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Diagnosis and prognosis for the maintenance of complex systems

Pauline Ribot[1,2] , Yannick Pencolé[1,2] and Michel Combacau[1,2]
[1] CNRS; LAAS; 7 avenue du Colonel Roche, F-31077 Toulouse, France
[2] Université de Toulouse; UPS, INSA, INP, ISAE; LAAS; F-31077 Toulouse, France
{pribot,ypencole,combacau}@laas.fr

***Abstract*—This paper adresses the problem of maintenance of a complex and heterogeneous system like an aircraft. To optimise maintenance, it is required to embed in the system a health monitoring system that implements diagnostic and prognostic capabilities. This paper thus presents a formal characterisation of the diagnostic and prognostic problems in order to support the maintenance of a complex system.**

***Index Terms*—Diagnosis, prognosis, preventive maintenance, complex systems.**

I. INTRODUCTION

Nowadays, system maintenance is a key challenge for manufacturers that produce complex systems, like aircraft [1] or cars, as it is a way to improve the reliability, security, safety and the final cost of their products. In order to improve the maintenance capabilities of a complex system, an on-board health monitoring system must be deployed and provide enough informative pieces of data to decide which type of maintenance operations is required. These types of systems are assemblage of heterogeneous components (continuous, discrete, hybrid components) that require many different types of techniques in order to monitor their *health state*. To get a global health monitoring system, there are many difficulties to deal with.

First of all, in classical maintenance, the objective is to replace components (also called Line Replacement Units, LRU for short) of the system that are faulty in the sense that these components do not perform the set of functions they are supposed to implement. To figure out the set of faulty components, the monitoring system requires diagnostic capabilities to determine on-line the faulty components [1], [2]. The second difficulty is about the optimality of the maintenance. In order to decrease the maintenance cost, it is necessary to perform preventive maintenance [3], [4], [5] which also require to embed in the health monitoring system some prognostic capabilities in order to determine the *ageing state* of the system [6], [7]. Last difficulty is about the description of the system itself. Without a good level of abstraction in order to describe the system knowledge to be used in the health monitoring system, it is impossible to guarantee the global consistency of the health monitoring architecture. The objective is thus to get an abstracted description that is homogeneous.

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The objective of this paper is to present a formal characterisation of a generic health monitoring system for the global maintenance of a complex system. The originality of this work is to fully characterise the health monitoring problem as a combination of diagnostic and prognostic problems. From this characterisation follow the necessary interactions between diagnostic and prognostic problems in order to improve the maintenance quality of a complex system.

II. HEALTH MONITORING: GENERIC CHARACTERISATION

A. System model

From a theoretical viewpoint, modeling a dynamical system Σ consists in determining a set of algebraic relations $\mathcal{A}$ between a set of *parameters* $\mathcal{P}$ [2]. A system Σ can be defined by a 3-tuple $\Sigma = \langle \mathcal{P}, \mathcal{R}, \mathcal{A} \rangle$ with $\mathcal{P} = \{p^k\}$ the set of parameters, $\mathcal{R}$ the set of interval ranges of the parameter values and $\mathcal{A} = \{ar^k\}$ the set of algebraic relations between parameters. The range of a parameter p^k represents the set of correct values and is denoted $r(p^k)$. For instance, for a continuous parameter, $r(p^k) = [\alpha^k, \beta^k]$ with $(\alpha^k, \beta^k) \in \mathbb{R}^2$ and $\alpha^k \leq \beta^k$. An algebraic relation ar^k is a mapping on the power set of parameters usually written as

$$p^j = ar^k(p^i, \ldots, p^m). \quad (1)$$

B. Components

For many complex systems, it is impossible to model the system in a monolithic way, it is required to apply a compositional approach: the system Σ is then modeled as an assemblage of N components: $Comps = \{C^1, \ldots, C^N\}$. A component is an entity that is in charge of implementing a set of elementary functions also called processes in [8]. In order to accurately define the concepts related to the diagnosis and prognosis problems, it is necessary to refine the description level of the functional structure of a component by the introduction of a structural and a behavioral model of the component [8].

With the introduced notations, a component C^i is defined by a 3-tuple $C^i = \langle \mathcal{P}^i, \mathcal{R}^i, \mathcal{A}^i \rangle$ with $\mathcal{P}^i = \{p^{i,k}\} \subseteq \mathcal{P}$, $\mathcal{R}^i = \{r^{i,k}\} \subseteq \mathcal{R}$, and $\mathcal{A}^i = \{ar^{i,k}\} \subseteq \mathcal{A}$. Naturally, the system is

[2]For the sake of generality and simplicity throughout this paper, the notion of parameters will be used to denote any type of variables in the model.

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built by assembling a given set of components:

$$\Sigma = \langle \mathcal{P}, \mathcal{R}, \mathcal{A} \rangle = \langle \bigcup_i \mathcal{P}^i, \bigcup_i \mathcal{R}^i, \bigcup_i \mathcal{A}^i \rangle. \quad (2)$$

1) Structural model of a component: Structural models describe the possible set of interactions between components. Usually, interactions of a component are modeled with *ports*. A port can be modeled as an *input parameter* corresponding to an input flow of information or as an *output parameter*, corresponding to an output flow of information. Data flow must be considered at the model level. Naturally, in a real system, a data flow like *voltage, intensity, pressure, ...* corresponds to a physical flow. For a component C^i, three kinds of parameters are distinguished:

- $\mathcal{IP}^i$: $p^{i,k}$ is an *input parameter* of C^i (noted $ip^{i,k}$) iff it belongs at least to another component and if the mechanisms implemented by C^i cannot modify its value. Formally, $ip^{i,k}$ belongs to $\mathcal{IP}^i$ iff $\exists C^j, j \neq i | ip^{i,k} \in \mathcal{P}^j \wedge \nexists ar^{i,k} | ip^{i,k} = ar^{i,k}(p^{i,1} \dots p^{i,m})$.
- $\mathcal{OP}^i$: $p^{i,k}$ is an *output parameter* of C^i (noted $op^{i,k}$) iff it belongs at least to another component and if its value results from the mechanisms implemented by C^i. Formally, $op^{i,k}$ belongs to $\mathcal{OP}^i$ iff $\exists C^j, j \neq i | op^{i,k} \in \mathcal{P}^j \wedge \exists ar^{i,k} | op^{i,k} = ar^{i,k}(p^{i,1} \dots p^{i,m})$.
- $\mathcal{PP}^i$: $p^{i,k}$ is a *private parameter* (noted $pp^{i,k}$) iff it belongs to C^i only. Formally, $pp^{i,k}$ belongs to $\mathcal{PP}^i$ iff $\nexists C^j, j \neq i | pp^{i,k} \in \mathcal{PP}^j$.

The value of a private parameter $pp^{i,k}$ can be modified by the component mechanisms. Private parameters are required for the generality of the model. For instance, without private parameters, the description of a dynamical system would be impossible.

2) Functional view of a component: A component C^i has to provide a set of basic/elementary functions $\mathcal{FU}^i$ that are structurally composed in order to perform a system function (functional model) [9]. These functions rely on the behavior of the component and can be modeled as *functional conditions* on the behavioral model (see [8] for more details). A functional condition associated to the basic function $Fu^{i,j} \in \mathcal{FU}^i$ is thus a property defined as an algebraic relation on the parameters of the component C^i and that can be written in as follows:

$$Fu^{i,j} \equiv (op^{i,j} = ar^{i,j}(ip^{i,1}, \dots, ip^{i,l_i}, pp^{i,1}, \dots, pp^{i,r_i})) \quad (3)$$

in which $op^{i,j} \in \mathcal{OP}^i$, $ip^{i,j} \in \mathcal{IP}^i$ and $pp^{i,j} \in \mathcal{PP}^i$.

These relations describe the behavioral model of the component when the function is available on it, in particular they describe the preconditions, postconditions and range of any realisations of the function based on the parameters involved in the relations. If this knowledge is about the nominal model of the component, it is available after the design stage of the component. Knowledge about faults comes from the safety analysis.

Figure 1 represents a component C^1 with three input parameters $ip^{1,1}$, $ip^{1,2}$, $ip^{1,3}$, two output parameters $op^{1,1}$, $op^{1,2}$ and two private parameters $pp^{1,1}$, $pp^{1,2}$. The relations

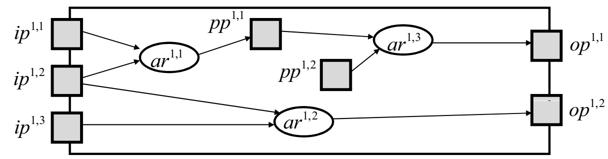

Fig. 1. Component modeling with parameters

$ar^{1,1}$, $ar^{1,2}$ and $ar^{1,3}$ model the conditions of two basic functions $Fu^{1,1}, Fu^{1,2} \in \mathcal{FU}^1$ implemented by C^1:

$$(Fu^{1,1}) \equiv (op^{1,1} = ar^{1,3}(pp^{1,2}, ar^{1,1}(ip^{1,1}, ip^{1,2}))),$$
$$(Fu^{1,2}) \equiv (op^{1,2} = ar^{1,2}(ip^{1,2}, ip^{1,3})).$$

C. Operational modes

An *operational mode* of a system is defined by the model that represents the set of available system functions at a given time. When the system is in a given mode, every parameter is in its range and every algebraic relation of the model is satisfied. The set of operational modes is a partition of the set of possible behaviors of the system. Formally, the model of mode x is $\Sigma_x = \langle \mathcal{P}, \mathcal{R}_x, \mathcal{A}_x \rangle$ (with $\mathcal{P} = \{p^{i,k}\}, \mathcal{R}_x = \{r_x^{i,k}\}, \mathcal{A}_x = \{ar_x^{i,k}\}$), then the system is in mode x iff

$$\forall i, \forall k, \begin{cases} op^{i,k} = ar_x^{i,k}(ip^{i,1}, \dots, ip^{i,l_i}, pp^{i,1}, \dots, pp^{i,r_i}) \\ p^{i,k} \in r_x(p^{i,k}) \end{cases}$$

The nominal operational mode is defined by the nominal model Σ_n that comes from the design step. In many cases, it is also possible to get fault models for a set of *anticipated faults*. As for the nominal knowledge, a fault model $\Sigma_f = \langle \mathcal{P}, \mathcal{R}_f, \mathcal{A}_f \rangle$ defines the fault mode f. When the fault f is present, not only the range of the parameters can be different but also the algebraic relations can be totally different from the nominal mode. It is possible that in a fault mode, some of the function conditions still hold (thus some of the basic functions are working properly).

D. Diagnosis characterisation

In the diagnosis community literature, the objective of diagnosis is often defined as determining the current *mode* of every component that is consistent with the whole set of available observations (sensors values, symptoms of failure...) [10]. Based on the functional view and the knowledge about the behavior of a component presented in the previous section, it is possible to distinguish three kinds of component modes:

1) C^i is in *nominal mode* n iff

$$\forall k, \begin{cases} op^{i,k} = ar_n^{i,k}(ip^{i,1}, \dots, ip^{i,l_i}, pp^{i,1}, \dots, pp^{i,r_i}) \\ p^{i,k} \in r_n(p^{i,k}) \end{cases}$$

2) C^i is in a faulty mode f iff

$$\forall k, \begin{cases} op^{i,k} = ar_f^{i,k}(ip^{i,1}, \dots, ip^{i,l_i}, pp^{i,1}, \dots, pp^{i,r_i}) \\ p^{i,k} \in r_f(p^{i,k}) \end{cases}$$

If C^i is in a faulty mode, it means that a fault has occurred on it due to an internal problem; at least a private parameter is not in the range of the nominal operational mode: $\exists j, pp^{i,j} \notin r_n(pp^{i,j})$. The component does not provide all the basic functions.

3) C_i is in *abnormal mode*, if there exists at least one input parameter that is not in the range of a function condition and consequently this condition is not satisfied: $\exists j, ip^{i,j} \notin r_n(ip^{i,j})$. It must be noticed that, in this case, the input parameter that is out of range gets its value from another component that cannot be itself in the nominal mode (the output parameter is out of range).

According to these definitions, a component C^i is associated to a set of modes $\mathcal{M}^i = \{m_n^i, m_a^i, m_{f1}^i, ..., m_{fk}^i\}$, each mode m_x^i being characterised by a specific model Σ_x^i. At any time, any component is in one mode only. The mode of the system is then a N-tuple of component modes $m_s^\Sigma = \langle m_{s1}^1, m_{s2}^2, ..., m_{sN}^N \rangle$. When a fault occurs, the system mode changes, and thus, during its operation (from start to end), the system follows a sequence of modes $m_0^\Sigma, m_1^\Sigma, ..., m_f^\Sigma$ where the system changes from mode m_j^Σ to m_{j+1}^Σ at time t_{j+1}.

A *diagnosis candidate* for the system Σ at a given time $t_{diag} \in [t_j, t_{j+1}[$ is a set $\langle m_j^1 ... m_j^i, ..., m_j^N \rangle$ of modes that are globally consistent with the set of available observations from time 0 to t_{diag}. Finally, the *global diagnosis* Δ^Σ at time t_{diag} returned by the diagnostic function is the set of possible current diagnostic candidates at time t_{diag}. It follows that the diagnostic function is in charge of determining the sequence of modes of Σ based on the available observations from time 0 to t_{diag}.

E. Prognosis characterization

Given the previous context, it is also easy to characterise the result of a prognostic function. At a given time $t_{prog} \in [t_j, t_{j+1}[$, the ultimate aim of a prognostic function is to determine the sequence of modes $m_{j+1}^\Sigma, ... m_f^\Sigma$. Thus, a system prognosis Π^Σ consists of a set of mode sequences $m_{j+1}^\Sigma, ..., m_f^\Sigma$ that are consistent with all the mode sequences determined by the diagnostic function at time $t_{diag} \leq t_{prog}$.

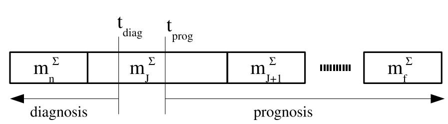

Fig. 2. Diagnosis and Prognosis

In order to predict the future mode sequence, a set of *ageing laws* (or ageing models) must be accurately known to compute the occurrence time of the next fault. Generally, ageing laws are known for specific parameters and come from safety analysis. These laws are dependent of the mode in which the components operate. This knowledge can be modeled by a mapping *lm* that associates an ageing law $ag_x^{i,k}$ to a private parameter $pp^{i,k}$ supposed to be in mode m_x^i:

$$\begin{cases} \mathcal{PP}^i \times \mathcal{M}^i \xrightarrow{lm} \mathcal{AG}^i \\ (pp^{i,k}, m_x^i) \xrightarrow{lm} lm(pp^{i,k}, m_x^i) = ag_x^{i,k}. \end{cases}$$

In which $ag_x^{i,k}$ stands for "ageing law for parameter k of component i in mode m_x^i".

The determination of the prognosis result is recursive.

1) From the system mode m_{j-1}^Σ estimated by the diagnostic function, determine the time t_j of a mode change to m_j^Σ and initiate the prognosis $\Pi_j^\Sigma = \langle m_j^\Sigma \rangle$.
2) From the last mode contained in Π_j^Σ, determine the ageing laws $ag_j^{i,k}$ of the system parameters and compute the time t_{j+1} of the next mode change m_{j+1}^Σ. $\Pi_{j+1}^\Sigma = \langle \Pi_j^\Sigma . m_{j+1}^\Sigma \rangle$.
3) $j \leftarrow j + 1$; return to step 2.

Note that this basic principle generates a trajectory of system modes in a time horizon $[t_j, t_e]$ with t_e a priori fixed. Here the prognosis result is a unique trajectory that corresponds to the most likely one (see the next section) relying on the fact that the system is supposed to be in mode m_{j-1}^Σ before time t_j.

III. PROGNOSIS METHODS

A. Introduction

As written previously, prognostics basically consists in estimating at time $t_{prog} \geq t_j$ the time $t_{j+1} \geq t_{prog}$ of the future mode change relying on the ageing models available in the system mode m_j^Σ. A set of ageing models (or also called life models in [11]) is available for each private parameter $pp^{i,k} \in \mathcal{PP}^i$. In the following, $pp^{i,k}$ will be generically noted pp for the sake of simplicity. Ageing models of a private parameter describe the evolution of the parameter value pp with environmental constraints (like temperature, humidity, vibration, stress conditions) [9]. It follows that an ageing model $ag_x^{i,k}$ can be represented as an algebraic relation depending on operational conditions of the system mode m_x^Σ. According to the current system mode m_i^Σ that defines the range of the private parameters of the system, the well-suited ageing models are selected for each private parameter $pp \in \mathcal{PP}$. The remaining time until the parameter pp becomes faulty is noted $RTF(pp)$. A fault probability for each private parameter in $\mathcal{PP}$ is established from the selected ageing models. Basically, let f_{pp} denote a probability density function (pdf for short) representing the fault probability of a private parameter pp in the mode m_x^Σ, P_{max} be the maximal fault probability value acceptable for the parameter pp, the *remaining time to fault* (RTF for short) of pp consists then in determining the time t_p for which the fault probability has reached the threshold P_{max}:

$$RTF(pp) = t_p \text{ such that } \int_0^{t_p} f_{pp}(t)dt = P_{max}. \quad (4)$$

It follows that $t_{j+1} = t_{prog} + min(RTF(pp), pp \in \mathcal{PP})$ and the system mode m_{j+1}^Σ is such that pp is out of range.

B. Generic modeling for prognostics

Private parameters usually represent physical attributes of the component; they are totally heterogeneous. The difficulty is then to find out a common representation of the prognosis for any private parameter that also must be as flexible as possible to represent any type of probability density functions. For these reasons, the Weibull distribution is often used [2], [7], [12], [13], whose probability density function is:

$$W(t, \beta, \eta, \theta) = \frac{\beta}{\eta}\Big(\frac{t-\theta}{\eta}\Big)^{(\beta-1)} e^{-(\frac{t-\theta}{\eta})^{\beta}}, \qquad (5)$$

where β characterises the shape of the distribution, η characterises the scale and finally θ characterises the location of the distribution. For a given private parameter $pp \in \mathcal{PP}^i$ and a given probability threshold P_{max}, we then get:

$$RTF(pp) = t_p \text{ such that } \int_0^{t_p} W(t, \beta_{pp}, \eta_{pp}, \theta_{pp})dt = P_{max}. \qquad (6)$$

β_{pp}, η_{pp}, θ_{pp} fully characterise the failure probability distribution and thus model the way the parameter *pp* is ageing. The description of the characteristics $\beta_{pp}, \eta_{pp}, \theta_{pp}$ relies on the ageing models associated to *pp*. Whatever the technique used to obtain such models, the available knowledge is then characterised by algebraic relations that define $\beta_{pp}, \eta_{pp}, \theta_{pp}$: $\beta_{pp} = ar_\beta(ip^{i,1}, \ldots, ip^{i,n}), \theta_{pp} = ar_\theta(ip^{i,1}, \ldots, ip^{i,n}), \eta_{pp} = ar_\eta(ip^{i,1}, \ldots, ip^{i,n})$.

C. Functional Prognosis

From the maintenance point of view, prognostics is the process of predicting the *remaining useful life* (RUL for short) of the overall system in order to improve the scheduling of maintenance actions. The RUL of the system is defined as the remaining time until the system cannot perform successfully its full set of complex functions that are resulting from the composition of the basic functions $\mathcal{FU}^i$ of any component $C^i \in Comps$ (see section II-B2). In systems like aircraft for instance, the system functions usually rely on redundant implementations of the basic functions (the functions of one component can be performed by another one). That is the reason why prognostics relies on the functional view of the system [5], [14] (see Section II-B2). In order to get the prognosis about the availability of any system function in the future, it is required to get the prognosis about the availability of every basic function $F \in \mathcal{FU}^i$ implemented on every component C^i of the system. The ageing of F naturally depends on the ageing of the private parameters of the component that implements F. The remaining time until a basic function F becomes failed is noted $ettf(F)$ which stands for *estimated time to failure*:

$$ettf(F) = t_f \text{ such that } \int_0^{t_f} W(t, \beta_F, \eta_F, \theta_F) = P_F, \qquad (7)$$

where P_F is a probability threshold and $\beta_F, \eta_F, \theta_F$ are fully determined by a combination of the characteristics $\{\beta_{pp}, \eta_{pp}, \theta_{pp}\}_{pp \in PP(F)}$. By extension, the $ettf(F_{sys})$ of any system function F_{sys} is defined as $ettf(F)$ where F is the basic function whose availability is necessary for F_{sys} to be available and whose $ettf(F)$ is minimal. The RUL of a component is then $RUL(C^i) = min(ettf(Fu^{i,j}), Fu^{i,j} \in \mathcal{FU}^i)$ and the RUL of the system is estimated to the minimal $ettf(F_{sys})$. Finally, maintenance can be scheduled based on the $ettf(F_{sys})$ and consists at least in replacing a component C^i implementing the basic function $F \in \mathcal{FU}^i$ by a new one so that $ettf(F_{sys})$ is increased.

IV. The three-tank system example

Our study is illustrated with the example of a three-tank system, see Figure 3. The system consists of three cylindrical tanks T_1, T_2 and T_3 which a cross section S_1, S_2 and S_3. The water level H_1, H_2 and H_3 in the three tanks are measured by sensors. The tanks are connected in series with one another by three cylindrical pipes (p_{13}, p_{32} and p_{20} with a cross section S_{13}, S_{32} and S_{20}). The water flow Q_{13} in the pipe p_{13} and Q_{32} in the pipe p_{32} is controlled by two electro-valves V_1 and V_2. The tank T_1 is filled by a pump P_1 that is governed by a level controller and delivers a flow Q_1. In the tank T_2, there is a outflow pipe p_20. The water from p_20 is collected in a big tank.

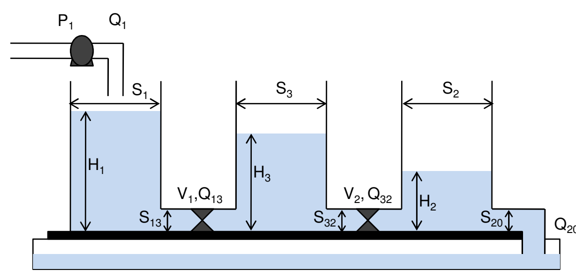

Fig. 3. The three-tank system

A. System Modeling

1) Structural modeling: The three-tank system, denoted Σ is composed of seven components that interact with each other:

$$Comps = \{P_1, T_1, T_2, T_3, p_{13}, p_{32}, p_{20}\}. \qquad (8)$$

Each component is modeled by a set of parameters, for instance:

$$\begin{aligned}
P_1: &\quad (\mathcal{IP}^{P_1} = \{v_{Q_1}\}, \mathcal{OP}^{P_1} = \{Q_1\}, \mathcal{PP}^{P_1} = \{f_{P_1}\}) \\
T_1: &\quad (\mathcal{IP}^{T_1} = \{Q_1, Q_{13}\}, \mathcal{OP}^{T_1} = \{H_1\}, \\
&\qquad \mathcal{PP}^{T_1} = \{S_1, f_{T_1}\}) \\
p_{13}: &\quad (\mathcal{IP}^{p_{13}} = \{H_1, H_3\}, \mathcal{OP}^{p_{13}} = \{Q_{13}\}, \\
&\qquad \mathcal{PP}^{p_{13}} = \{f_{V_1}, f_{V_2}, S_{13}\})
\end{aligned} \qquad (9)$$

Two interacting components share at least one output/input parameter (as $H_1, Q_{13}, \ldots$). The pipe p_{13} (resp. p_{32}) and the valve V_1 (resp. V_2) are considered as only one component. That means they would be removed simultaneously by a maintenance operator.

2) Functional modeling: The system components implement a set of basic functions in order to realize the system function that is represented by the following set of constraints:

$$\begin{gathered} H_1 < 0.6m,\ H_2 < 0.6m,\ H_3 < 0.6m, \\ H_1 > H_3 > H_2. \end{gathered} \tag{10}$$

Analytical relations model the conditions of basic functions for each system component:

$$\begin{aligned} Fu^{P_1} &\equiv Q_1 = ar^{P_1}(v_{Q_1}, f_{P_1}), \\ Fu^{T_1} &\equiv H_1 = ar^{T1}(Q_1, Q_{13}, S_1, f_{T_1}), \\ Fu^{T_3} &\equiv H_3 = ar^{T_3}(Q_{13}, Q_{32}, S_3, f_{T_3}), \\ Fu^{T_2} &\equiv H_2 = ar^{T_2}(Q_{32}, Q_{20}, S_2, f_{T_2}), \\ Fu^{p_{13}} &\equiv Q_{13} = ar^{p_{13}}(H_1, H_3, f_{V_1}, f_{V_2}, S_{13}), \\ Fu^{p_{32}} &\equiv Q_{32} = ar^{p_{32}}(H_3, H_2, f_{V_1}, f_{V_2}, S_{32}), \\ Fu^{p_{20}} &\equiv Q_{20} = ar^{p_{20}}(H_2, S_{20}). \end{aligned} \tag{11}$$

The functional and structural modeling of the system is illustrated in Figure 4. The set of system components with their private parameters is represented as well as the component connections through output/input parameters.

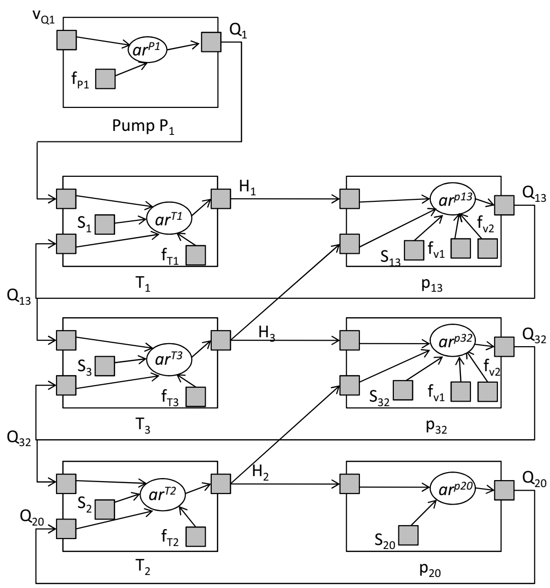

Fig. 4. Three-tank system modeling with parameters

B. Operational modes

According to the interval range of the value of private parameters involved in the algebraic relations, these relations model the component modes. The pump behavior is modeled by the following equations:

$$ar^{P_1} : Q_1(t) = \begin{cases} v_{Q_1}(1 - f_{P_1}) & if \quad 0 < v_{Q_1} < Q_{1_{max}}, \\ 0 & if \quad v_{Q_1} \leq 0, \\ Q_{1_{max}}(1 - f_{P_1}) & if \quad v_{Q_1} > Q_{1_{max}}. \end{cases} \tag{12}$$

In nominal mode $m_n^{P_1}$, the pump delivers a flow Q_1 that is assumed to be proportional to the controller output (voltage) v_{Q_1} and limited $Q_1 < Q_{1_{max}}$, then $r_n^{P_1}(f_{P_1}) = \{0\}$. In fault mode $m_f^{P_1}$, P_1 does not deliver a flow Q_1 whereas the voltage is different from zero, then $r_f^{P_1}(f_{P_1}) = \{1\}$.

The water level in the tanks can be calculated through mass balance equations :

$$\begin{aligned} ar^{T_1} &: \dot{H}_1 = (Q_1 - Q_{13} - f_{T_1})/S_1, \\ ar^{T_3} &: \dot{H}_3 = (Q_{13} - Q_{23} - f_{T_3})/S_3, \\ ar^{T_2} &: \dot{H}_2 = (Q_{23} - Q_{20} - f_{T_2})/S_2. \end{aligned} \tag{13}$$

In fault mode $m_f^{T_i}$, a leak in the tank T_i is modeled by the private parameter f_{T_i} such that $r_f^{T_i}(f_{T_i}) = \{1\}$, for $i = \{1, 2, 3\}$. In nominal mode $m_n^{T_i}$, with no water leak, $r_n^{T_i}(f_{T_i}) = \{0\}$.

Using the Torricelli law, flows through the pipes can be calculated :

$$\begin{aligned} ar^{p_{13}} &: Q_{13} = V_1 S_{13}\ sign(H_1 - H_3)\sqrt{2g\ |H_1 - H_3|}, \\ ar^{p_{32}} &: Q_{32} = V_2 S_{32}\ sign(H_3 - H_2)\sqrt{2g\ |H_3 - H_2|}, \\ ar^{p_{20}} &: Q_{20} = S_{20}\sqrt{2g\ H_2}. \end{aligned} \tag{14}$$

The behavior of pipes p_{13} and p_{32} depends on the status of their associated valve $V_{(i=1,2)}$ that is modeled by

$$V_i = \begin{cases} 0 + f_{V_1} & if \quad v_{V_i} \leq 0, \\ 1 - f_{V_2} & if \quad v_{V_i} > 1. \end{cases} \tag{15}$$

where v_{V_i} is a voltage provides by a switch controller. In nominal mode $m_n^{p_{ij}}$: $r_n^{p_{ij}}(V_i) = \{0, 1\}$ and $r_n^{p_{ij}}(f_{V_1}) = r_n^{p_{ij}}(f_{V_1}) = \{0\}$. $V_i = 0$ means that the valve i is closed and $V_i = 1$ means that it is opened. In fault mode $m_{f_1}^{p_{ij}}$, the valve is stucked open, then $r_{f_1}^{p_{ij}}(f_{V_1}) = \{1\}$. In fault mode $m_{f_2}^{p_{ij}}$, the valve stucked close and $r_{f_2}^{p_{ij}}(f_{V_2}) = \{1\}$.

C. Fault Diagnosis

Diagnosis problem consists in assigning a mode to each component of the system in order to be globally consistent with the system observations. For example, at time t_{diag}, a global diagnosis may contain only one diagnosis candidate for the system Σ such that

$$\Delta^{\Sigma} = < m_f^{P_1}, m_n^{T_1}, m_n^{T_3}, m_n^{T_2}, m_n^{p_{13}}, m_n^{p_{32}}, m_n^{p_{20}} > . \tag{16}$$

The pump P_1 is in fault mode f whereas all other system components are in their nominal mode. The system mode will change with the occurrence of the next fault on a system component.

D. Prognosis

When the system Σ is in nominal mode m_n^{Σ} at time t_j, the future mode m_{j+1}^{Σ} can be predicted from ageing laws of component private parameters. An ageing law of a private parameter $pp^{i,k}$ in a mode m_j^i is represented by an algebraic relation denoted by $ag_j^{i,k}$.

In the nominal mode, a mechanical parameter like f_{P_1} of P_1 may be damaged by the voltage or the utilisation time: $f_{P_1} = ag_n^{P_1}(v_{Q_1}, t)$. The valves V_1 and V_2 are damaged by three stresses: polluted water (model by ρ_{liquid}) that clogs the valve, the number of valve switchings (N_{sw}) and a high level flow (Q_{ij}) in the pipe. Ageing laws model the evolution of private parameters f_{V_1} and f_{V_2}: $f_{V_i} = ag_n^{V_i}(\rho_{liquid}, N_{sw}, Q_{ij})$. We do not consider the ageing of the tank sections because $S_i >> S_{ij}$ (the effect of clogging on the tank section is not significant).

This set of laws associated to the nominal mode are used by the prognosis function to compute the time of the next fault occurence (system mode change).

In a fault mode in which the pump does not deliver a sufficient flow, another set of ageing laws must be used. This set of ageing laws must be determined by considering physical laws like:

- a pipe is progressively clogged by a flow of polluted water;
- a pipe clogging is increased by a low level flow through the pipe.
- ...

In this fault mode, the flow through the pipe is at a level abnormally low, thus the clogging rate of the pipe S_{13} is greater than in the nominal mode.

We consider that two parameters can be correlated if they are involved in a same algebraic relation $ar^{i,k}$ (that also expressed functional dependances). By analysing the set of algebraic relations, a network of parameter dependances can be extracted. Figure 5 shows the dependance network of our example.

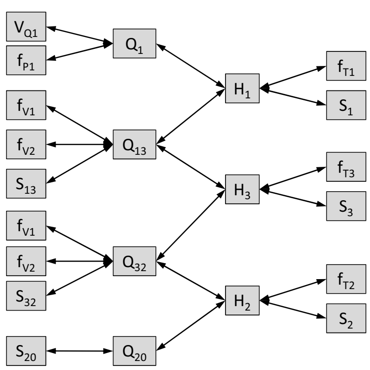

Fig. 5. Dependence of parameter ageing law

On line, if the global diagnosis determines that P_1 is in fault mode $m_f^{P_1}$, the ageing models associated to this mode are used for each private parameter of these components in order to compute a prognosis taken the fault mode into account.

V. Conclusion

This paper presents a formal characterisation of the diagnosis and the prognosis in order to support preventive maintenance of complex systems. A generic system modeling is proposed in order to define the modes of a system that represent the common support to the diagnostic and prognostic problems. As complex systems are composed of heterogeneous components, prognostic methods have to handle with different types of ageing models. Then a generic prognostic function is introduced that takes the heterogeneity and the functional view of the system into account in order to estimate the system remaining useful life.

Future works will focus on two aspects. We would precise how to map an ageing law to a parameter according to the system mode. The case where the global diagnosis contains several diagnostic candidates must be considered. Then several ageing models are selected for each private parameter according to the set of possible system modes. We would like to find an intelligent way for choosing the most suited model for every system parameter.

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