Proceedings of the 2009 IEEE International Conference on Systems, Man, and Cybernetics
San Antonio, TX, USA - October 2009

Fuzzy Large-Scale Systems Stabilization with Nonlinear State Feedback Controller

Iman Zamani
Electrical Engineering Department
The Amirkabir University of Technology
Tehran, Iran
zamaniiman@aut.ac.ir

Nasser Sadati
Electrical and Computer Engineering Department
The University of British Columbia
Vancouver, BC Canada
sadati@ece.ubc.ca

***Abstract*— This paper investigates the stability and stabilization problem of fuzzy large-scale systems in which the system is composed of a number of Takagi-Sugeno fuzzy subsystems with interconnection. Instead of fuzzy parallel distributed compensation (PDC) design, nonlinear state feedback controllers are used in the stabilization of the overall large-scale system. Based on Lyapunov criterion, some conditions are derived under which the whole fuzzy large-scale system is stabilized asymptotically.**

Keywords*— *Fuzzy large-scale system, nonlinear controller, T-S fuzzy systems, stability analysis

I. INTRODUCTION

Stability analysis and stabilization of fuzzy large-scale systems for both discrete and continuous cases were discussed by some researches[1]-[4]. In this work, a fuzzy large-scale system consisting of J subsystems $S_i (i = 1,2,\cdots,J)$ with interconnection is considered. Each subsystem is decomposed into a set of fuzzy regions, were in each region there is a local dynamic behavior of the subsystems described by a T-S fuzzy model.

Motivated from the work in [5] for fuzzy systems, the stability analysis and controller design of continuous-time fuzzy large-scale system using Lyapunov function is considered in this paper. Based on Lyapunov and relevant theory such as Lasalle's theorem, a nonlinear state feedback controller for each subsystem is designed such that the whole closed loop large-scale system under some sufficient conditions is stabilized asymptotically. These conditions are derived in the form of matrix inequalities. The main advantages of the proposed approach are low computation, optional feedback gains and some merits which are discussed in the next sections.

The article is organized as follows; the considered system and problem formulation are discussed in Section II. In Section III, nonlinear state feedback controllers are designed and the main results are proposed. Section IV provides an example to illustrate the applicability of the proposed controller design. Finally, some concluding remarks are given in Section V.

II. PRELIMINARIES

Consider a fuzzy large-scale system (S) which consists of J interconnected subsystems $S_i\ (i = 1,2,\cdots,J)$. The i th fuzzy subsystem S_i is described by the following rule-based equations

$$S_i^l = \begin{cases} \text{if } x_{i1}(t) \text{ is } M_i^1 \text{ and } \cdots\ x_{in_i}(t) \text{ is } M_i^{n_i} \\ \text{then } \dot{x}_i(t) = A_i^l x_i(t) + B_i^l u_i(t) + \sum_{\substack{j=1 \\ j\neq i}}^{J} f_{ij}^l\big(x_j(t)\big) \end{cases}$$
$$(i = 1,2,\cdots,J),\ (j = 1,2,\cdots,J),\ (l = 1,2,\cdots,r_i) \quad (1)$$

where

S_i^l	is the lth rule of S_i.
$u_i(t) \in \mathbb{R}^{m_i}$	is the control input of S_i at time t with appropriate length.
$x_i(t) \in \mathbb{R}^{n_i}$	the State vector of the ith subsystem $x_i(t) = \left[x_{i1}(t), x_{i2}(t), \cdots, x_{in_i}(t)\right]^T$.
$f_{ij}^l\big(x_j(t)\big)$	the interconnection between ith and jth subsystem in the lth rule of S_i.
r_i	the number of rules in subsystem S_i.
n_i	the Number of states in subsystem S_i.
$\big(A_i^l, B_i^l\big)$	the system matrices of rule l in subsystem S_i that are controllable.
M_i^k	the grade of membership of $x_{ik}(t)$, $(k = 1,2,\cdots,n_i)$.

If we utilize the standard fuzzy inference method i.e., singleton, minimum or product fuzzy inference, and central-average deffuzzifier, (1) can be inferred as

$$\dot{x}_i(t) = \sum_{l=1}^{r_i} \mu_i^l\big(x_i(t)\big)\big(A_i^l x_i(t) + B_i^l u_i(t)\big) + \sum_{\substack{j=1 \\ j\neq i}}^{J} \sum_{l=1}^{r_i} \mu_i^l(x_i(t)) f_{ij}^l\big(x_j(t)\big) \quad (2)$$

where

$$\mu_i^{\,l}(x_i(t)) = {w_i^{\,l}}\Big/{\textstyle\sum_{j=1}^{r_i} w_i^{\,j}},\; w_i^{\,l} = \textstyle\prod_{k=1}^{n_i} M_i^k(x_{ik}(t)) \ge 0$$

and $\sum_{l=1}^{r_i} \mu_i^{\,l}(x_i(t)) = 1$. $\mu_i^{\,l}(x_i(t))$ is the firing strength of lth rule of the ith subsystem. It is assumed that the fuzzy sets are normalized, were two examples of normalized fuzzy sets are shown in Fig. 1. Also it is assumed that $f_{ij}^l(x_j(t)) = C_{ij}^l x_j(t)$, where C_{ij}^l is a constant matrix with appropriate dimension.

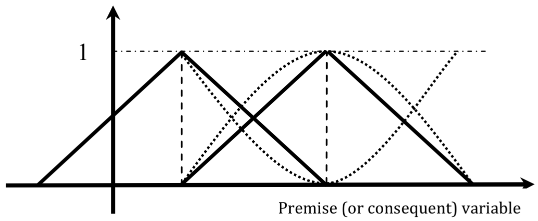

Figure 1. Two types of normalized fuzzy sets; solid line shows the normalized triangular fuzzy sets and dash line shows the other normalized fuzzy sets

III. NONLINEAR STATE FEEDBACK CONTROLLERS

In this section, we will consider the controller design for fuzzy large-scale system (3). For stabilization, a nonlinear state feedback controller is considered as follows

$$u_i(t) = -\sum_{k=1}^{c_i} m_i^k(t) K_i^k x_i(t) \tag{3}$$

in which

$$\sum_{k=1}^{c_i} m_i^k(t) = 1 \qquad \& \qquad 0 \le m_i^k(t) \le 1$$
$$(i = 1,2,\cdots,J \quad k = 1,2,\cdots,c_i) \tag{4}$$

and K_i^k is the state feedback gain with appropriate dimension. $m_i^k(t)$ is a nonlinear function defined by (5. a) and (5. b), and c_i is an optional number for designing the controller in each subsystem. ((5. a) and (5. b) are shown at the end of this page and the next page). It should be noted that if the control law based on PDC was used, then the proof of stabilization by Lyapunov method had much difficulty in the derivation of $\dot{V} < 0$. Hence, by proposing the new approach, using combination of (2) and (3), the closed-loop fuzzy subsystem becomes

$$\dot{x}_i(t) = \sum_{l=1}^{r_i}\sum_{k=1}^{c_i}\sum_{\substack{j=1\\ j\ne i}}^{J} \mu_i^l(x_i(t)) m_i^k(t)\left(\frac{1}{J-1} Y_i^{lk} x_i(t) + C_{ij}^l x_j(t)\right) \tag{6}$$

where $Y_i^{lk} = A_i^l - B_i^l K_i^k$.

Now, our task is to design K_i^k in such a way that the whole fuzzy large-scale system is stabilized asymptotically and exponentially. The stability condition of fuzzy large-scale system (6), can be summarized by the following theorem.

Theorem 3.1: The fuzzy large-scale system (2) is stabilized asymptotically by the nonlinear state feedback controller (3), if there exist symmetric positive matrices P_i , positive scalars η_i^l and the state gains K_i^1 such that the following conditions are satisfied:

$$-Q_i^{l1} \le -\eta_i^l I_i \tag{7.a}$$
$$-M < 0 \tag{7.b}$$

where

$$I_i = \begin{bmatrix} 1 & & & 0 \\ & 1 & & \\ & & \ddots & \\ 0 & & & 1 \end{bmatrix}_{n_i \times n_i}, (l = 1,2,\cdots,r_i),\; (i = 1,2,\cdots,J)$$

$$,\qquad {Y_i^{lk}}^T P_i + P_i Y_i^{lk} = -Q_i^{lk} \qquad , \qquad (k = 1,2,\cdots,c_i)$$

n_i is the number of states of S_i , $\eta_i = \min_l(\eta_i^l) > 0$,

$$M = [m_{ij}] = \begin{cases} m_i^1(t)\eta_i & i = j \\ -m_i^1(t) r_i \lambda_i \beta_{ij} & i \ne j \end{cases},\; \beta_{ij} = \max_l(\|C_{ij}^l\|)(n_i + n_j),$$

$\lambda_i = \max(\mathrm{eig}(P_i))$.

Proof: Consider the Lyapunov function $V(t)$ for the fuzzy large-scale system, given by

$$V(t) = \sum_{i=1}^{J} V_i(t) = \sum_{i=1}^{J} x_i^T(t) P_i x_i(t) \tag{8}$$

Now, by taking the derivative of $V_i(t)$ along the trajectories of ith subsystem, we get

$$\dot{V}_i(t) = \dot{x}_i^T(t) P_i x_i(t) + x_i^T(t) P_i \dot{x}_i(t) \tag{9}$$

$$m_i^1(t) = \begin{cases} 1 - \dfrac{\left(\sum_{k=2}^{c_i}\left(\sum_{l=1}^{r_i}\sum_{\substack{j=1\\ j\ne i}}^{J} \mu_i^l(x_i(t))\left(\frac{1}{J-1} x_i(t)^T Q_i^{lk} x_i(t) - F_{ij}^l(t)\right)\right)\right)}{\sum_{h=1}^{c_i}\sum_{l=1}^{r_i}\sum_{\substack{j=1\\ j\ne i}}^{J}\left|\mu_i^l(x_i(t))\left(\frac{1}{J-1} x_i(t)^T Q_i^{lh} x_i(t) - F_{ij}^l(t)\right)\right|} & ,\text{if} \displaystyle\sum_{h=1}^{c_i}\sum_{l=1}^{r_i}\sum_{\substack{j=1\\ j\ne i}}^{J}\left|\mu_i^l(x_i(t))\left(\frac{1}{J-1} x_i(t)^T Q_i^{lh} x_i(t) - F_{ij}^l(t)\right)\right| \ne 0 \\ \dfrac{1}{c_i} & \text{if} \displaystyle\sum_{h=1}^{c_i}\sum_{l=1}^{r_i}\sum_{\substack{j=1\\ j\ne i}}^{J}\left|\mu_i^l(x_i(t))\left(\frac{1}{J-1} x_i(t)^T Q_i^{lh} x_i(t) - F_{ij}^l(t)\right)\right| = 0 \end{cases} \tag{5.a}$$

Now since

$$F_{ij}^{l}(t) = x_j^T(t){C_{ij}^{l}}^{T}P_i x_i(t) + x_i(t)^T P_i C_{ij}^{l} x_j(t) \qquad (10)$$

(9) can be rewritten as

$$\dot{V}_i(t) = -\sum_{l=1}^{r_i}\sum_{k=1}^{c_i}\sum_{\substack{j=1\\ j\neq i}}^{J} \mu_i^l\big(x_i(t)\big)m_i^k(t)\left(\frac{1}{J-1}x_i(t)^T Q_i^{lk} x_i(t) - F_{ij}^{l}(t)\right) \qquad (11)$$

$$= -\left(m_i^1(t)\left(\sum_{l=1}^{r_i}\sum_{\substack{j=1\\ j\neq i}}^{J}\mu_i^l\big(x_i(t)\big)\left(\frac{1}{J-1}x_i(t)^T Q_i^{l1} x_i(t) - F_{ij}^{l}(t)\right)\right)\right) - \left(\sum_{k=2}^{c_i} m_i^k(t)\left(\sum_{l=1}^{r_i}\sum_{\substack{j=1\\ j\neq i}}^{J}\mu_i^l\big(x_i(t)\big)\left(\frac{1}{J-1}x_i(t)^T Q_i^{lk} x_i(t) - F_{ij}^{l}(t)\right)\right)\right) \qquad (12)$$

So,

$$\dot{V}_i(t) = -\left(m_i^1(t)\left(\sum_{l=1}^{r_i}\sum_{\substack{j=1\\ j\neq i}}^{J}\mu_i^l\big(x_i(t)\big)\left(\frac{1}{J-1}x_i(t)^T Q_i^{l1} x_i(t) - F_{ij}^{l}(t)\right)\right)\right) - \left(\sum_{k=2}^{c_i}\left(\frac{\left(\sum_{l=1}^{r_i}\sum_{\substack{j=1\\ i\neq j}}^{J}\mu_i^l\big(x_i(t)\big)\left(\frac{1}{J-1}x_i(t)^T Q_i^{lk} x_i(t) - F_{ij}^{l}(t)\right)\right)^2}{\sum_{h=1}^{c_i}\sum_{l=1}^{r_i}\sum_{\substack{j=1\\ j\neq i}}^{J}\left|\mu_i^l\big(x_i(t)\big)\left(\frac{1}{J-1}x_i(t)^T Q_i^{lh} x_i(t) - F_{ij}^{l}(t)\right)\right|}\right)\right) \qquad (13)$$

Therefore,

$$\dot{V}(t) = \sum_{i=1}^{J}\dot{V}_i(t) \le -\sum_{i=1}^{J} m_i^1(t)\left(\sum_{l=1}^{r_i}\sum_{\substack{j=1\\ j\neq i}}^{J}\mu_i^l\big(x_i(t)\big)\left(\frac{1}{J-1}x_i(t)^T Q_i^{l1} x_i(t) - F_{ij}^{l}(t)\right)\right)$$

$$= -\sum_{i=1}^{J} m_i^1(t)\left(\sum_{l=1}^{r_i}\mu_i^l\big(x_i(t)\big)\left(x_i(t)^T Q_i^{l1} x_i(t)\right) - \sum_{l=1}^{r_i}\sum_{\substack{j=1\\ j\neq i}}^{J}\mu_i^l\big(x_i(t)\big)F_{ij}^{l}(t)\right) \qquad (14)$$

Since $\sum_{l=1}^{r_i} \grave{\imath}_i^l\big(x_i(t)\big) = 1$ and $\eta_i = \min_l(\eta_i^l)$, we can conclude that

$$-\sum_{l=1}^{r_i}\mu_i^l\big(x_i(t)\big)\left(x_i(t)^T Q_i^{l1} x_i(t)\right) \le -\eta_i\|x_i\|^2\sum_{l=1}^{r_i}\mu_i^l\big(x_i(t)\big) = -\eta_i\|x_i\|^2 \qquad (15)$$

also

$$m_i^k(t) = \begin{cases} \dfrac{\sum_{l=1}^{r_i}\sum_{\substack{j=1\\ j\neq i}}^{J}\mu_i^l\big(x_i(t)\big)\left(\frac{1}{J-1}x_i(t)^T Q_i^{lk} x_i(t) - F_{ij}^{l}(t)\right)}{\sum_{h=1}^{c_i}\sum_{l=1}^{r_i}\sum_{\substack{j=1\\ j\neq i}}^{J}\left|\mu_i^l\big(x_i(t)\big)\left(\frac{1}{J-1}x_i(t)^T Q_i^{lh} x_i(t) - F_{ij}^{l}(t)\right)\right|}, & \text{if } \sum_{h=1}^{c_i}\sum_{l=1}^{r_i}\sum_{\substack{j=1\\ j\neq i}}^{J}\left|\mu_i^l\big(x_i(t)\big)\left(\frac{1}{J-1}x_i(t)^T Q_i^{lh} x_i(t) - F_{ij}^{l}(t)\right)\right| \neq 0 \\ \dfrac{1}{c_i} & \text{if } \sum_{h=1}^{c_i}\sum_{l=1}^{r_i}\sum_{\substack{j=1\\ j\neq i}}^{J}\left|\mu_i^l\big(x_i(t)\big)\left(\frac{1}{J-1}x_i(t)^T Q_i^{lh} x_i(t) - F_{ij}^{l}(t)\right)\right| = 0 \end{cases} \qquad (5.b)$$

$$\sum_{l=1}^{r_i} \mu_i^l(x_i(t)) F_{ij}^l(t)$$

$$= \sum_{l=1}^{r_i} \mu_i^l(x_i(t)) \left(x_j^T(t) {C_{ij}^l}^T P_i x_i(t) + x_i(t)^T P_i C_{ij}^l x_j(t) \right)$$
$$\leq r_i \|P_i x_i\| \, \|C_{ij}^l x_j\| (n_i + n_j)$$
$$\leq r_i \sqrt{(C_{ij}^l x_j)^T C_{ij}^l x_j} \sqrt{\lambda_{max}(P_i^T P_i)} \|x_i\| \, \|x_j\| (n_i + n_j)$$
$$\leq r_i \|x_i\| \, \|x_j\| \|P_i\|_2 \max_l(\|C_{ij}^l\|) (n_i + n_j) \quad (16)$$

Now based on (15) and (16), it is clear that

$$\dot{V}(t) \leq \sum_{i=1}^{J} m_i^1(t) \left(-\eta_i \|x_i\|^2 + \sum_{\substack{j=1 \\ j \neq i}}^{J} r_i \|x_i\| \, \|x_j\| \|P_i\|_2 \max_l(\|C_{ij}^l\|) (n_i + n_j) \right) \quad (17)$$

Therefore,

$$\dot{V}(t) \leq \sum_{i=1}^{J} m_i^1(t) \left(-\eta_i \|x_i\|^2 + \sum_{j=1}^{J} r_i \|x_i\| \, \|x_j\| \lambda_i \, \beta_{ij} \right) \quad (18)$$

where

$$\|P_i\|_2 = \lambda_i = \lambda_{max}(P_i), \beta_{ij} = max_l(\|C_{ij}^l\|) (n_i + n_j)$$

Following [6], the right-hand side of (18) is quadratic in terms of $\{\|x_1\| \quad \|x_2\| \quad \cdots \quad \|x_J\|\}$, which can be rewritten as

$$\text{right}-\text{hand side of } (18) = -[\|x_1\| \quad \|x_2\| \quad \cdots \quad \|x_J\|] \times M \times \begin{bmatrix} \|x_1\| \\ \|x_2\| \\ \vdots \\ \|x_J\| \end{bmatrix} \quad (19)$$

Now if (15) and (18) hold, then we get $\dot{V}(t) < 0$. Now, if $m_i^k(x_i(t)) = \frac{1}{c_i}$, then

$$\mu_i^l(x_i(t)) \left(\frac{1}{J-1} x_i(t)^T Q_i^{lk} x_i(t) - F_{ij}^l(t) \right) = 0$$

As it can be seen, the above procedure is more simpler and it can be easily resulted that $\dot{V}(t) \leq 0$. With using the Laslle's principle, since the limit set includes only the trivial trajectory $x(t) \equiv 0$, and since the limit set only includes the origin, the origin is asymptotically stable. Thus, the proof is completed. ■

Remark 3.1: Considering (7. b), we can use Sylvester criterion [7] for positive definite matrices. It is easy to show that these conditions are independent of $m_i^1(t)(i = 1,2,\cdots,J)$ and we can get η_i $(i = 1,2,\cdots,J)$ easily.

Exponential stability is another important subject in stability analysis, which is concerned with asymptotic stability and decay rate. To treat the exponential stability analysis, an approach is often adopted, namely exponential stability theorem [8].

From the proposed method and the theorem, it is easy to conclude that

I. This theorem and the proposed method are innovative, usable and applicable with low amount of computation. This method does not include some restrictive conditions such as norm bounded, and also this is not a pre-designed method, i.e. it is not necessary to check the stability for pre-designed system that requires trial-and-error for controller design.

II. (7.a) is checked only for Q_i^{l1} and K_i^1. Therefore K_i^k is optional (for $k \neq 1$) and doesn't affect the stability. The number of state feedback gains which must be designed decreases drastically. Based on the authors experience, $K_i^k (k \neq 1)$ affect the type of responses i.e., the amount of oscillation, damping, settling time, etc. Therefore, the designer has more flexibility in designing.

IV. ILLUSTRATIVE EXAMPLE

Here, an example is presented to verify the results of the proposed stabilization procedure of fuzzy large-scale systems. We consider the stability of the following fuzzy large-scale system composed of two subsystems described by

- Subsystem S_1:

Rule1: if $x_{11}(t)$ is M_1^2 and $x_{12}(t)$ is M_1^1

$$\text{then} \quad \dot{x}_1(t) = A_1^1 x_1(t) + B_1^1 u_1(t) + \sum_{\substack{j=1 \\ j \neq 1}}^{2} C_{1j}^1 x_j(t)$$

Rule2: if $x_{11}(t)$ is M_1^2 and $x_{12}(t)$ is M_1^2

$$\text{then} \quad \dot{x}_1(t) = A_1^2 x_1(t) + B_1^2 u_1(t) + \sum_{\substack{j=1 \\ j \neq 1}}^{2} C_{1j}^2 x_j(t)$$

Rule3: if $x_{11}(t)$ is M_1^1 and $x_{12}(t)$ is M_1^1

$$\text{then} \quad \dot{x}_1(t) = A_1^3 x_1(t) + B_1^3 u_1(t) + \sum_{\substack{j=1 \\ j \neq 1}}^{2} C_{1j}^3 x_j(t)$$

in which $A_1^1 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, A_1^2 = \begin{bmatrix} 0 & 1 \\ -1.2 & 1 \end{bmatrix}, A_1^3 = \begin{bmatrix} 0 & 3 \\ 1 & 0 \end{bmatrix}$, $B_1^1 = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix}, B_1^2 = \begin{bmatrix} 1.6 \\ 0 \end{bmatrix}, B_1^3 = \begin{bmatrix} 0 \\ 1.2 \end{bmatrix}$. Moreover, the interconnection matrices among the two subsystems are given by $C_{12}^1 = \begin{bmatrix} 0.1 & 0 \\ -0.6 & 0 \end{bmatrix}, C_{12}^2 = \begin{bmatrix} 0.01 & 0.01 \\ -0.6 & 0 \end{bmatrix}$, $C_{12}^3 = \begin{bmatrix} -0.1 & 0.21 \\ -0.6 & 0 \end{bmatrix}$. Also $\beta_{12} = \max_l(\|C_{12}^l\|)\,(n_1 + n_2)$ $= \max\{\|C_{12}^1\|, \|C_{12}^2\|, \|C_{12}^3\|\} \times (n_1 + n_2) = 0.6094$, $r_1 = 3$ and $x_1(t) = \begin{bmatrix} x_{11}(t) \\ x_{12}(t) \end{bmatrix}$. The normalized membership functions of subsystem 1 are shown in Fig. 2.

- Subsystem S_2:

Rule1: if $x_{21}(t)$ is M_2^1 and $x_{22}(t)$ is M_2^1

$$\text{then } \dot{x}_2(t) = A_2^1 x_2(t) + B_2^1 u_2(t) + \sum_{\substack{j=1 \\ j\neq 2}}^{2} C_{2j}^1 x_j(t)$$

Rule2: if $x_{21}(t)$ is M_2^2 and $x_{22}(t)$ is M_2^1

$$\text{then } \dot{x}_2(t) = A_2^2 x_2(t) + B_2^2 u_2(t) + \sum_{\substack{j=1 \\ j\neq 2}}^{2} C_{2j}^2 x_j(t)$$

in which $A_2^1 = \begin{bmatrix} 0 & 1.5 \\ 4 & 0 \end{bmatrix}, A_2^2 = \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix}$, $B_2^1 = \begin{bmatrix} 0 \\ 0.56 \end{bmatrix}, B_2^2 = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix}$. Moreover, the interconnection matrices among the two subsystems are given as

$$C_{21}^1 \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \qquad C_{21}^2 = \begin{bmatrix} 0.1 & 0.1 \\ 0.1 & 0.1 \end{bmatrix}$$

also

$$\beta_{21} = \max_l(\|C_{21}^l\|)\,(n_2 + n_1) \max\{\|C_{21}^1\|, \|C_{21}^2\|\}\,(n_2 + n_1) = 0.2000, \; r_2 = 2 \text{ and } x_2(t) = \begin{bmatrix} x_{21}(t) \\ x_{22}(t) \end{bmatrix}.$$

The normalized membership functions of subsystem 2 are shown in Fig. 3.

First, we design a nonlinear state feedback controller for each subsystem so to make the fuzzy large-scale system becomes stable. For each subsystem, let us set $c_1 = c_2 = 2$. By using the above procedure, we can have

$$K_1^1 = [2.0734, -0.1452],$$

$$q = \begin{bmatrix} 5.5420 & 0.3937 \\ 0.3937 & 2.8138 \end{bmatrix} = P^{-1} > 0$$

which implies $\eta_1 = (0.1786)^2 = (\lambda_{min}(P))^2 = 0.0319$, $\lambda_{max}(P) = 0.3626$.

Also

$$\eta_2 = 24.0959 \Longrightarrow \begin{vmatrix} (\lambda_{min}(P_i))^2 & -r_1\lambda\beta_{12} \\ -r_2\lambda\beta_{21} & \eta_2 \end{vmatrix} = \begin{vmatrix} 0.0319 & -3 \times 0.3626 \times 0.6094 \\ -2 \times 0.3626 \times 0.2000 & \eta_2 \end{vmatrix} > 0$$

and the state feedback gain is obtained as $K_2^1 = [5.1682 \quad 43.5930]$. Since K_1^2 and K_2^2 are optional, by choosing $K_1^2 = [1,1]$ and $K_2^2 = [0.01, 0.01]$, the procedure is completed. Hence, the overall closed-loop fuzzy large-scale system is stabilized asymptotically. The complete simulation results with initial condition $x_1(0) = [0.3, -0.5]^T$, $x_2(0) = [0.8, -0.9]^T$ are shown in Figs. 4-7, respectively. Also for comparison, the responses without controllers ($u_i(t) \equiv 0$ $(i = 1,2)$) are shown.

V. CONCLUSION

In this paper, a new approach is used to stabilize the fuzzy large-scale systems in sense of Lyapunov Stability. Under some sufficient conditions, the nonlinear state feedback controllers are developed to stabilize the fuzzy large-scale system exponentially. Since in the real world, almost all systems are complex, the extracted results in this paper can be widely used in the practical applications. An example was also presented to demonstrate the applicability and effectiveness of the proposed approach.

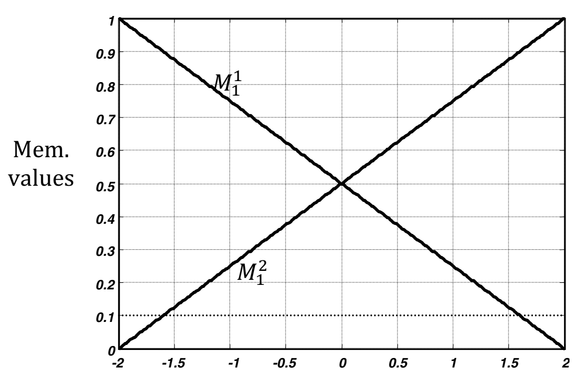

Figure 2. Membership functions of subsystem S_1

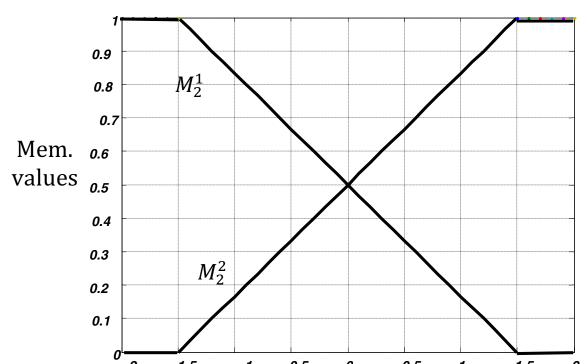

Figure 3. Membership function of subsystem S_2

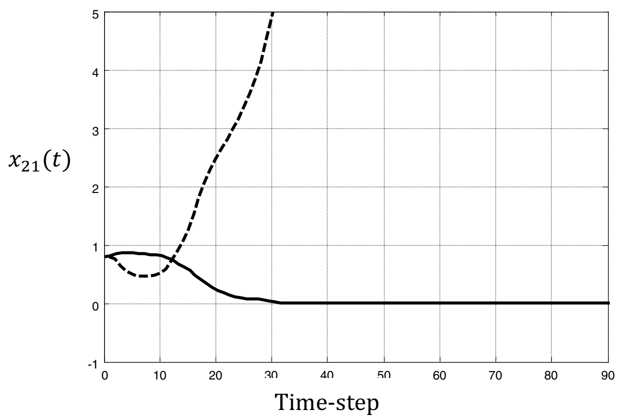

Figure 4. Response $x_{11}(t)$ of subsystem1 when $u(t) \equiv 0$ (dash-line) and with controller (solid-line) $x_{11}(0) = 0.3$

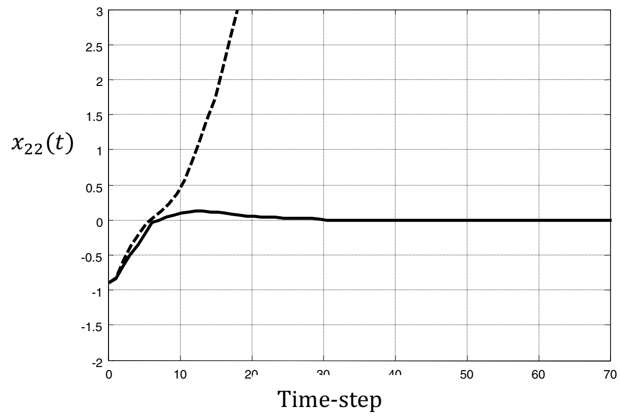

Figure 5. Response $x_{12}(t)$ of subsystem1 when $u(t) \equiv 0$ (dash-line) and with controller (solid-line) $x_{12}(0) = -0.5$

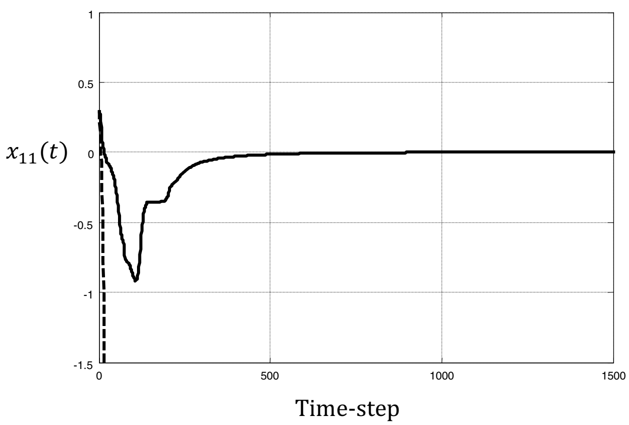

Figure 6. Response $x_{21}(t)$ of subsystem1 when $u(t) \equiv 0$ (dash-line) and with controller (solid-line) $x_{21}(0) = 0.8$

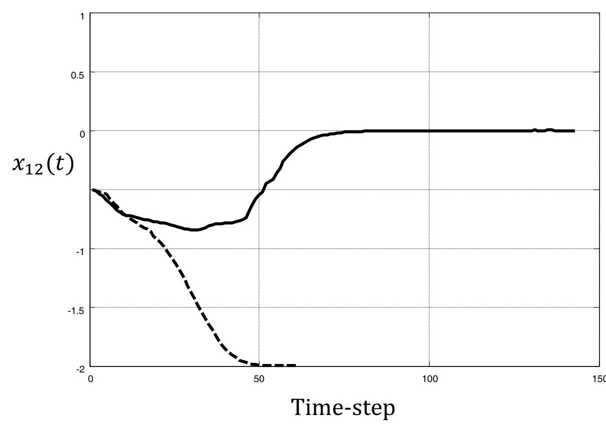

Figure 7. Response $x_{22}(t)$ of subsystem1 when $u(t) \equiv 0$ (dash-line) and with controller (solid-line) $x_{22}(0) = -0.9$

REFERENCES

[1] W. J. Wang and W. W. Lin "Decentralized PDC for large-scale T–S fuzzy systems," IEEE Trans. Fuzzy Syst., vol. 13, no. 6, Dec 2005.

[2] W. J. Wang and L. Luoh "Stability and stabilization of fuzzy large-scale systems "IEEE Trans. Fuzzy Syst., vol. 12, no. 3, June 2004.

[3] F. H. Hsiao and J. D. Hwang "Stability analysis of fuzzy large-scale systems," IEEE Trans. Syst., Man, Cybern. vol. 32, no. 1, Feb 2001.

[4] H. Zhang, C. Li, and X. Liao "Stability Analysis and H∞ Controller Design of Fuzzy Large-Scale Systems Based on Piecewise Lyapunov Functions," IEEE Trans. Syst., Man, Cybern. B, Cybern., vol. 36, no. 3, June 2006.

[5] H. K. Lam, F. H. Leung, and P. K. S. Tam,″ Nonlinear state feedback controller for nonlinear systems: stability analysis and design based on fuzzy plant model,″ IEEE Trans. Fuzzy Syst., vol. 9, no. 4, Aug 2001.

[6] H. K. Khalil, Nonlinear Systems, 3rd ed. Upper saddle River, NJ: Prentice-Hall, 2002.

[7] K. Ogata, Discrete Time Control Systems. Published 1995, Prentice Hall.

[8] M. Bodson., Stability, Convergence, and Robustness of Adaptive Systems, Doctoral thesis, College of Engineering, University of California, Berkeley (1986).