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Spectrum sensing with multiple antennas

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***Abstract*—The primary signal detection is an essential operation for the secondary spectrum usage. In this paper we extend the covariance based detection for multiple-antenna receiver. The proposed method uses the noise power estimation and does not suffer from the noise level uncertainty. We analyze the detection algorithm and compute the distribution of the decision variable not only for the pure noise case but also for the jointly presence of primary signal and noise. By deriving the distribution of the primary signal in noise we are able to use the detection probability as the detector design constraint. Our analysis takes advantage of the high amount of samples available at the detector. The proposed method sets the foundation for the analysis of any detector that involves cross correlation, summing and squaring operations of samples.**

***Index Terms*—Signal detection, probability, covariance analysis**

I. INTRODUCTION

The simplest detector of a signal with unknown structure is a radiometer. The foundations of radiometer analysis have been established already long ago [1]. Recently, it has received a renewed attention as a means to detect unknown primary user's transmission in cognitive radio networks [2]. Unfortunately the primary user transmission has to be detected at signal levels that are in the same order as the noise estimation error. At those low levels the lack of knowledge of precise noise power value prevents the radiometer to detect the signal no matter how many measurements it collects [3]. Similar imperfections limit the performance of detectors with multiple antennas where each antenna is considered as independent detector [4].

For detecting primary signals at the low SNR environment the signal cyclostationary properties can be used. However, that approach requires some a priori knowledge about the signal and the method itself is relative complex. In addition, it is sensitive to particular detector's imperfections [5].

An altenative approach is to use a vector processing. For instance, the covariance-based [6] and the eigenvalue-based [7] detection remedy the limitations of the power detector by estimating the noise power and compensating it. Unfortunately, the analysis in [6] and [7] derive the distribution of the decision variable only in the presence of noise. Essentially, we do not know what is the decision variable distribution if the signal is present. Because of that we cannot set the decision threshold based on the detection probability requirement. One has to design the decision algorithm using the false alarm rate as the design parameter. From the cognitive radio perspective, this is a major drawback because a decision algorithm must fulfill a minimum detection probability requirement.

Despite the covariance matrix being Wishart distributed, we do not use the Wishart matrix properties: eigenvalues or determinants. Computing such quantities adds unnecessary complexity to the test statistics. At the same time, the test statistic can be analyzed only for the pure noise case. Instead, we propose a simple covariance matrix based test that we are able to model analytically.We calculate the test variable statistics by making the transformation of the distributions at each step of the statistics' computation.

While looking at a spectrum sensing detector, one may notice the availability of tens of thousands of samples. One could imagine that a large amount of samples gives the opportunity to use central limit theory in approximation of distribution of decision variables. In this paper we look how this approximation can be applied to a covariance matrix based detector. Here the covariance matrix is computed between the samples from different antennas. In case of pure noise the distributions of matrix elements are easy to derive. We also go through all the steps for computing the covariance matrix elements distribution in the presence of the signal in noise. The computed distributions motivate us to propose an effective detection algorithm. The proposed detector demonstrates as good detection performance as the detectors in [6] [7]. Moreover we are able to compute all the necessary distributions required for the detector performance estimation.

While the covariance matrix can be computed also for signals from one antenna [6] we apply the analysis here to a multi-antenna based detectors. We see numerous benefits by using multi-antenna based detectors. Firstly, in this paper we see how the noise uncertainty can be addressed by computing the cross-correlation between the measurements collected from different antennas. Secondly, the multi-antenna scheme allows to reduce the measurement time since the total amount of the samples is scaled by the amount of antennas.

The contributions of this paper are summed up below:

- We extend the covariance-based detector for multi-antenna receiver.
- For covariance-based detection, we derive the decision variable distribution for the case of signal in noise. As a result, we can use the detection probability requirement for setting the decision threshold.
- We investigate the noise uncertainty impact to the detector's performance and give guidelines of how to control the detection probability in case of noise uncertainty.

The simple power detector is not able to discriminate between

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the primary system transmission and transmission of other secondary users. However, the secondary transmitters are a new system. While deploying those transmitters we are free to require them to communicate additional information that simplifies their separation from primary systems. Because of that a simple power detection is still an attractive primary system detection method.

The rest of the paper is organised as follows: In section II we introduce our system model and in section III we describe the proposed detector. In section IV. we derive the distribution of the covariance matrix elements, while in section V we continue with the distribution of the decision variable. Before evaluating the performance of the proposed detector we introduce the noise uncertainty model in section VI. In section VII. we show that we can predict the detector's performance sufficiently well. In addition, it is illustrated that the detector is resilient to the noise uncertainty. Finally, in section VIII we conclude the paper and summarize the main results.

II. System model

Usually, the signal detection problem is defined in the hypothesis testing framework [8]. Based on the collected channel measurements we need to separate between two possible cases: the measured samples contain both signal and noise: hypothesis H_1; or they contain pure noise: hypothesis H_0. Consider a model where the received waveform at the i^{th} antenna is

$$y_i(t) = \begin{cases} \Re\{[s_i(t) + \eta_i(t)]e^{j2\pi f_c t}\} & H_1 \\ \Re\{\eta_i(t)e^{j2\pi f_c t}\} & H_0, \end{cases} \quad (1)$$

where $s_i(t)$ and $\eta_i(t)$ stand for the received signal and the white Gaussian noise process. The received signal has central frequency f_c and positive bandwidth W.

The detector has N_a antennas and N complex samples are collected by each of them. The received waveform at the output of every antenna is filtered by the ideal bandpass filter of positive bandwidth W and sampled at the Nyquist rate $T_s = 1/W$. For convenience, we denote the collected samples as: $y_i(n) = y_i(nT_s)$ with $1 \leq n \leq N$. The total sensing time equals N/W and $N \cdot N_a$ complex samples will be used to estimate the covariance matrix.

Before showing how to calculate the elements of the covariance matrix, we introduce our signal model. In the analysis below we consider only phase difference between the different antennas. The primary signal collected by the i^{th} antenna is: $s_i(n) = s_a(n)e^{j\cdot\phi_{d,i}(n)}$, where $s_a(n)$ is the amplitude and $\phi_{d,i}(n)$ describes the time dependent phase shift. If the antennas are located sufficiently near to each other one reflected signal copy arriving from one direction has the same Doppler frequency and different initial phase at each antenna. At the receiver we have multiple such copies. Because we sum together multiple reflected copies with different initial phases in each antenna the resulting phase change in each antennas is different.

In this paper we derive the analytical model of the detector for the case of highly correlated signal at the different antennas. Simulations indicate that the proposed detector has a good performance also in the case of independent fading samples between the antennas. However, the detailed analytical study of that case has been left for future research.

We propose to compute the main diagonal elements and the off-diagonal elements of the covariance matrix as

$$\mathbf{R} = \begin{cases} r_{ii} = \frac{1}{N}\sum_{n=1}^{N} y_i(n)y_i^\dagger(n) = \frac{1}{N}\sum_{n=1}^{N} |y_i(n)|^2 & i = j \\ r_{ij} = \frac{1}{N}\sum_{n=1}^{N/M} \left|\sum_{m=1}^{M} y_i(m)y_j^\dagger(m)\right|^2 & i \neq j. \end{cases} \quad (2)$$

One can see that the main diagonal elements r_{ii} correspond to the sum of the powers of the samples: this is the decision variable used in the basic radiometer. The computation of the off-diagonal elements is based on the cross correlation between the samples collected from the different antennas: the samples are grouped, inside the groups the cross correlation is computed and then squared. Finally, we sum the squares of the sums of the group elements and normalize with the total number of samples.

The slightly complex computations for off-diagonal elements is related to the need to cope with different phase change at different antennas. The signal model predicts that over the N samples a phase difference between the antennas will change $\phi_{d,ij}(n) = \phi_{d,i}(n) - \phi_{d,j}(n)$. After cross correlating the samples from the antennas i^{th} and the j^{th} we have

$$\begin{aligned} \sum_{n=1}^{N} y_i(n)\, y_j^\dagger(n) = \; & \sum_{n=1}^{N} s_i(n)\, s_j^\dagger(n) + \sum_{n=1}^{N} s_i(n)\, \eta_j^\dagger(n) \\ & + \sum_{n=1}^{N} s_j^\dagger(n)\, \eta_i(n) + \sum_{n=1}^{N} \eta_j^\dagger(n)\, \eta_i(n), \end{aligned} \quad (3)$$

where the phase change of the term $s_i(n)s_j^\dagger(n)$ over the N samples is $\phi_{d,ij}(n)$. Here $\dagger$ stands for the complex conjugate. Because of different phase differences the amplitudes are not summed together in phase. In the worst cases after summing over N samples the signal samples with opposite phases compensate each other. This means that for the hypothesis H_1 the off-diagonal elements of $\mathbf{R}$ will contain only the noise power terms. In order to avoid such averaging out for the off-diagonal elements we sum together the powers of the samples. The increased complexity can be balanced by avoid squaring every sample but a set of M samples. As seen in (2) we first split the received samples into in N/M groups of M samples. The size M of the group is selected such that inside it the phase does not change significantly. Therefore, in the presence of a signal in noise the off-diagonal elements of the matrix $\mathbf{R}$ will contain also the term

$$\begin{aligned} \left|\sum_{m=1}^{M} s_i(m)\, s_j^\dagger(m)\right|^2 &= \left|\sum_{m=1}^{M} s_a^2(m)\, e^{j\phi_{d,ij}(m)}\right|^2 \\ &= \left|e^{j\phi_{d,ij}(m)}\right|^2 \left|\sum_{m=1}^{M} s_a^2(m)\right|^2 \\ &= \left|\sum_{m=1}^{M} s_a^2(m)\right|^2, \end{aligned} \quad (4)$$

where we assume that the $\phi_{d,ij}(m)$ is constant over the set of M samples.

Notice that for the case analyzed below we actually do not need this sub-sum over M samples. For highly correlated signals also the phase changes could be assumed to be equal and $\phi_{d,ij}(m) = 0$. However the analysis with sub-sum over M turns out to be just slightly more complex than without it. By including this sum here first it makes the detector to operate in the fading environment and it is easier to extend the analysis described here to incorporate the fading environment

In the analysis below we assume the signal and the noise to be independent. The signal has a constant amplitude and its power is P_s. The noise is additive white Gaussian. If N_0 denotes its one-sided power spectral density, the noise variance at the output of the receiver filter equals $\sigma_n^2 = N_0 \cdot W$. While the signal from different antennas is correlated the noise samples are assumed to be uncorrelated.

III. Detector

In order to separate between the two hypotheses the detector computes a decision variable L and compares its value with a predefined decision threshold ν. The decision level is set in a way to satisfy either the detection or the false alarm requirement. In this section we outline the way to construct the decision variable L, while in the next section we derive the decision variable's distribution under the two hypotheses, $p(L|H_0), p(L|H_1)$. Such distributions are important to estimate in order to set accurately the decision threshold.

One can deduce from (2) and (3) that in the presence of noise only the square of the mean of main diagonal elements of $\mathbf{R}$ is nearly the same as the mean of the off-diagonal elements. On the other hand, this dependency does not exist in the case of signal plus noise. (This observation is elaborated in the next section). Motivated by this remark, we propose to make the separation of the two hypotheses based on the decision variable L computed as a difference between the mean of the off-diagonal elements and the mean of the squares of the main diagonal elements

$$L = \overline{r_{ij}} - \overline{r_{ii}^2}, \tag{5}$$

where the two means are computed from the matrix $\mathbf{R}$ as:

$$\overline{r_{ij}} = \frac{2}{N_a(N_a-1)} \sum_{i<j} r_{ij}, \tag{6}$$

$$\overline{r_{ii}^2} = \frac{1}{N_a} \sum_{i=j} |r_{ii}|^2 . \tag{7}$$

Notice that the matrix $\mathbf{R}$ is symmetric. In computing $\overline{r_{ij}}$ we consider only independent elements and therefore averaging only over the lower triangular of $\mathbf{R}$ elements. Hereafter, we mean by the averaging over off diagonal elements - averaging over lower triangular of the correlation matrix.

IV. Distributions

In order to derive the distribution of the decision variable L, we need to calculate the distributions of the covariance matrix elements.While calculating the distribution for the off-diagonal elements, we use the normal product distribution that is, the distribution of the product of two normal distributions. The first two moments of the product of two normally distributed random variables X_1, X_2 are [9]

$$\begin{aligned} E\left[X_1 X_2\right] &= \mu_1\mu_2 + \rho \cdot \sigma_1\sigma_2, \\ \sigma_{X_1X_2}^2 &= \mu_1^2\sigma_2^2 + \mu_2^2\sigma_1^2 + \sigma_1^2\sigma_2^2 + 2\rho\mu_1\mu_2\sigma_1\sigma_2 + \rho^2\sigma_1^2\sigma_2^2, \end{aligned} \tag{8}$$

where μ_1, μ_2 are the individual means, σ_1, σ_2 the standard deviations and ρ the correlation coefficient.

For later reference, we also give the dependency between the distribution parameters of a normal random variable $X = \mathcal{N}(\mu, \sigma)$ and the sum of the squares of k such variables $Y = \sum_k X^2$. As it is known Y follows chi-square distribution with k degrees of freedom. The mean and the variance of the distribution are [10]

$$\begin{aligned} E(Y) &= k\sigma^2 + k\mu^2, \\ \sigma_Y^2 &= 2k\sigma^4 + 4k\sigma^2\mu^2. \end{aligned} \tag{9}$$

By using (8) and (9), we follow the operations in (2) and outline how to evaluate the mean and variance of elements in $\mathbf{R}$. The corresponding results are collected into Table I. Below, we describe the operations for computing the terms of $\mathbf{R}$ and the corresponding transformations of the distributions.

A. Distribution of $\mathbf{R}$ elements under hypothesis H_0

Under hypothesis H_0 the measured samples contain only noise: $y_i(n) = \eta_i(n)$.

$p(r_{ii}|H_0)$: The main diagonal elements describe the noise power at the i^{th} antenna. The power of one complex noise sample has chi-square distribution with two degrees of freedom. The diagonal elements are computed as the mean of the powers of N such complex samples. Therefore, the main diagonal elements follow Chi-square distribution with $2N$ degrees of freedom.

$p(r_{ij}|H_0)$: The off-diagonal elements are computed based on the cross correlation of two noise sequences from different antennas. We modified the usual cross correlation computation by splitting it up to N/M blocks. Each block contains M terms. Every term is the product of two complex Gaussian random variables with zero mean. By setting $\rho = 0$ into (8), we find that the product follows the complex normal product distribution with zero mean and variance σ_n^4. By assuming a sufficiently large M, the sum of those terms can be approximated by a complex Gaussian distribution with zero mean and variance $M\sigma_n^4$. Since the sum approaches Gaussian distribution the power of it approaches chi-square distribution. Finally, the sum of N/M blocks is described by a chi-squared distribution with $2N/M$ degrees of freedom.

B. Distribution of $\mathbf{R}$ elements under hypothesis H_1

Under hypothesis H_1 the measured samples contain both noise and signal: $y_i(n) = s_i(n) + \eta_i(n)$. We assume a constant amplitude signal with complex power P_s.

$p(r_{ii}|H_1)$: The main diagonal elements describe the power of the samples collected by the i^{th} antenna. Because of the constant signal amplitude, the power of one complex sample follows non-central chi-square distribution with two degrees of freedom. Therefore, the main diagonal elements follow non-central chi-square distribution with $2N$ degrees of freedom. Due to the presence of signal the mean and variance of the distribution are different compared to those under H_0.

$p(r_{ij}|H_1)$: The off-diagonal elements are computed based on the cross correlation between samples collected by different antennas. According to (8) the multiplication of two samples follows the complex normal product distribution with mean P_s and variance $\sigma_n^4 + 2\sigma_n^2 P_s$. The cross correlation can be described by the convolution of M such distributions. For sufficiently large M we propose to approximate the convolution integral with the complex normal distribution. The mean and the variance of the normal approximation are MP_s and $M\sigma_n^4 + 2M\sigma_n^2 P_s$ respectively. One can deduce that the mean of the cross correlation is contributed by the first term of (3) while the variance from the last three terms. Finally, the sum of N/M cross correlation blocks is described by a chi-square distribution with $2N/M$ degrees of freedom.

To sum up, the distribution of the diagonal elements under both hypotheses follows the chi-square distribution. Regarding the off-diagonal elements their exact distribution is difficult to derive. Based on the central limit theorem, we propose to approximate it by the chi-square distribution. In order to make the central limit theorem applicable we assume that the sum of a large number of normal product distributions can be well approximated by the normal distribution. Such an approximation has been verified in [9].

V. Distribution of decision variable

By comparing the terms in Table (I) we notice that under H_0 the mean of the off-diagonal elements is the square of the mean of the main diagonal elements. Such relation does not exist under H_1. The proposed decision variable in (5) takes advantage of this dependency. Firstly, we compute the average over the off-diagonal elements $(\overline{r_{ij}})$. Secondly, we square the main diagonal elements $(\overline{r_{ii}^2})$ and substract the average of the squares from the off-diagonal average. Because of the substraction, the decision variable L would have a mean located close to zero under H_0 but away from zero under H_1.

By computing the average over the off-diagonal elements the quality of our estimates is improved. Recall that the off-diagonal elements have chi-square distribution and all of them have the same mean and variance. Their average would also follow chi-square distribution with the same mean (as in rows 2 and 4 of Table (I)) and variance scaled down with the normalization factor.

The diagonal elements are identically chi-square distributed. Because of large degrees of freedom of this chi-square distribution, we can approximate it by the normal distribution. After squaring (now assumed to be normally distributed) main diagonal element we can compute new mean and variance of the corresponding chi-square distribution. By summing over squared main diagonal elements we increase the degrees of freedom of the chi-square distribution. The normalization by the number of elements keeps the mean of the sum to be same as the mean of one element and the variance is reduced by the normalization factor (number of antennas).

After making those operations for diagonal elements under H_0 is (use the terms in first row of Table I)

$$\begin{aligned} E[\overline{r_{ii}^2}|H_0] &= \sigma_n^4 + \tfrac{\sigma_n^4}{N\cdot N_a}, \\ Var[\overline{r_{ii}^2}|H_0] &= 2\tfrac{\sigma_n^8}{N\cdot N_a}(\tfrac{1}{N\cdot N_a}+2) \end{aligned} \quad (10)$$

and under H_1 is (use the terms in the third row of Table I)

$$\begin{aligned} E[\overline{r_{ii}^2}|H_1] &= (\sigma_n^2+p_s)^2 + \tfrac{1}{N\cdot N_a}(\sigma_n^2+\sigma_n^2 p_s), \\ Var[\overline{r_{ii}^2}|H_1] &= \tfrac{2\left(\sigma_n^4+2P_s\sigma_n^2\right)^2}{(N\cdot N_a)^2} + \tfrac{4\left(\sigma_n^4+2P_s\sigma_n^2\right)\left(\sigma_n^2+P_s\right)^2}{NN_a}. \end{aligned} \quad (11)$$

To sum up, both random variables $\bar{r_{ij}}$ and $\bar{r_{ii}^2}$ are chi-square distributed. The distribution of the difference of these variables (5) can be therefore described by the difference of their means and by the sum of their variances.

Under H_0 the decision variable distribution $p(L|H_0)$ has the mean and variance

$$\begin{aligned} \mu_{H_0} &= -\tfrac{\sigma_n^4}{N\cdot N_a}, \\ \sigma_{H_0}^2 &= \tfrac{M\sigma_n^4}{N\cdot\frac{N_a(N_a-1)}{2}} + \tfrac{2\sigma_n^8}{N\cdot N_a}(\tfrac{1}{N\cdot N_a}+2). \end{aligned} \quad (12)$$

Under H_1 the decision variable distribution $p(L|H_1)$ has the mean and variance

$$\begin{aligned} \mu_{H_1} &= (M-1)\,P_s^2 - \tfrac{\sigma_n^4+2P_s\sigma_n^2}{N_a\cdot N}, \\ \sigma_{H_1}^2 &= \tfrac{2}{N_a(N_a-1)}\tfrac{M\left(\sigma_n^4+2P_s\sigma_n^2\right)^2+2M^2P_s^2\left(\sigma_n^4+2P_s\sigma_n^2\right)}{N} \\ &+ \tfrac{2\left(\sigma_n^4+2P_s\sigma_n^2\right)^2}{(N_a\cdot N)^2} + \tfrac{4\left(\sigma_n^4+2P_s\sigma_n^2\right)\left(\sigma_n^2+P_s\right)^2}{N_aN}. \end{aligned} \quad (13)$$

VI. Noise uncertainty

Errors in the noise level estimation can significantly reduce the radiometer's performance [3]. For real system implementation we need to guarantee the detection performance even in the case of certain receiver imperfections. Our algorithm attempts to bypass the noise uncertainty issue by removing the noise impact: substraction between squared diagonal and off-diagonal elements. While for sufficiently high amount of collected samples the proposed detector could reach any required detection target the noise uncertainty still degrades the test performance. Next, we show how to incorporate the receiver's imperfections into the system design.

Usually the noise variance is not known precisely but only within some tolerance limits $\pm\Delta$ dB. The actual noise level lies in the interval $\left[\frac{1}{\delta}\cdot\sigma_n^2, \delta\cdot\sigma_n^2\right]$ where $\delta = 10^{\Delta/10}$. A common approach to model the noise uncertainty is to use robust statistics and design the system for the worst case scenario [11]. The new decision threshold can be computed by incorporating the noise uncertainty into (12) and (13) .

We see in (12) and (13) that the noise power contributes to the mean and variance of the test statistic under both hypotheses. A simple analysis indicates that the means are closest to each other when the noise uncertainty is at its

TABLE I
MEAN AND VARIANCE OF TERMS IN MATRIX $\mathbf{R}$ IN CASE OF PURE NOISE: H_0, OR SIGNAL AND NOISE: H_1

	mean	variance
$p(r_{ii}\|H_0)$	σ_n^2	$\frac{\sigma_n^4}{N}$
$p(r_{ij}\|H_0)$	σ_n^4	$\frac{M\cdot\sigma_n^8}{N}$
$p(r_{ii}\|H_1)$	$\sigma_n^2 + P_s$	$\frac{\sigma_n^4+2\sigma_n^2 P_s}{N}$
$p(r_{ij}\|H_1)$	$2P_s\sigma_n^2 + \sigma_n^4 + MP_s^2$	$\frac{M}{N}\left(\left(2P_s\sigma_n^2+\sigma_n^4\right)^2 + 2M\left(2P_s\sigma_n^2+\sigma_n^4\right)P_s^2\right)$

maximum level $\sigma_u^2 = \delta \cdot \sigma_n^2$. At this level also the variances are maximized. We can conclude that the worst case detector performance occurs when the noise power has the maximum value in the uncertainty interval. Based on this argument we can upper bound the detection error by using in the detection threshold computation the noise level σ_u^2.

VII. NUMERICAL EXAMPLES

Because of the sufficient number of samples we approximate the distribution of L with a normal distribution

$$\begin{aligned} p(L|H_0) &\approx \mathcal{N}\left(\mu_{H_0}, \sigma_{H_0}^2\right), \\ p(L|H_1) &\approx \mathcal{N}\left(\mu_{H_1}, \sigma_{H_1}^2\right), \end{aligned} \tag{14}$$

with mean and variance as in (12) and (13).

In practical detectors the Gaussian approximation simplifies significantly the computation of the decision variable and due to that the implementation of the detector [12].

By using Gaussian distribution, the two computed moments are sufficient for estimating the detector's performance. The computed ROC curves follow closely the simulation results (Fig. 1). The discrepancies arise at the tails of the distributions where the Gaussian approximation is not very good.

So far, the common target in covariance-based detector algorithms has been to constrain the false alarm probability. In spectrum sensing context the constraint has to be imposed on the detection probability. By using (14) the false alarm and the detection probability are:

$$P_f = P(H_1|H_0) = Q\left(\frac{\nu - \mu_{H_0}}{\sigma_{H_0}}\right), \tag{15}$$

$$P_d = P(H_1|H_1) = Q\left(\frac{\nu - \mu_{H_1}}{\sigma_{H_1}}\right), \tag{16}$$

where $Q(\cdot)$ is the tail probability of the zero-mean and unit-variance Gaussian random variable.

As it is expected, the computed miss probability ($P_m = 1 - P_d$) matches very well the simulation results when there is perfect noise estimation (Fig. 2). In the presence of noise level uncertainty we propose to design the detector based on the worst case scenario. In section VI we argue that the worst detection performance occurs when the noise variance is maximized inside the uncertainty interval. Therefore, we compute the decision threshold by using the maximum possible value of the noise variance. The simulation should describe the realistic system where the noise variance can take any value within the uncertainty interval. We do that by selecting the noise variance to be uniformly distributed between its minimum and maximum possible value. Because of that the simulated miss probability would be always smaller than the design constraint imposed on the miss probability (Fig. 2). The simulations indicate that the threshold is set rather pessimistically but it guarantees the required performance.

Selecting the decision threshold ν based on the detection probability (or miss probability P_m) does not allow to limit the false alarm. However, given the detection probability we could estimate the corresponding false alarm (Fig. 3). If the noise level is known exactly the equations match the simulations very well. In case of noise uncertainty the false alarm computation does not guarantee the upper bound for low SNR values. However the prediction is very close to simulated results and can be used as a basis for analytical studies.

Notice also an interesting behavior when designing the system under noise uncertainty. If we design the decision threshold for a fixed false alarm (or miss probability) we cannot guarantee the lower bound for the detection probability (or false alarm correspondingly) for all the ranges of SNRs (Fig. 4). For instance, consider the case when the target false alarm is set such that it captures more than half of the $p(L|H_1)$ distribution. The detection probability becomes less than 0.5 (the decision threshold is set in the "other" side of the $p(L|H_1)$). Because of that the computed threshold becomes upper bound for P_d. For the SNR values where the $P_d > 0.5$ the computed probability is again a lower bound. The discrepancy between the computed and the simulated results is because the computaion gives a bound and the simulations use uniform noise variance distribution.

VIII. CONCLUSION

In this paper we proposed a detection algorithm for a sensor with multiple antennas. The detection algorithm is constructed such that an analytical prediction of its performance can be derived. While deriving the analytical model of the detector we show how to use constructively the fact that the detector could collect a high amount of samples.

The simulations show that the computed performance values match very well the simulation results. Another benefit of the analysis is the ability to compute not only the false alarm but also the detection probability. For spectrum sensing applications, the detection probability is more important

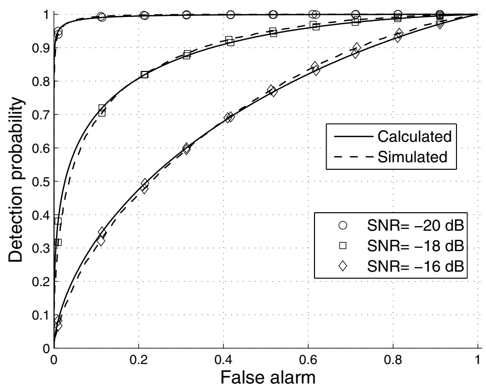

Fig. 1. Computed and simulated ROC curves of the dectector with 4 antennas. The system parameters are: $N_a = 4$, $N = 10000$.

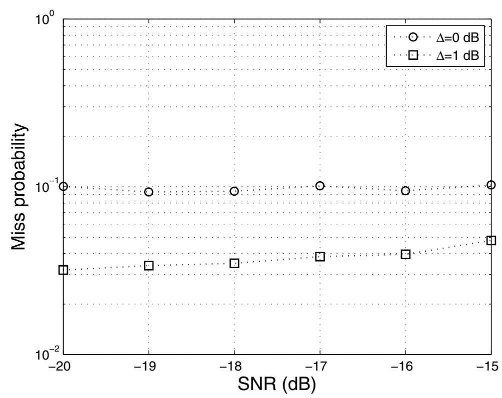

Fig. 2. Simulated miss probability. The decision threshold is selected such that $P_m = 0.1$. The system parameters are: $N_a = 4$, $N = 10000$.

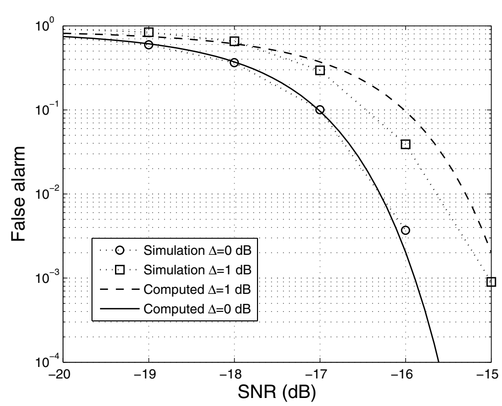

Fig. 3. False alarm if the miss probability is constrained such that $P_m = 0.1$. The system parameters are: $N_a = 4$, $N = 10000$.

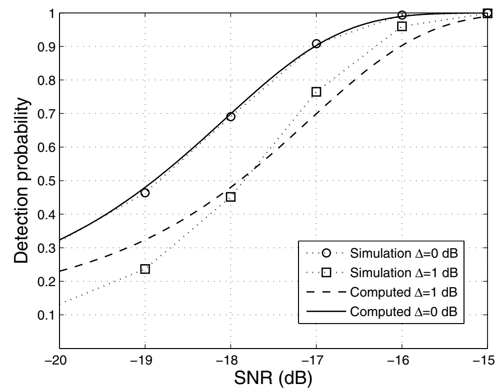

Fig. 4. Detection probability if the false alarm is constrained such that $P_f = 0.1$. The system parameters are $N_a = 4$ $N = 10000$.

quantity compared to the false alarm. In addition, the proposed detection algorithm attempts to compensate for the noise level uncertainty at the detector. Our analytical model allows to find a decision threshold that incorporates the noise uncertainty. The computed decision threshold includes a margin that can be used for coping with the receiver imperfections.

The proposed approach can be utilized for the analysis of more complex detectors. In the future research we plan to extend the analysis in order to incorporate channels with independently fading signals between the different antennas.

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