

$$\Delta:[0,T-1]\rightarrow S\times[-0.5,0.5)$$
$$\Delta(\beta)=(S_i,\alpha_i)=\begin{cases}S_i & i=round(\beta)\\ \alpha_i=\beta-i & \alpha_i\in[-0.5,0.5)\end{cases} \tag{2}$$

where $round()$ is "rounding" integer operation.

Definition 3: Assume that linguistic assessment set $S=\{S_0,S_1,\cdots,S_T\}$, (S_i,α_i) is a 2-tuple semantic. Then, There is an inverse function Δ^{-1} that can make the 2-tuple semantic converted into the corresponding numerical 2-tuple semantic $\beta\in[0,T-1]$, that is :

$$\Delta^{-1}:S\times[-0.5,0.5)\rightarrow[0,T-1]$$
$$\Delta^{-1}(S_i,\alpha_i)=i+\alpha_i=\beta \tag{3}$$

Assume that (S_i,α_i) and (S_j,α_j) are any two 2-tuple semantic, they have blew attributes：

1) Ordering

When $i\geq j$, it has S_i"≥"S_j, where "≥" is the meaning of “better than or equal to”.

When $i=j$:

a) if $\alpha_i>\alpha_j$,then (S_i,α_i)">"(S_j,α_j);

b) if $\alpha_i=\alpha_j$, then (S_i,α_i)"="(S_j,α_j), symbol "=" represent “equal to”;

c) if $\alpha_i<\alpha_j$, then (S_i,α_i)"<"(S_j,α_j); symbol "<" represent “inferior to”.

2) Reversibility

There is a reversible operator called "neg". $neg((S_i,\alpha_i))=\Delta(T-1-\Delta^{-1}((S_i,\alpha_i)))$, here T represent the element’s number in set S.

3) Maximizing Operation

When (S_i,α_i)">"(S_j,α_j), $\max\{(S_i,\alpha_i),(S_j,\alpha_j)\}=(S_i,\alpha_i)$.

4) Minimizing Operation

When (S_i,α_i)">"(S_j,α_j), $\min\{(S_i,\alpha_i),(S_j,\alpha_j)\}=(S_j,\alpha_j)$.

Definition 4: Distance (d,α_d) between any two 2-tuple semantic (S_i,α_i) and (S_j,α_j) can be defined as:

$$(d,\alpha_d)=\Delta(\left|\Delta^{-1}(S_i,\alpha_i)-\Delta^{-1}(S_j,\alpha_j)\right|) \tag{4}$$

where $d\in S$, $\alpha_d\in[-0.5,0.5)$.

Definition 5: Assume $(b_1,\alpha_1),(b_2,\alpha_2),\cdots(b_m,\alpha_m)$ is a group of 2-tuple semantic to be gathered, then the number average operator $\overline{B}^e$ based on 2-tuple semantic is defined as follows:

$$\overline{B}^e=(\overline{b},\overline{\alpha})=\Delta(\sum_{i=1}^{m}\tfrac{1}{m}\Delta^{-1}(b_i,\alpha_i)) \tag{5}$$

where $\overline{b}\in S,\overline{\alpha}\in[-0.5.0.5)$.

Definition 6: Assume $(b_1,\alpha_1),(b_2,\alpha_2),\cdots(b_m,\alpha_m)$ is a group of 2-tuple semantic to be gathered, $R=(r_1,r_2,\cdots,r_m)$ is the corresponding weight vector, then the weighted number average operator $\hat{B}^e$ based on 2-tuple semantic is defined as follows:

$$\hat{B}^e=(\hat{b},\hat{\alpha})=\Delta(\sum_{i=1}^{m}[r_i\times\Delta^{-1}(b_i,\alpha_i)]) \tag{6}$$

where $\hat{b}\in S,\hat{\alpha}\in[-0.5.0.5)$.

III. Grey Cluster Analysis

Grey cluster analysis is a method based on matrices of grey incidences or whiteization weight function of grey numbers, to classify the observation indices or observational objects into definable classes [9].

In grey cluster decisions, most used method are whitenization weight function of upper measure, whitenization weight function of moderate measure and whitenization weight function of lower measure.

Assume that there are m indicators, and in grey cluster decisions, there are s classes.

(1) The whitenization weight function of upper measure is given by,

$$f_u^v(x)=\begin{cases}0 & x<x_u^v(1)\\ \dfrac{x-x_u^v(1)}{x_u^v(2)-x_u^v(1)} & x\in[x_u^v(1),x_u^v(2)]\\ 1 & x\geq x_u^v(2)\end{cases} \quad (7)$$

where $0\leq v\leq s-1$, $0\leq u\leq m-1$.

(2) The whitenization weight function of moderate measure is given by,

$$f_u^v(x)=\begin{cases}0 & x\notin[x_u^v(1),x_u^v(4)]\\ \dfrac{x-x_u^v(1)}{x_u^v(2)-x_u^v(1)} & x\in[x_u^v(1),x_u^v(2)]\\ \dfrac{x_u^v(4)-x}{x_u^v(4)-x_u^v(2)} & x\in[x_u^v(2),x_u^v(4)]\end{cases} \quad (8)$$

where $0\leq v\leq s-1$, $0\leq u\leq m-1$.

(3) The whitenization weight function of lower measure is given by,

$$f_u^v(x)=\begin{cases}0 & x\notin[0,x_u^v(4)]\\ 1 & x\in[0,x_u^v(3)]\\ \dfrac{x_u^v(4)-x}{x_u^v(4)-x_u^v(3)} & x\in[x_u^v(3),x_u^v(4)]\end{cases} \quad (9)$$

where $0\leq v\leq s-1$, $0\leq u\leq m-1$.

In assessment progress, according to the whitenization weight function, clustering coefficient for the u^{th} indicator v^{th} subclasses of objects is $\sigma_u^v=f_u^v(x)$.

Then clustering coefficient vector for u^{th} indicator of objects is $\sigma_u=(\sigma_u^0,\sigma_u^1,\cdots,\sigma_u^{s-1})$.

Definition 7: Assume that $\max\limits_{0\leq v\leq s-1}\{\sigma_u^v\}=\sigma_u^{v^*}$, then the object is belonged to v^{*th} grey subclasses for u^{th} indicator.

IV. WEIGHT BASED ON ENTROPY THEORY

The definition of the entropy comes from thermodynamics, which describes the phenomenon in the hydronium or molecule's movement. Since being introduced into information theory by Shannon, it has been applied extensively in engineering technology, social economy and other fields [10]-[12]. According to the principle of information theory, the information is a measurement of the system order degree while the entropy is a measurement of the system orderless degree. The absolute values of them are equal but their symbols are opposite [13].

Assume that there are n objects, m evaluate index, data matrix of original evaluate value $X=(x_{ij})_{n\times m}$, $x_{ij}\geq 0$, $i=1,2,\cdots,n$, $j=1,2,\cdots,m$. Because the variable value range in entropy is between 0 and 1, we need to pretreatment the original evaluate value up to the requirement. We deal it with the unitary method, i.e. $p_{ij}=x_{ij}\Big/\sum_{i=1}^{n}x_{ij}$ to get the matrix P. It should be noted that P has no probability meaning.

The information entropy of the index x_i in the system is:

$$E_j=-k\times\sum_{i=1}^{n}(p_{ij}\ln p_{ij}) \quad (10)$$

Where $k=1/\ln n$.

Definition 8: Define that w_i as the entropy weight of the j^{th} index. Then

$$w_i=(1-E_j)\Big/(m-\sum_{j=1}^{m}E_j) \quad (11)$$

We can see from above that: the smaller the information entropy of the index is, the bigger the variation degree of its index value is, and the bigger its weight is. Whereas the bigger the information entropy of the index is, the smaller the variation degree of its index value is, and the smaller its weight is. Extremely, if the information entropy of the index $E_{\max}=1$, which indicates that the index values for each object are equal, it can be deleted. In conclusion, we can use entropy to calculate the weight of each index according to the variation degree of each index value.

V. 2-TUPLE LINGUISTIC ASSESSMENT BASED ON GREY CLUSTER

It can be seen from 2-tuple linguistic assessment method and grey cluster analysis method that: 2-tuple linguistic assessment method can mass linguistic assessment information; but the quantitative assessment data has been clustered through gray whitenization weighted functions and then gains the results. There are often both quantitative assessment data and qualitative results in real system. Therefore, it can be a combination of the two methods in order to achieve the combination of qualitative and quantitative. Here we can use method of changing the linguistic rating information into quantitative data, or use method that transferring quantitative data into linguistic rating information. Now we will use method of changing gray cluster analysis result into 2-tuple linguistic assessment information. 2-tuple linguistic assessment method based on grey cluster analysis is proposed.

A. Changing Grey Cluster Information into 2-tuple Linguistic Assessment Information

In order to changing grey cluster information into 2-tuple linguistic assessment information, let $s = T$, where s is the number of grey classes, and T represent the element's number in linguistic assessment set S. Assume that there is a object belong to grey classes $v*$ for u th indicator, which is σ_u^{v*}, then $b_u = v*$. The value of α_u needs using different calculation method according to different type whitenization weight functions.

(1) for the whitenization weight function of upper:

$$\alpha_u = \begin{cases} \sigma_u^{v*} - 1 & x \le x_u^v(2) \\ 0 & x > x_u^v(2) \end{cases} \tag{12}$$

where $0 \le v \le s-1$, $0 \le u \le m-1$.

(2) for the whitenization weight function of moderate:

$$\alpha_u = \begin{cases} \sigma_u^{v*} - 1 & x \le x_u^v(2) \\ 1 - \sigma_u^{v*} & x > x_u^v(2) \end{cases} \tag{13}$$

where $0 \le v \le s-1$, $0 \le u \le m-1$.

（3）for the whitenization weight function of lower:

$$\alpha_u = \begin{cases} 0 & x \le x_u^v(3) \\ 1 - \sigma_u^{v*} & x > x_u^v(3) \end{cases} \tag{14}$$

where $0 \le v \le s-1$, $0 \le u \le m-1$.

In this way, the Gray Cluster results can be translated into 2-tuple linguistic assessment information (b_u, α_u).

B. 2-tuple Linguistic Assessment Method based on Grey Cluster Decision

Step 1: According to the quantitative rating data, gray cluster analysis method is used in order to obtain the class that the object for the indicator belong to. Then change the grey class into 2-tuple linguistic assessment information using the method proposed above.

Step 2: Using qualitative indicators, expert give linguistic assessment.

Step 3: According to equation (1), change language assessment information b_{ij}^k into 2-tuple form, which is $(b_{ij}^k, 0)$ using transfer function θ.

Step 4: Calculate of the weighted compositive assessment information of the object.

According to the weighted average operator equator (6), calculate the weighted assessment information of the object (b, α).

$$(b, \alpha) = \Delta(\sum_{j=1}^{n} [r_j \times \Delta^{-1}(b_j, \alpha_j)]) \tag{15}$$

where $j = 1, 2, \cdots, n$.

Step 5: Get the comprehensive language rank information of the object.

We can see from the above algorithm that the assessment model based on grey cluster analysis and 2-tuple linguistic information can handle assessment the system in which there are both qualitative indicators as well as quantitative indicators.

VI. Conclusions

Based on the Characteristics of 2-tuple language rating information model and grey cluster analysis theory, this paper proposed a 2-tuple linguistic rating method based on grey cluster analysis, the weight is calculated based entropy theory. It can handle assessment indicator system that both have qualitative indicators and quantitative indicators. At the same time it also made full use of the characteristics of grey cluster analysis which can deal with "less data" and "poor information" condition well.

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