Proceedings of the 2009 IEEE International Conference on Systems, Man, and Cybernetics
San Antonio, TX, USA - October 2009

Stable Fuzzy Self–Tuning PID Control of Robot Manipulators

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***Abstract*—In this paper we present a fuzzy self-tuning algorithm to select the Proportional, Integral and Derivative gains of a PID controller according to the actual state of a robotic manipulator. The stability via Lyapunov theory for the closed loop control system is also analyzed and shown that is locally asymptotically stable for a class of gain matrices depending on the manipulator states. This feature increases the potential of the PID control scheme to handle practical constraints in actual robots such as presence of actuators with limited torque capabilities. Experimental results on a two degrees of freedom robot arm shown the usefulness of the proposed approach.**

***Index Terms*—Self-tuning PID, stability, Robots, Fuzzy control.**

I. INTRODUCTION

The conventional PID controllers have been extensively used in industrial robots due to their design simplicity, and effectiveness. Although the PID control is a popular strategy, it lacks of a global asymptotic stability proof [1], [2]. Regarding the stability of the PID controller, the stability proofs shown until now are only valid in a local sense [3] or, in the best of the cases, in a semiglobal sense [4], [5]. It is worth noting that the stability analysis of the closed loop system driven by the PID control is usually carried out by considering a constant selection of the controller gains. This characteristic may limit the application of this controller in case where in addition to asymptotic stability, it is required to maintain performance for the control system. In order to get high performance or broach real constraints of actual manipulators such as actuator capabilities, it may be necessary to have variable gains for these controllers [6]. Several practical techniques have been suggested to choose adequate values for the controller gains which depend on the robot configuration such as gain scheduling, fuzzy control and neural networks approach [7], [8], [9]. Fuzzy control is a control theory arising in recent years. A review of the specialized literature during the last two decades reveals a remarkable amount and variety of fuzzy adaptive realizations. Our approach is inspired by a previous work presented in [6] and [10]. The structure of the fuzzy controller used in [6] is based on fuzzy tuning PD algorithms to select the Proportional and Derivative gains according to the actual position errors. In this paper we use the potential of fuzzy self-tuning schemes in order to design a methodology for on-line selection of the proportional, derivative and integral gains for the PID controller. In addition, to guarantee Lyapunov stability for the closed-loop system, this approach ensures also practical performances beyond the standard non-fuzzy scheme. Specifically, the proposed control scheme solves the regulation problem of robot manipulators constrained to deliver torques inside prescribed limits according to the actuator capabilities. In order to illustrate the performance of this control scheme, we present experimental results on a vertical two degrees of freedom direct-drive robot. Also, in this paper is shown that the proposed tuning method yields a locally asymptotically stable closed-loop system not only for constant positive definite gain matrices, but also for a class of manipulator state dependent gain matrices. This is a theoretical result with useful implications to handle real constraint of robot manipulators such as torque capability limitations of their actuators.

Throughout this paper, we use the notation $\lambda_{\rm m}\{A\}$ and $\lambda_{\rm M}\{A\}$ to indicate the smallest and largest eigenvalues, respectively, of a symmetric positive definite bounded matrix $A(\boldsymbol{x})$, for any $\boldsymbol{x} \in \mathbb{R}^n$. The norm of vector $\boldsymbol{x}$ is defined as $\|\boldsymbol{x}\| = \sqrt{\boldsymbol{x}^T\boldsymbol{x}}$.

II. ROBOT DYNAMICS

In the absence of friction and other disturbances, the dynamics of a serial n-link rigid robot can be written as [11]:

$$M(\boldsymbol{q})\ddot{\boldsymbol{q}} + C(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{\tau} \qquad (1)$$

where $\boldsymbol{q}$ is the $n \times 1$ vector of joint displacements, $\dot{\boldsymbol{q}}$ is the $n \times 1$ vector of joint velocities, $\boldsymbol{\tau}$ is the $n \times 1$ vector of applied torques, $M(\boldsymbol{q})$ is the $n \times n$ symmetric positive definite manipulator inertia matrix, $C(\boldsymbol{q},\dot{\boldsymbol{q}})$ is the $n \times n$ matrix of centripetal and Coriolis torques, and $\boldsymbol{g}(\boldsymbol{q})$ is the $n \times 1$ vector of gravitational torques obtained as the gradient of the robot potential energy $\mathcal{U}(\boldsymbol{q})$, i.e. $\boldsymbol{g}(\boldsymbol{q}) = \frac{\partial \mathcal{U}(\boldsymbol{q})}{\partial \boldsymbol{q}}$.

III. PID CONTROL WITH NONLINEAR GAINS

In this section we introduce a new PID controller whose main feature is that stability hold even though the parameters depend on the robot state. A generalization of the classical linear PID controller can be obtained by allowing nonlinear proportional $K_p(\tilde{\boldsymbol{q}})$, integral $K_i(\boldsymbol{\omega})$ and derivative $K_v(\tilde{\boldsymbol{q}})$ gain matrices as function matrices of the robot configuration. This leads to the following proposed control law

$$\boldsymbol{\tau} = K_p(\tilde{\boldsymbol{q}})\tilde{\boldsymbol{q}} - K_v(\tilde{\boldsymbol{q}})\dot{\boldsymbol{q}} + K_i(\boldsymbol{\omega})\int_0^t \tilde{\boldsymbol{q}}(\sigma)\ d\sigma \qquad (2)$$

where $K_p(\tilde{\boldsymbol{q}}),\ K_v(\tilde{\boldsymbol{q}})$ and $K_i(\boldsymbol{\omega})$ are positive definite diagonal $n \times n$ matrices, whose entries are denoted by $k_{p_i}(\tilde{q}_i)$, $k_{v_i}(\tilde{q}_i)$ and $k_{i_i}(\omega_i)$ respectively, and $\tilde{\boldsymbol{q}} = \boldsymbol{q}_d - \boldsymbol{q}$ denotes the position error vector and $\dot{\boldsymbol{\omega}} = \alpha\tilde{\boldsymbol{q}} - \dot{\boldsymbol{q}}$, with $\alpha > 0$.

For stability analysis purposes, the control law (2) can be rewritten as

$$\boldsymbol{\tau} = K_p^{'}(\tilde{\boldsymbol{q}})\tilde{\boldsymbol{q}} - K_v(\tilde{\boldsymbol{q}})\dot{\boldsymbol{q}} + K_i^{'}(\boldsymbol{\omega})\int_0^t (\alpha\tilde{\boldsymbol{q}}(\sigma) + \dot{\tilde{\boldsymbol{q}}}(\sigma))\ d\sigma \quad (3)$$

where: $K_p^{'}(\tilde{\boldsymbol{q}}) = K_p(\tilde{\boldsymbol{q}}) - \frac{K_i(\boldsymbol{\omega})}{\alpha}$, $K_i^{'}(\boldsymbol{\omega}) = \frac{K_i(\boldsymbol{\omega})}{\alpha}$ with: $\alpha > \frac{\lambda_{\mathrm{M}}\{K_i\}}{\lambda_{\mathrm{m}}\{K_p\}}$.
This latter condition ensures that $K_p^{\prime}(\tilde{\boldsymbol{q}}) > 0$. The α constant is introduced in order to make easier the stability analysis and will be used as a parameter of the Lyapunov function.

Assumption 1. There exist positive constants k_{pl_i}, k_{pu_i}, k_{vl_i}, k_{vu_i}, k_{il_i} and k_{iu_i} such that Lemma 1 introduced by [14] can be applied. That is:

$$\frac{1}{2}\tilde{\boldsymbol{q}}^T K_{pu}^{'}\tilde{\boldsymbol{q}} \geq \int_0^{\tilde{\boldsymbol{q}}} \boldsymbol{\xi} K_p^{'}(\boldsymbol{\xi})\, d\boldsymbol{\xi} \geq \frac{1}{2}\tilde{\boldsymbol{q}}^T K_{pl}^{'}\tilde{\boldsymbol{q}} \qquad (4)$$

$$\frac{1}{2}\tilde{\boldsymbol{q}}^T K_{vu}\tilde{\boldsymbol{q}} \geq \int_0^{\tilde{\boldsymbol{q}}} \boldsymbol{\xi} K_v(\boldsymbol{\xi})\, d\boldsymbol{\xi} \geq \frac{1}{2}\tilde{\boldsymbol{q}}^T K_{vl}\tilde{\boldsymbol{q}} \qquad (5)$$

$$\frac{1}{2}\boldsymbol{\omega}^T K_{iu}^{'}\boldsymbol{\omega} \geq \int_0^{\boldsymbol{\omega}} \boldsymbol{\xi} K_i^{'}(\boldsymbol{\xi})\, d\boldsymbol{\xi} \geq \frac{1}{2}\boldsymbol{\omega}^T K_{il}^{'}\boldsymbol{\omega} \qquad (6)$$

where $K_{pu}^{'}$, $K_{pl}^{'}$, K_{vu}, K_{vl}, $K_{iu}^{'}$, $K_{il}^{'}$ are $n \times n$ constant positive definite diagonal matrices whose entries are $k_{pu_i}^{'}, k_{pl_i}^{'}, k_{vu_i}, k_{vl_i}, k_{iu_i}^{'}, k_{il_i}^{'}$ respectively, with $i = 1, 2, ...n$.

Assumption 2. In an ϵ-neighborhood $N(\boldsymbol{\omega}, \epsilon) = \{\boldsymbol{\omega} \in \mathbb{R}^n : \|\ \boldsymbol{\omega}\| < \epsilon\}$ of $\boldsymbol{\omega} = 0$, the integral gain matrix is constant, that is, $K_i^{'}(\boldsymbol{\omega}) = K_{il}^{'}$, where $K_{il}^{'} \in \mathbb{R}^{n\times n}$ is a diagonal positive definite constant matrix.

The practical usefulness of this control strategy will become clear later when a fuzzy self–tuning algorithm will be introduced.

The closed- loop system is obtained substituting the control law (3) into the robot dynamics (1). This can be written as

$$\frac{d}{dt}\begin{bmatrix}\tilde{\boldsymbol{q}}\\ \dot{\boldsymbol{q}}\\ \\ \dot{\boldsymbol{\omega}}\end{bmatrix} = \begin{bmatrix} -\dot{\boldsymbol{q}} \\ M^{-1}\Big[K_p^{'}(\tilde{\boldsymbol{q}})\tilde{\boldsymbol{q}} - K_v(\tilde{\boldsymbol{q}})\dot{\boldsymbol{q}} - C(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} - \boldsymbol{g}(\boldsymbol{q}) \\ + K_i^{'}(\boldsymbol{\omega})\boldsymbol{\omega} + \boldsymbol{g}(\boldsymbol{q}_d)\Big] \\ \alpha\,\tilde{\boldsymbol{q}} - \dot{\boldsymbol{q}}\end{bmatrix} \qquad (7)$$

where $\boldsymbol{\omega}$ is defined as:

$$\boldsymbol{\omega}(t) = \int_0^t \left[\alpha\,\tilde{\boldsymbol{q}}(\sigma) + \dot{\tilde{\boldsymbol{q}}}(\sigma)\right]\, d\sigma - K_{il}^{'-1}\boldsymbol{g}(\boldsymbol{q}_d)$$

and we have used the Assumption 2 in such a way that (7) becomes an autonomous nonlinear differential equation whose origin $[\,\tilde{\boldsymbol{q}}^T \quad \dot{\boldsymbol{q}}^T \quad \boldsymbol{\omega}^T\,]^T = \boldsymbol{0} \in \mathbb{R}^{3n}$ is an equilibrium point.

IV. Stability analysis

In this section we show that the stability also holds for a class of nonconstant state-depending proportional, integral and derivative gain matrices. More specifically, consider the control law (3) corresponding to a PID control scheme with nonlinear gain matrices

A. Lyapunov function candidate

In order to study stability of equilibrium point obtained above we propose the following Lyapunov function candidate:

$$\begin{aligned} V(\tilde{\boldsymbol{q}},\dot{\boldsymbol{q}},\boldsymbol{\omega}) \quad = \quad & \int_{\boldsymbol{0}}^{\tilde{\boldsymbol{q}}} \boldsymbol{\xi}^T K_p^{'}(\boldsymbol{\xi})\, d\boldsymbol{\xi} - \mathcal{U}(\boldsymbol{q}_d) + \mathcal{U}(\boldsymbol{q}) \\ & + \boldsymbol{g}(\boldsymbol{q}_d)^T\tilde{\boldsymbol{q}} + \frac{1}{2}\dot{\boldsymbol{q}}^T M(\boldsymbol{q})\dot{\boldsymbol{q}} \\ & -\alpha\tilde{\boldsymbol{q}}^T M(\boldsymbol{q})\dot{\boldsymbol{q}} + \alpha\int_{\boldsymbol{0}}^{\tilde{\boldsymbol{q}}} \boldsymbol{\xi}^T K_v(\boldsymbol{\xi}), d\boldsymbol{\xi} \\ & + \int_{\boldsymbol{0}}^{\omega} \boldsymbol{\xi}^T K_i^{'}(\boldsymbol{\xi})\, d\boldsymbol{\xi} \end{aligned} \qquad (8)$$

Under Assumption 1 we have $\alpha\int_{\boldsymbol{0}}^{\tilde{\boldsymbol{q}}} \boldsymbol{\xi}^T K_v(\boldsymbol{\xi})\, d\boldsymbol{\xi} - \frac{\alpha^2}{2}\tilde{\boldsymbol{q}}^T M(\boldsymbol{q})\tilde{\boldsymbol{q}} \geq \frac{\alpha}{2}\tilde{\boldsymbol{q}}^T\left[K_{vl} - \alpha M(\boldsymbol{q})\right]\tilde{\boldsymbol{q}}$. Also note that under Assumption 1 the function in (8) can be lower bounded as:

$$\begin{aligned} V(\tilde{\boldsymbol{q}},\dot{\boldsymbol{q}},\boldsymbol{\omega}) \quad \geq \quad & V_a(\tilde{\boldsymbol{q}},\dot{\boldsymbol{q}},\boldsymbol{\omega}) = \frac{1}{2}\tilde{\boldsymbol{q}}^T K_{pl}^{'}\tilde{\boldsymbol{q}} - \mathcal{U}(\boldsymbol{q}_d) \\ & + \mathcal{U}(\boldsymbol{q}) + \frac{1}{2}\left[-\dot{\boldsymbol{q}} + \alpha\tilde{\boldsymbol{q}}\right]^T M(\boldsymbol{q})\left[-\dot{\boldsymbol{q}} + \alpha\tilde{\boldsymbol{q}}\right] \\ & + \frac{\alpha}{2}\tilde{\boldsymbol{q}}^T\left[K_{vl} - \alpha M(\boldsymbol{q})\right]\tilde{\boldsymbol{q}} \\ & + \frac{1}{2}\boldsymbol{\omega}^T K_{il}^{'}\boldsymbol{\omega} + \boldsymbol{g}(\boldsymbol{q}_d)^T\tilde{\boldsymbol{q}} \end{aligned} \qquad (9)$$

Now, we will give sufficient conditions to make $V_a(\tilde{\boldsymbol{q}},\dot{\boldsymbol{q}},\boldsymbol{\omega})$ be a positive definite function.

More flexibility is possible in choice of the gains if conditions for stability are expressed in terms of conditions at individual joints instead of expressing them as a single condition relating the largest and the smallest eigenvalues, or norms, of gain and parameter matrices [16], the key is the use of a different criterion to ensure positive definiteness of a matrix: a symmetric matrix is positive definite if it is strictly diagonally dominant and its diagonal entries are positive [12]. In agreement with [14] the Lyapunov function candidate (8) is a globally positive definite function under the condition:

$$k_{pl_i} > \sum_{j=1}^{n} \max_q\left|\frac{\partial g_i(q)}{\partial q_j}\right| \qquad (10)$$

and α chosen in such way that it satisfies:

$$\frac{k_{vl_i}}{\sum_{j=1}^{n}\max_q|M_{ij}(q)|} > \alpha > \frac{k_{il_i}}{k_{pl_i} - \sum_{j=1}^{n}\max_q|\partial g_i(q)/\partial q_j|} \qquad (11)$$

Thus, $V(\tilde{\boldsymbol{q}},\dot{\boldsymbol{q}},\omega)$ introduced in (8) is a globally positive definite and radially unbounded function.

B. Time derivative of the Lyapunov function candidate

The time derivative of the Lyapunov function candidate (8) along the trajectories of the closed loop equation (7) is

$$\begin{aligned}\dot V(\tilde{\boldsymbol q},\dot{\boldsymbol q},\omega) \;=\; & -\dot{\boldsymbol q}^T[K_v(\tilde{\boldsymbol q})-\alpha M(\boldsymbol q)]\dot{\boldsymbol q}-\alpha\tilde{\boldsymbol q}^T C(\boldsymbol q,\dot{\boldsymbol q})^T\dot{\boldsymbol q}\\ & -\alpha\tilde{\boldsymbol q}^T[K_p(\tilde{\boldsymbol q})-K_i(\boldsymbol\omega)/\alpha]\tilde{\boldsymbol q}\\ & -\alpha\tilde{\boldsymbol q}^T[\boldsymbol g(\boldsymbol q_d)-\boldsymbol g(\boldsymbol q)]\end{aligned}$$

where we have used the Leibnitz' rule for differentiation of integrals. Again, following similar steps given in [16], note that we can write $K_v(\tilde{\boldsymbol q}) = K_{v\rho}(\tilde{\boldsymbol q}) + K_{v\sigma}(\tilde{\boldsymbol q})$, where $K_{v\rho}(\tilde{\boldsymbol q})$ and $K_{v\sigma}(\tilde{\boldsymbol q})$ are two diagonal positive definite matrices, because $K_v(\tilde{\boldsymbol q})$ is a diagonal positive definite matrix. Hence, we can write $\dot{\boldsymbol q}^T[K_v(\tilde{\boldsymbol q})-\alpha M(\boldsymbol q)]\dot{\boldsymbol q} = \dot{\boldsymbol q}^T K_{v\sigma}(\tilde{\boldsymbol q})\dot{\boldsymbol q} + \dot{\boldsymbol q}^T[K_{v\rho}(\tilde{\boldsymbol q}) - \alpha M(\boldsymbol q)]\dot{\boldsymbol q}$. Suppose that $K_{v\rho}(\tilde{\boldsymbol q})$ is chosen such that $K_{v\rho}(\tilde{\boldsymbol q}) - \alpha M(\boldsymbol q)$ is positive definite, i.e.:

$$\frac{k_{v\rho l_i}}{\sum_{j=1}^n \max_q |M_{ij}(q)|} > \alpha, \qquad i=1,\cdots,n, \tag{12}$$

where $k_{v\rho l_i}$ is the i-th entry of a positive definite diagonal matrix $K_{v\rho l}$ satisfying $K_{v\rho}(\tilde{\boldsymbol q}) > K_{v\rho l}$ for all $\tilde{\boldsymbol q}\in\mathbb{R}^n$. Hence, we can write:

$$\dot{\boldsymbol q}^T[K_v(\tilde{\boldsymbol q})-\alpha M(\boldsymbol q)]\dot{\boldsymbol q} \ge \dot{\boldsymbol q}^T K_{v\rho}(\tilde{\boldsymbol q})\dot{\boldsymbol q} \ge \lambda_{\rm m}\{K_{v\rho_l}\}\,\|\dot{\boldsymbol q}\|^2$$

Following the same steps as in [16], we have $-\alpha[\boldsymbol g(\boldsymbol q_d) - \boldsymbol g(\boldsymbol q)]^T\tilde{\boldsymbol q} = -\alpha\tilde{\boldsymbol q}^T[\partial\boldsymbol g(\boldsymbol\vartheta)/\partial\boldsymbol\vartheta]|_{\boldsymbol\vartheta=\boldsymbol\xi}\ \tilde{\boldsymbol q}$, for some $\boldsymbol\xi$ belonging to the line that joins $\boldsymbol q_d$ and $\boldsymbol q$. Hence, according to these facts as well as the properties of the dynamic model [13] we find the following bound:

$$\begin{aligned}\dot V(\tilde{\boldsymbol q},\dot{\boldsymbol q},\omega) \le\; & -\;[\lambda_{\rm m}\{K_{v\rho l}\}-\alpha k_c\,\|\tilde{\boldsymbol q}\|]\,\|\dot{\boldsymbol q}\|^2\\ & -\;\alpha\tilde{\boldsymbol q}^T\left[K_p(\tilde{\boldsymbol q})-\frac{K_i(\boldsymbol\omega)}{\alpha}+\frac{\partial\boldsymbol g(\boldsymbol\vartheta)}{\partial\boldsymbol\vartheta}|_{\boldsymbol\vartheta=\boldsymbol\xi}\right]\ \tilde{\boldsymbol q}\end{aligned} \tag{13}$$

The matrix $K_p(\tilde{\boldsymbol q}) - K_i(\boldsymbol\omega)/\alpha + \frac{\partial\boldsymbol g(\boldsymbol\vartheta)}{\partial\boldsymbol\vartheta}|_{\boldsymbol\vartheta=\boldsymbol\xi}$ is positive definite [14]. Let us define η as the radius of an open ball D around the origin of the state space:

$$D = \{\boldsymbol x := [\tilde{\boldsymbol q}^T\ \dot{\boldsymbol q}^T\ \boldsymbol\omega^T]^T \in \mathbb{R}^{3n} : \|\boldsymbol x\| < \eta\}$$

in which $\dot V(\tilde{\boldsymbol q},\dot{\boldsymbol q},\boldsymbol z)$ is negative semidefinite. Using η instead of $\|\dot{\boldsymbol q}\|$ in (13) we can write the conditions for negative semidefiniteness of $\dot V(\tilde{\boldsymbol q},\dot{\boldsymbol q},\boldsymbol z)$ in the domain D as:

$$\frac{\lambda_{\rm m}\ \{K_{v\rho}(\tilde{\boldsymbol q})\}}{k_c\eta} > \alpha > \frac{k_{il_i}}{k_{pl_i} - \sum_{j=1}^n \max_q |\partial g_i(q)/\partial q_j|}, \tag{14}$$

Finally, by invoking LaSalle invariance principle ensures local asymptotic stability provided that (10), (11), (12), and (14) are satisfied.

So far, we have proved the following:

Proposition 1. Consider the robot dynamical model (1) together with the control law (3). Under conditions (10), (11), (12), (14), there always exist appropriate proportional $K_p(\tilde{\boldsymbol q})$, derivative $K_v(\tilde{\boldsymbol q})$ and integral $K_i(\boldsymbol\omega)$ gain matrices, such that the equilibrium $\left[\tilde{\boldsymbol q}^T\ \ \dot{\boldsymbol q}^T\ \ \boldsymbol\omega^T\right]^T = \boldsymbol 0 \in \mathbb{R}^{3n}$ of the closed-loop system (7) is locally asymptotically stable. A Lyapunov function to prove this is given by (8).

V. Fuzzy approach for self–tuning the PID controller gains

Fuzzy logic is a suitable approach as a mechanism to determine the nonlinear gains of the PID control scheme according to previous practical specifications. This is because the input–output characteristics of fuzzy logic systems could be easily suited in order to fulfill the stability requirements established in Proposition 1, namely

1) $k_{pl_i}(\tilde q_i) > \sum_{j=1}^n \max_q \left|\frac{\partial g_i(\boldsymbol q)}{\partial q_j}\right|$ for all $\tilde q_i \in \mathbb{R}$, and $i = 1,\cdots,n$
2) $k_{il_i}(\omega_i) > 0$ for all $\omega_i \in \mathbb{R}$, and $i = 1,\cdots,n$
3) $k_{vl_i}(\tilde q_i) > \frac{\sum_{j=1}^n \max_q |M_{ij}(q)| K_{ii}}{K_{Pi} - \sum_{j=1}^n \max_q |\partial g_i(q)/\partial q_j|}$ for all $\tilde q_i \in \mathbb{R}$, and $i = 1,\cdots,n$

In order to tune the proportional gains $k_{pl_i}(\tilde q_i)$, the integral gain $k_{il_i}(\omega_i)$ and the derivative gains $k_{vl_i}(\tilde q_i)$ according to the state $|\tilde q_i|$ and $|\omega_i|$, in this paper we define one conceptual Fuzzy Logic Tuner (FLT). In summary, $3n$ elementary FLT will be involved in computation of n proportional gains $k_{pi}(\tilde q_i)$, n integral gains $k_{ii}(\omega_i)$ and n derivative gains $k_{vi}(\tilde q_i)$.

A. Basic fuzzy logic tuner

Having in mind the real–time implementation of the fuzzy self–tuning algorithm, a quite simple approach to design the FLT has been adopted [10]. Let the conceptual FLT have one input $|x|$ and the corresponding output y. The FLT can be seen as a static mapping H defined by

$$\begin{aligned} H:\ \ \mathbb{R}_+ &\to \mathbb{R}\\ |x| &\mapsto y\end{aligned}$$

The universes of discourse of $|x|$, and y are partitioned into three fuzzy sets: B (Big), M (Medium), and S (Small) with each attribute being described by a membership function. We shall employ trapezoidal membership functions for input variables and singleton for output variables. In order to simplify notation, let us use the following convention. With reference to Fig. 1, the corresponding Small, Medium and Big membership functions for the input variable x are denoted respectively by

$$\begin{aligned}&\mu^S(|x|;p_1,p_2),\\ &\mu^M(|x|;p_1,p_2,p_3,p_4),\\ &\mu^B(|x|;p_3,p_4).\end{aligned}$$

From Fig. 1, it is worth noting that the above chosen fuzzy sets can be seen as real function of the input $|x|$, which are themselves related by the following expressions:

$$\begin{aligned}\mu^M(|x|;p_1,p_2,p_3,p_4) = 1-\mu^S(|x|;p_1,p_2), &\quad \text{for } |x|<p_3\\ \mu^B(|x|;p_3,p_4) = 1-\mu^M(|x|;p_1,p_2,p_3,p_4), &\quad \text{for } |x|\ge p_3.\end{aligned}$$

With reference to the output variable y (see Fig. 2), the singleton membership functions corresponding to Small, Medium and Big are represented by $\mu_y^S(\cdot;k_1)$, $\mu_y^M(\cdot;k_2)$ and $\mu_y^B(\cdot;k_3)$.

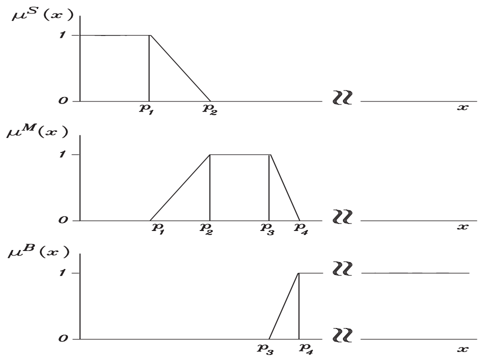

Figure 1. Input membership functions

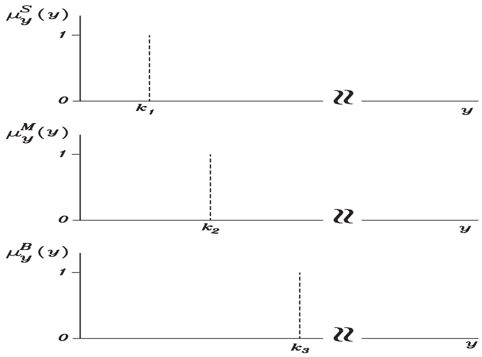

Figure 2. Output membership functions

B. Rule base and inference method

Since we attempt real-time implementation of the fuzzy self–tuning algorithm, this should have as few rules as possible in order to reduce the computational effort. The selected rules using Mamdani's method are the following:

$$\begin{array}{l} \texttt{IF} \quad \mu^S(|x|\,;p_1,p_2) \quad \texttt{THEN} \quad \mu_y^B(\cdot;k_3) \\ \texttt{IF} \quad \mu^M(|x|\,;p_1,p_2,p_3,p_4) \quad \texttt{THEN} \quad \mu_y^M(\cdot;k_2) \\ \texttt{IF} \quad \mu^B(|x|\,;p_3,p_4) \quad \texttt{THEN} \quad \mu_y^S(\cdot;k_1). \end{array}$$

The first rule specifies that for a small position error we should apply a big k_p in order to reduce still more this error. The second one specifies that for a medium position error we must apply a medium k_p, and the last rule specifies that for a big position error we need to apply a small k_p in order to avoid torque saturation. With regard to derivative gain k_v, a similar criterion is used taking into account that for big position errors it is suitable to have small damping, to avoid an oscillatory response. For the case of integral gains a similar criterion is used taking into account that for big $|\boldsymbol{\omega}|$ it is suitable to have small k_i to avoid undesirable oscillations. Evaluation of the rules using Mamdani's product inference and singleton fuzzifier (no premise connective is needed since we have only one input) leads to a real-valued vector function $\boldsymbol{h}(|x|\,,\cdot) : \mathbb{R}_+ \times \mathbb{R} \rightarrow \mathbb{R}^3$ given by

$$\boldsymbol{h}(|x|\,,\cdot) = \begin{bmatrix} h_1(|x|\,,\cdot) \\ h_2(|x|\,,\cdot) \\ h_3(|x|\,,\cdot) \end{bmatrix}$$

where $\boldsymbol{h}(|x|\,,\cdot)$ has the following property: $1 \geq h_i(|x_i|\,,\cdot) \geq 0$ with i = 1,2,3.

Owing to singleton membership functions, for a given $s \in \mathbb{R}$ we have

$$\boldsymbol{h}(|x|\,,\cdot) \;=\; \begin{bmatrix} \mu^S(|x|\,;p_1,p_2)\mu_y^S\delta(s-k_3) \\ \mu^M(|x|\,;p_1,p_2,p_3,p_4)\mu_y^M\delta(s-k_2) \\ \mu^B(|x|\,;p_3,p_4)\mu_y^B\delta(s-k_1) \end{bmatrix} \quad (15)$$

where $\delta(\cdot)$ denotes the Dirac function. The following feature of Dirac functions will be evoked later. For a real -valued function $\psi : \mathbb{R} \rightarrow \mathbb{R}$ and any $s_0 \in \mathbb{R}$, the Dirac function has the following property:

$$\int_{-\infty}^{\infty} \psi(s)\delta(s-s_0)ds = \psi(s_0). \quad (16)$$

C. Defuzzification method

The defuzzification strategy chosen in this paper is the center of area method [15]. Therefore, the output y can be computed as:

$$y = \frac{\sum_{i=1}^{3} \int_{-\infty}^{\infty} h_i(|x|\,,s)sds}{\sum_{i=1}^{3} \int_{-\infty}^{\infty} h_i(|x|\,,s)ds} \quad (17)$$

Invoking this formula and (15)–(16), the defuzzification procedure reduces to

$$y = \boldsymbol{k}^T \frac{\boldsymbol{\mu}(|x|\,;\boldsymbol{p})}{\|\boldsymbol{\mu}(|x|\,;\boldsymbol{p})\|_1} \quad (18)$$

where $\boldsymbol{p} = \{p_1,p_2,p_3,p_4\}$ and $\boldsymbol{k} = \{k_1,k_2,k_3\}$, $\|\cdot\|_1$ stands for the 1 norm, and $(\cdot)^T$ denotes transpose and

$$\boldsymbol{\mu}(|x|\,;\boldsymbol{p}) \;=\; \begin{bmatrix} \mu^B(|x|\,;p_3,p_4) \\ \mu^M(|x|\,;p_1,p_2,p_3,p_4) \\ \mu^S(|x|\,;p_1,p_2) \end{bmatrix}$$

Therefore, for a given input $|x|$ the output y can be computed straightforward from (18).

As it was pointed out in [10]. This FLT has the feature that under simple conditions its output y is upper and lower bounded by strictly positive constants. This feature allows to fulfill completely Assumptions 1 and 2. A block diagram of the self–tuning PID control scheme is depicted in Fig. 3.

D. Tuning of the gains

As previously described, the basic FLT is invoked to determine the proportional, integral and derivative gains. Thus, a set of $3n$ FLTs for $i = 1, \cdots, n.$ are defined, that is

$$\begin{array}{rccc} H_{k_{pi}}: & \mathbb{R}_+ & \rightarrow & \mathbb{R} \\ & |\tilde{q}_i| & \mapsto & k_{pi} \\ H_{k_{ii}}: & \mathbb{R}_+ & \rightarrow & \mathbb{R} \\ & |\omega_i| & \mapsto & k_{ii} \\ H_{k_{vi}}: & \mathbb{R}_+ & \rightarrow & \mathbb{R} \\ & |\tilde{q}_i| & \mapsto & k_{vi} \end{array}$$

and

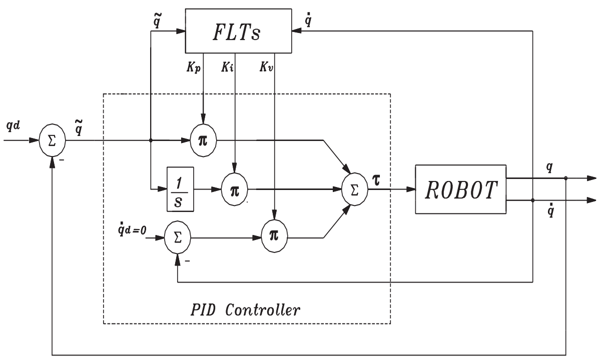

Figure 3. Block diagram fuzzy self-tuning PID control

Figure 4. Experimental robot arm

VI. EXPERIMENTAL RESULTS

In order to illustrate the performance of the proposed fuzzy self-tuning approach with respect to the fixed parameters PID control scheme, some experiments were carried out on a well-identified robot arm [17]. The system corresponds to a direct drive vertical robot arm as shown in Fig. 4. The maximum torque provided by the actuators is given by $|\tau_1| \leq 200$ [Nm] and $|\tau_2| \leq 15$ [Nm].

With the end of supporting the effectiveness of the proposed controller we have used a squared signal whose amplitude is increased or decreased in magnitude every two seconds. More specifically, the robot task is coded in the following desired joint positions $q_{d1}(t) = \{45\ 90\ 45\}$ degrees and $q_{d2}(t) = \{90\ 45\ 90\}$ degrees.

Above position references are piecewise constant and really demand large torques to reach the amplitude of the respective requested step. In order to compare and evaluate the effectiveness of the proposed controller, the experiment was conducted for two controllers: the proposed fuzzy self-tuning PID control scheme and the classical PID control scheme. Fuzzy partitions of the universes of discourse of the errors $|\tilde{q}_1|$ and $|\tilde{q}_2|$ are characterized, respectively, by the sets

$P(|\tilde{q}_1|) = \{0,\ 2,\ 4,\ 10,\ 30,\ 180\}$ [deg]
$P(|\tilde{q}_2|) = \{0,\ 2,\ 4,\ 10,\ 30,\ 180\}$ [deg]

where $P(|\tilde{q}_i|) = \{p_1, p_2, ..., p_4\}$ denotes the supports of the membership function of $\tilde{q}_i$ according to our convention (see Fig. 1), while fuzzy partitions of the state variables $|\omega_1|$ and $|\omega_2|$ are characterized, respectively, by the sets

$P(|\omega_1|) = \{0, 0.573, 5.73, 14.325, 28.65, 85.95\}$ [deg]
$P(|\omega_2|) = \{0, 5.73, 28.65, 57.3, 114.6, 286.5\}$ [deg].

The fuzzy partitions of the universe of discourse of the proportional, integral and derivative gains are determined taking into account the desired accuracy and stability conditions given in Proposition 1. The partition of the universe of discourses for the proportional gains were

$\boldsymbol{k}_{p1} = \{2.97,\ 10.47,\ 17.45\}$ [Nm/deg]
$\boldsymbol{k}_{p2} = \{0.15,\ 0.87,\ 1.75\}$ [Nm/deg]

where $\boldsymbol{k}_{pi} = \{k_1, k_2, k_3\}$ denotes the supports of the membership function of output k_{pi} according to our convention (see Fig. 2). Following similar criteria the partition of the universe of discourse for the integral and derivative gains k_{ii} and k_{vi} were chosen as follows:

$\boldsymbol{k}_{i1} = \{1.57,\ 1.92,\ 2.36\}$ [Nm/(deg sec)]
$\boldsymbol{k}_{i2} = \{0.0087,\ 0.035,\ 0.087\}$ [Nm/(deg sec)]

$\boldsymbol{k}_{v1} = \{1.66,\ 2,\ 2.5\}$ [Nm sec/deg]
$\boldsymbol{k}_{v2} = \{0.041,\ 0.26,\ 0.5\}$ [Nm sec/deg]

The above fuzzy partitions ensure that the fuzzy self-tuners deliver proportional, integral and derivative gains in agreement with conditions of Proposition 1, thus the origin of the state space of the closed loop system is locally asymptotically stable. The experimental results are depicted in Figs. (5)-(8). They show the desired and actual joint positions and the applied torques for the Fuzzy self–tuning PID control and classical PID control. From Figures (5)-(6), one can observe that the transient responses for the classical PID control, in each change of the step magnitude, of the links are not really good and the accuracy of positioning is not satisfactory.

The proposed fuzzy self-tuning PID controller was tested under the same desired task. The transient responses q_1 and q_2 are shown in Figs. (5)-(6) comparing these position errors with those obtained for the case of fixed gains, we see that the accuracy and the transient are improved by using the fuzzy self-tuning algorithm. Applied torque τ_1 and τ_2 are sketched in Figs. (7)-(8) these figures show the evolution of the applied torques; both remain within the torque actuator limits.

VII. CONCLUSIONS

In this paper we have proposed a fuzzy adaptation scheme for tuning the proportional, integral and derivative state dependent gains of a PID controller for robot manipulators. Besides, a locally asymptotically stability proof for the proposed fuzzy self–tuning PID control for robot manipulators is presented. The proposed approach allows to consider important practical features in real robots such as to achieve desired accuracy and to avoid the work of the actuator torques beyond their capabilities. The performance of the proposed fuzzy scheme has been verified by means of real time experimental tests on a two degree of freedom direct drive robot arm.

ACKNOWLEDGMENT

This research was funded by CONACyT Mexico postdoctoral fellowship grant No. 290536, CONACyT SNI Project No. 89605, ITESM e-Robots Research chair and Dirección General de Educación Superior Tecnológica (DGEST) Mexico.

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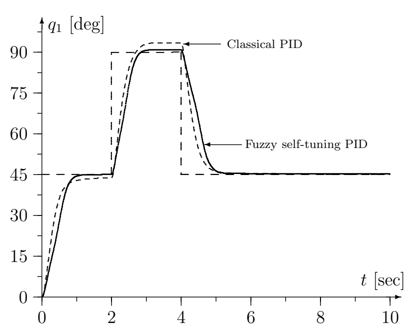

Figure 5. Desired and actual positions 1 for the fuzzy self-tuning PID control and classical PID

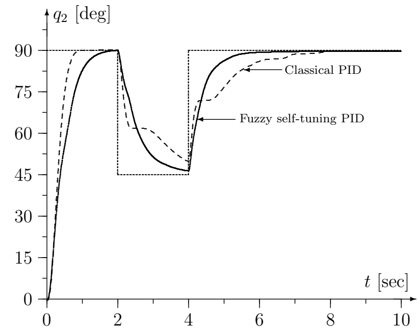

Figure 6. Desired and actual positions 2 for the fuzzy self-tuning PID control and classical PID

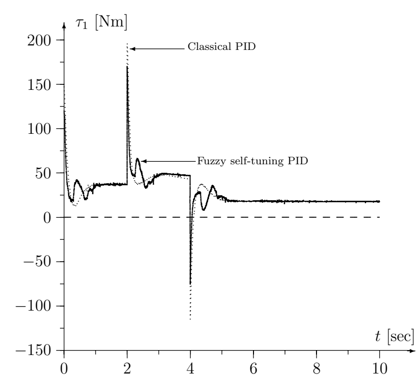

Figure 7. Applied torque to the joint 1 for the fuzzy self-tuning PID control and classical PID

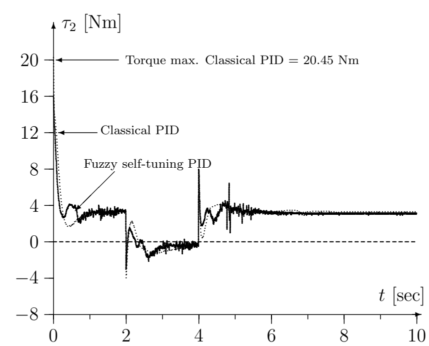

Figure 8. Applied torque to the joint 2 for the fuzzy self-tuning PID control and classical PID