Proceedings of the 2009 IEEE International Conference on Systems, Man, and Cybernetics
San Antonio, TX, USA - October 2009

A Modeling and Aggregation Approach for Analyzing Resilience of Manufacturing Enterprises

Yao Hu, Jingshan Li, Lawrence E. Holloway
Department of Electrical and Computer Engineering
University of Kentucky
Lexington, KY, USA
yao.hu@uky.edu, jingshan@engr.uky.edu, holloway@engr.uky.edu

***Abstract*—We consider modeling of manufacturing enterprises as networks which process, store, and transport materials between enterprise nodes in order to feed customer demand. Each node within the network is represented as a dynamic model with associated costs of production and inventory. Examples of disruptions for such an enterprise could be weather events, material shortages, equipment disasters, or labor events. Using the dynamic model, we consider major disruptions within this network. We present aggregation methods which can support the analysis to evaluate the impact of these disruptions, and develop control strategies that reduce the impact of the disruption.**

***Index Terms*—Resilience, Manufacturing, Modeling.**

I. Introduction

This paper extends our previous research on modeling and analysis of resilience within manufacturing enterprises [1]. We consider a manufacturing enterprise as an interconnection of suppliers, producers, and consumers working together to provide a mixture of certain goods or services to end customers. A variety of events may occur to disrupt the smooth flow of material through this interconnected network. This may range from natural events (such as hurricanes, floods, ice storms, or tornadoes), to accidents (such as fires, power outages, or equipment failures), to man-made events (such as strikes, terrorism, wars, epidemics, and bankrupt suppliers). The way a manufacturing enterprise responds to a disruptive event is important for the profitability and perhaps even survivability of the enterprise. For example, a fire in Philips New Mexico plant caused $40 million loss sales of high-margin, high-tech chips, and direct damage to the plant of 39 million Euro insurance settlement [2]. An 18 day labor strike in 1996 at a General Motors brake supplier plant led to the idling of 26 assembly plants, and caused an estimated reduction of $900 million in quarterly earnings [3]. The ability of an enterprise to withstand potentially high-impact disruptive events is known as its resilience, which is characterized by the redundant or absorbing capability of the enterprise to the event to "dampen" its impact, and its recovery capability, i.e., quickly resume production or transportation by redistributing its resources.

After the terrorist attack in 2001, research related to resilient enterprises has received more attention. Paper [2] uses resilience to describe the ability to bounce back from disruptions and disasters by building in redundancy and flexibility. It includes the analysis of their successes, failures, preparations, and methods to reduce vulnerability and increase supply chain flexibility. Many other investigations based on case studies can also be found in [5]-[7]. These studies are more qualitative or descriptive oriented.

Much of the quantitative studies on resilience are focused on optimal network design for better resilience, such as using Tabu search in [8] or stochastic hybrid genetic algorithms in [9]. Cascading failure in complex networks due to redistribution of load is discussed in [10]. Analysis on vulnerability of critical infrastructure is presented in [11]. With attacker-defender models, a model is set up for a complete infrastructure system, including how losses of the system's assets reduce the system's value to society, or how improvements in the system mitigate lost value. Most of the studies on manufacturing enterprise resilience focus on supply chain networks. Paper [6] studies supply chain management to respond to terrorist attacks. Risk management to reduce supply chain vulnerability is discussed in [7]. Case studies in [12] show that companies' resilience can be bolstered by either building in redundancy or in flexibility. Vulnerable options in supply chain is studied in [13] by using a single-period, multi-stage model of a two-echelon supply chain with competing risky suppliers and single manufacturer. Paper [14] discusses the importance of decoupling disruption and recurrent supply risk during mitigation strategy planning.

In spite of these efforts, the study on resilience in manufacturing enterprises is still limited. A systematic method to quantitatively study the resilience of a manufacturing enterprise by addressing its redundancy, recovery capabilities, and its control policies has not been well developed to date. The tools that allow the modeling, analysis and simulation of resilience in a manufacturing enterprise are not available.

The goal of this research is to provide some principles and tools to help general enterprises to deal with the problems of disruptions, and enhance the resilience of enterprises to them. The research is focused on the control issue within the dynamic process, in order to allow the system to respond appropriately during the disruption.

The remainder of the paper is structured as follows: An enterprise model and problem formulation are introduced in Section II. Using this model, a simple system is analyzed in Section III. The approach of aggregation is described in Section IV. Finally, Section V presents the conclusions. Due to page limitation, all proofs are omitted and can be found in [15].

978-1-4244-2794-9/09/$25.00 ©2009 IEEE

II. System Description And Problem Formulation

A. Network System Model

A manufacturing enterprise can be structured as a network of nodes defining the supply chain from sources to customer. The nodes on the network represent facilities, operations, transportation links, sources, and consumers. Each node has dynamics that describe the transformation or flow of product, the use of resources, the capacity of operations, and the cost and responsiveness to change.

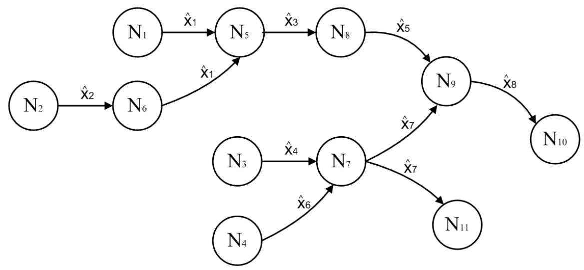

Fig. 1. A manufacturing enterprise network

Node: Each node represents an operation to transform one or more inventories into one or more other inventories. In the model, we have n_p *inventories*, each is denoted as $\hat{x}_i$, where $i = 1, 2, ..., n_p$. A $(n_p \times 1)$ vector $x(k)$ is used to represent the quantity of all n_p products in the system at the end of a time period k. The element of $x(k)$, $x_i(k)$, stands for the quantity of $\hat{x}_i$ at time k. An operation of $node\ i$ is represented by a $(n_p \times 1)$ vector b_i. For example,

$$b_1 = \left[\begin{array}{ccccc} -1 & -2 & 1 & 0 & 0 \end{array} \right]^T.$$

The operation b_1 represents an assembly of one of product $\hat{x}_1$ with two of product $\hat{x}_2$ to produce one of $\hat{x}_3$.

Network Matrix and Operation Vector: Let n_o indicate the number of nodes or *operations*. Define the $(n_p \times n_o)$ matrix $B = [b_1, b_2, \cdots, b_o]$. Let the $(n_o \times 1)$ vector $u(k)$ represent the production operation executed at time period k, such that the value of the element $u_i(k)$ will be the number of times that operation of $node\ i$ occurs over time period k. We assume there is no loss or spoilage of inventory. This then gives us the inventory update equation:

$$x(k+1) = x(k) + Bu(k). \quad (1)$$

There are n_r resources (such as equipment or facilities) needed in operations. The use of resources in operations is represented by the $(n_r \times n_0)$ nonnegative matrix R. There are n_d types of final products which are demanded. Then D is a $n_d \times n_o$ matrix mapping the operations that represent a removal of a product from within the system to a customer out of the system.

Constraints: There are several constraints for $x(k)$ and $u(k)$. We only consider four kinds of constraints. *No Backorder Constraint* states that inventory of each item can never be negative; *Capacity Constraint* states operations cannot exceed resource limits over any time period; *Demand Constraint* states that product sold cannot exceeds demand; *Operation Direction Constraint* states that operation direction can only be from raw material to product. These four constraints can be described as:

$$\text{NB-Constraint:} \quad (\forall k): \quad x(k) \geq 0; \quad (2)$$

$$\text{Capacity Constraint:} \quad (\forall k): \quad Ru(k) \leq c(k); \quad (3)$$

$$\text{Demand Constraint:} \quad (\forall k): \quad Du(k) \leq d(k); \quad (4)$$

$$\text{Operation Direction Constraint:} \quad (\forall k): \quad u(k) \geq \overrightarrow{0}, \quad (5)$$

where $c(k)$ is a $n_r \times 1$ vector indicating the capacity limit of each resource, and $d(k)$ is a $n_d \times 1$ vector indicating the expected amount of each demand at time k.

Disruption: A disruption is an undesired reduction in the capacity of a resource, represented as a reduction in $c(k)$ over a period of time. A disruption thus reduces the operations that can be done under the Capacity Constraint, and thus potentially or eventually reduces the ability of the system to satisfy demand. The system can be considered as having at least two states: *Nominal State* and *Disruption State*. The nominal state is the state when capacity is not reduced by disruption. In the nominal state, for any resource i, $c_i(k) = \overline{c}_i$. We denote the nominal capacity vector as $\overline{c}$. The disruption state is the one when the capacity of some resource(s) is reduced by disruption. In the disruption state, the duration of disruption state is denoted as k_{dis}, and the starting point is denoted as K. In this paper, we assume that if resource i is reduced by disruption, the disruption results in a zero capacity of this resource, so that $c_i(k) = 0$ for $k \in [K, K + k_{dis})$.

Cost: In our objective function described in Section II-C, we consider that the total cost is made up of three kinds of costs: production cost $(q_u u(k))$, inventory storage cost $(q_x x(k))$, and lost demand cost $(d(k) - Du(k))$.

System Model: To conclude the items we model above, a system can be defined with B, R, D, q_u, q_x, c, and d. Such a system is denoted as $\mathcal{N} = (B, R, D, q_u, q_x, c, d)$, where c and d are functions of k, and $q_x \in \Re^{1 \times n_p}$ and $q_u \in \Re^{1 \times n_o}$.

B. Nodes Description

1) Parameters of Nodes: Each node represents an operation. For a node N_i, it's operation amount is $u_i(k)$. When N_i operates, four things can be influenced: inventory amount, production cost, resource used, and demand filled. These four changes are represented by the following:

$$\begin{aligned} \text{change of inventory: } Bu(k) &= [b_1 \quad b_2 \quad ... \quad b_n]u(k); \\ \text{resources used: } Ru(k) &= [r_1 \quad r_2 \quad ... \quad r_n]u(k); \\ \text{demand filled: } Du(k) &= [d_1^D \quad d_2^D \quad ... \quad d_n^D]u(k); \\ \text{production cost: } q_u u(k) &= [q_u(1) \quad q_u(2) \quad ... \quad q_u(n)]u(k). \end{aligned}$$

From the above, we can represent the parameters of node N_i as the constant vector:

$$P(N_i) = \begin{bmatrix} b_i & r_i & d_i^D & q_u(i) \end{bmatrix}^T.$$

Then the effect of the operation of N_i can be expressed as:

$$P(N_i)u_i(k) = \begin{bmatrix} b_i u_i(k) & r_i u_i(k) & d_i^D u_i(k) & q_u(i)u_i(k) \end{bmatrix}^T.$$

We focus on two node structures: Nodes with fixed operations ratios, and OR nodes.

a) Nodes with fixed operations ratio:

Definition 1 (Nodes with Fixed Operations Ratio)**.** *Consider a set of nodes $\{N_1, N_2, ..., N_m\}$, whose operations are $\{u_1(k), u_2(k), ..., u_m(k)\}$, respectively. If there exist constants $\hat{u}_i$ for $i = 1, 2, ..., m$, and $\forall k$:*

$$\frac{u_1(k)}{\hat{u}_1} = \frac{u_2(k)}{\hat{u}_2} = ... = \frac{u_m(k)}{\hat{u}_m} = u^r(k), \qquad (6)$$

then we refer to the set of nodes $\{N_1, N_2, ..., N_m\}$ as nodes with fixed operations ratio.

In most cases, such nodes appear when no inventory changes among them. For the set of nodes $\{N_1, N_2, ..., N_m\}$, if all of inventory $\hat{x}_i$ produced by N_a is fed to N_b ($a, b \in \{1, 2, ..., m\}$), $u_a(k)$ and $u_b(k)$ can be chosen such that x_i is not changed by N_a and N_b. Such nodes include the serial nodes where upstream nodes produce the exact amount of inventory required by the downstream nodes. Such nodes also occur in AND nodes where several nodes feed a single downstream node at a ratio of operations defined by the needs of downstream node operation. In either type of structure, if one node's operation in the subset of nodes is known, all other nodes' operations can be easily calculated.

Definition 2 (OR nodes)**.** *For a set of nodes $\{N_1, N_2, ..., N_m\}$, if*

1) *each node N_i produces the same type output inventory $\hat{x}_{out}$ and has no input inventory, and*
2) *all the $\hat{x}_{out}$'s produced by N_i are fed as inputs to the same node N_{m+1}, and*
3) *$\hat{x}_{out}$'s are the only inputs of N_{m+1},*

then the set of nodes $\{N_1, N_2, ..., N_m\}$ are defined as OR nodes.

In such a structure, the downstream node can select any of the OR nodes to complete its production, although different selections can lead to different costs.

C. Problem Formulation

The objective is to minimize the cost by applying $u(k)$, which is the control signal representing the operation commands applied to the system. Our goal is to find the optimal $u(k)$ to achieve the minimum cost. We evaluate the total cost within a time window defined by k_p and k_f, which are the past and future coverage, respectively. The objective function is the total cost over this time window, which is:

$$C = \sum_{k'=k-k_p}^{k+k_f} \{a_u q_u u(k') + a_x q_x x(k') + a_d[d(k') - Du(k')]\}. \qquad (7)$$

The scalar values a_u, a_x, a_s, and a_d allow us to weight various elements of the cost, and all of them are non-negative numbers. Determining the control u can thus be done by solving a linear integer program to minimize C.

In order to simplify the analysis in this paper, we introduce the following assumptions.

Assumption 1. *$a_d \gg a_u$, $a_d \gg a_x$, so that a necessary condition of minimum total cost is that lost demand cost is minimum.*

Assumption 2. *Assume $\overline{c} \gg Ru(k)$ when $k \notin [K, K + k_{dis})$, which means all resources are always sufficient in nominal state; $c_i(k) = 0$ when $k \in [K, K + k_{dis})$ and resource i is disrupted, which implies the amount of disrupted resource is zero in disruption state. We use R_i to denote the matrix mapping $u(k)$ to resource i.*

Assumption 3. *$k_f \geq 1$, i.e., the starting time of disruption can be known in advance, in k_f time units before disruption happens.*

Then the problem is formulated as below, with a second priority objective function that is a concern only after the first priority objective function is met.

$$\begin{aligned}
Minimize: \quad & \sum_{k'=k-k_p}^{k+k_f} \{a_d[d(k') - Du(k')]\}, \\
& \sum_{k'=k-k_p}^{k+k_f} \{a_u q_u u(k') + a_x q_x x(k')\}, \\
Subject\ to: \quad & x(k) \geq \overrightarrow{0}, \\
& u(k) \geq \overrightarrow{0}, \\
& R_i u(k) = 0 \ when \ k \in [K, K + k_{dis}), \\
& Du(k) \leq d(k), \\
& x(k+1) = x(k) + Bu(k), \\
& k_f \geq 1, \\
& k > k_p, \\
& k, k_p, k_f \in \mathbb{N},
\end{aligned}$$

where i is the index of the disrupted resource.

III. Analysis of a Simple System

In this section we develop a mathematical model for a simple system in order to calculate the control solution used to reduce the impact of the disruption. Our analysis shows the process of solving the optimal problem we formulate. The solution is the optimal control signal, which can help to analyze how the system performs and responds. As the simple system is a building block of more complex system, the results of the simple system are useful for the analysis of complex system.

Consider a simple system shown in Figure 2 below:

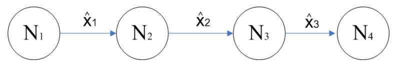

Fig. 2. A Simple System

The network matrix can be denoted as:

$$B = [b_1 \quad b_2 \quad b_3 \quad b_4] = \begin{bmatrix} m_1 & -m_1 & 0 & 0 \\ 0 & m_2 & -m_2 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}.$$

Demand is d, and demand matrix is $D = [0 \;\; 0 \;\; 0 \;\; 1]$. Initial inventory is

$$\begin{aligned} x(0) &= \overrightarrow{0}, \\ q_u &= [q_u(1) \;\; q_u(2) \;\; q_u(3) \;\; q_u(4)], \\ q_x &= [q_x(1) \;\; q_x(2) \;\; q_x(3)]. \end{aligned}$$

The goal is to analyze how the system will respond when disruption occurs at N_2, i.e., $R_2 = [0 \;\; r_2 \;\; 0 \;\; 0]$. The time when the disruption occurs is K. The length of the disruption can be predicted is denoted as k'_{dis}. Its value equals to the smaller one between k_f and the real length of the disruption.

To analyze this simple system, we consider several cases to determine the optimal control signal u (the operations of all the nodes) for the system under a disruption, which are summarized below:

Case 1: Nominal stage. $k < K - k_f$.

$$u(k) = [d \;\; d \;\; d \;\; d]^T.$$

Case 2: Preparing for disruption. $K - k_f \leq k \leq K - 2$.

$$u(k) = [d \;\; d \;\; d \;\; d]^T.$$

Case 3: Right before disruption. $k = K - 1$.

Case 3a: For $q_x(2)m_2 > q_x(3)$,

$$u(k) = [d + d \cdot k'_{dis} \;\; d + d \cdot k'_{dis} \;\; d + d \cdot k'_{dis} \;\; d]^T.$$

Case 3b: For $q_x(2)m_2 \leq q_x(3)$,

$$u(k) = [d + d \cdot k'_{dis} \;\; d + d \cdot k'_{dis} \;\; d \;\; d]^T.$$

Case 4: During disruption. $K \leq k \leq K + k'_{dis} - 1$.

Case 4a: For $q_x(2)m_2 > q_x(3)$,

$$u(k) = [0 \;\; 0 \;\; 0 \;\; d]^T.$$

Case 4b: For $q_x(2)m_2 \leq q_x(3)$,

$$u(k) = [0 \;\; 0 \;\; d \;\; d]^T.$$

We conclude the results as follows:

- To deal with the disruption, the system needs to build up $\hat{x}_2$ or $\hat{x}_3$. To choose which one depends on their corresponding inventory storage cost, which are $q_x(2)m_2$ and $q_x(3)$, respectively. The system will choose the cheaper one to build up.
- If $\hat{x}_3$ is built up, the operations of N_1, N_2 and N_3 will increase to $d + d \cdot k'_{dis}$ in the cycle right before disruption, and then decrease to 0 during the disruption. N_4 will keep the operation amount of d until the k'_{dis}-th cycle after disruption happens. The total cost increase is $a_x q_x(3) d \frac{(k'_{dis}+1)k'_{dis}}{2}$.
- If $\hat{x}_2$ is built up, the operations of N_1 and N_2 will increase to $d + d \cdot k'_{dis}$ in the cycle right before disruption, and then decrease to 0 during the disruption. N_3 and N_4 will keep the operation amount of d until the k'_{dis}-th cycle after disruption happens. The total cost increase is $a_x q_x(2) m_2 \cdot d \frac{(k'_{dis}+1)k'_{dis}}{2}$.
- If $q_x(2)m_2 = q_x(3)$, the system can build up either $\hat{x}_3$ or $\hat{x}_2$. The total cost will be identical for these two controls.

In order to achieve the lowest total cost, the system will choose one inventory to build up according to the storage cost. The amount is determined by the disruption duration. For the operations, related nodes will increase their work right before disruption to build up inventory, and during the disruption the necessary nodes will work to meet the demand.

When disruptions occur at other nodes, the analysis is similar.

IV. Analysis of Complex System

The analysis above shows that the approach for simple systems is available. To analyze complex systems, we present a method of Aggregation, where a group of nodes are combined to create a single node with similar identical behaviors to the original set of nodes. The aggregated network can then be analyzed using the results of simple systems.

In this section, first we propose the basic rules which the desired aggregation should satisfy. Then the method of aggregating nodes without disruption is introduced. Also, the disaggregation of the control law of such nodes is discussed.

A. Aggregate Nodes without Disruption

Suppose the nodes to be aggregated are N_1, N_2, ..., and N_m, and the aggregated node to represent these is denoted as N_{agg}. The aggregation is defined with a function $Agg(\cdot)$ over parameter vectors of $\{N_1, N_2, ..., N_m\}$ such that

$$P(N_{agg}) = Agg(P(N_1), P(N_2), ..., P(N_m)).$$

The parameters of N_{agg} are denoted by $P(N_{agg}) = [b_{agg}, q_u(agg), r_{agg}, d^D_{agg}]^T$. An aggregation function $Agg(P(N_1), P(N_2), ..., P(N_m))$ satisfies the *Aggregation Property* if for all the time k and operation sequence u (over nodes $N_1, ...N_m$), there exists a u_{agg} (over node N_{agg}) such that

1) The inventory change due to the production of N_{agg} is the same as the inventory change due to the production of $\{N_1, N_2, ..., N_m\}$:

$$b_{agg} u_{agg}(k) = [b_1 \; b_2 \; ... \; b_m][u_1(k) \; u_2(k) \; ... \; u_m(k)]^T.$$

2) The production cost of N_{agg} is the same as production cost of $\{N_1, N_2, ..., N_m\}$:

$$q_u(agg) u_{agg}(k) = [q_u(1) \;\; q_u(2) \;\; ... \;\; q_u(m)] \cdot \left[u_1(k) \;\; u_2(k) \;\; ... \;\; u_m(k)\right]^T.$$

3) The amount of resources used by N_{agg} is the same as the amount of resources used by $\{N_1, N_2, ..., N_m\}$:

$$r_{agg}u_{agg}(k) = [r_1 \ r_2 \ ... \ r_m][u_1(k) \ u_2(k) \ ... \ u_m(k)]^T.$$

4) The demand filled by N_{agg} is the same as the demand filled by $\{N_1, N_2, ..., N_m\}$:

$$d^D_{agg}u_{agg}(k) = [d^D_1 \quad d^D_2 \quad ... \quad d^D_m]\cdot \\ \begin{bmatrix}u_1(k) & u_2(k) & ... & u_m(k)\end{bmatrix}^T.$$

The *Aggregation Property* is summarized by:

$$\begin{aligned}[P(N_1) \ P(N_2) \ ... \ P(N_m)][u_1(k) \ u_2(k) \ ... \ u_m(k)]^T \\ = P(N_{agg})u_{agg}(k).\end{aligned} \tag{8}$$

This equation implies that the inventory amount, production cost, resource used and demand filled do not change between the system before or after aggregation. Thus, the behavior of N_{agg} is equivalent to the behavior of the set of nodes $\{N_1, N_2, ..., N_m\}$.

Then, we claim that an aggregation function $Agg(\cdot)$ satisfies the *Aggregation Property* if for all the time k and u, there exists a u_{agg} such that Equation (8) is satisfied. The operations $u(k)$ and $u_{agg}(k)$ correspond under aggregation function $Agg(\cdot)$ if Equation (8) is satisfied for all k.

In this paper, we present aggregation functions for two types of system structures: sets of nodes with Fixed Operations Ratios, and sets of OR nodes.

For nodes with fixed operations ratio, the aggregation is to add the parameters of all the nodes based on the fixed ratio.

Theorem 1 (Aggregation for nodes with fixed operations ratio)**.** *If a set of nodes $N_1, N_2, ..., N_m$ are nodes with fixed operations ratio defined by Definition 1, and if we define $Agg(\cdot)$ over these nodes as:*

$$\begin{aligned} &Agg(P(N_1), P(N_2), ..., P(N_m)) \\ :=&\hat{u}_1P(N_1) + \hat{u}_2P(N_2) + ...\hat{u}_mP(N_m) \\ =&[P(N_1) \ P(N_2) \ ... \ P(N_m)][\hat{u}_1 \ \hat{u}_2 \ ... \ \hat{u}_m]^T,\end{aligned} \tag{9}$$

then $Agg(\cdot)$ satisfies the Aggregation Property.

It follows from Theorem 1 and Equation (9) that

$$b_{agg} = \hat{u}_1b_1 + \hat{u}_2b_2 + ...\hat{u}_mb_m; \tag{10}$$

$$q_{u,agg} = \hat{u}_1q_u(1) + \hat{u}_2q_u(2) + ...\hat{u}_mq_u(m). \tag{11}$$

As we do not consider any disruptions within the aggregated nodes, every node can work perfectly and determine its operations based on the demand. All of the inventory produced by the upstream nodes will be used up by the downstream nodes. It is not necessary to build up inventory among them. The subsystem could respond fast enough to the change of demand. Therefore, in order to minimize the cost, the nodes in this subsystem will always keep a fixed operation ratio. If the operation of any of the nodes is known, we can easily obtain the operation of all other nodes by multiplying the fixed ratio. Therefore, it is possible to use only one operation to control all the nodes, which are working as a whole at the same time. The parameter vector of the aggregated node is the weighted sum of those of each node.

For OR nodes, the aggregation is carried out by selecting one node among them as the aggregated node. The node selected is the one which has the lowest cost to produce the same amount of output inventory.

Theorem 2 (Aggregation for OR nodes)**.** *Assume a set of nodes $N_1, N_2, ..., N_m$ are OR nodes defined by definition 2. $b_i(\hat{x}_{out})$ is the element in b_i associated with $\hat{x}_{out}$, which is the amount of $\hat{x}_{out}$ produced by N_i with unit operation. $q_u(i)$ is the unit production cost of N_i. If j is the index such that*

$$\frac{q_u(j)}{b_j(\hat{x}_{out})} \leq \frac{q_u(i)}{b_i(\hat{x}_{out})}, \quad \forall i,$$

and we define $Agg(\cdot)$ over these nodes as:

$$Agg(P(N_1), P(N_2), ..., P(N_m)) := P(N_j). \tag{12}$$

Then $Agg(\cdot)$ satisfies the Aggregation Property.

A result of Theorem 2 is

$$b_{agg} = b_j \qquad q_u(agg) = q_u(j). \tag{13}$$

In the case of OR nodes, as with the fixed production ratio nodes, it is not necessary to build up any inventory. The optional nodes can change operations identically. The only difference among them is in the production cost. Assume N_j has the lowest cost. At any time, if any other optional node is producing, we can shift its work to N_j so that the total production cost can be reduced. Thus, we can make the operations of all nodes be 0 except N_j.

The above approaches describe the aggregation in a single step. Sometimes, it is necessary to do multiple aggregations in a sequence. A useful property is found for multiple aggregations.

Definition 3 (Associativity)**.** *Suppose there are m nodes: $N_1, N_2, ..., N_m$. Let m_1, m_2, ... , m_g be integers such that:*

$$1 \leq m_1 < m_2 < ... < m_{g-1} < m_g = m,$$

then an aggregation function $Agg(\cdot)$ satisfies the Associativity property if:

$$\begin{aligned}P(N_{agg}) =&Agg(P(N_1), P(N_2), ..., P(N_m)) \\ =&Agg[Agg(P(N_1), P(N_2), ...P(N_{m_1})), \\ &\quad Agg(P(N_{m_1+1}), P(N_{m_1+2}), ..., P(N_{m_2})), \ldots, \\ &\quad Agg(P(N_{m_{g-1}+1}), P(N_{m_{g-1}+2}), ..., P(N_m))].\end{aligned}$$

Theorem 3 (Associativity)**.** *The method of aggregation given by Equations (9) and (12) in Theorems 1 and 2 satisfies associativity.*

Theorem 3 shows that the aggregation can be carried out recursively. It will make no difference whether all the nodes are aggregated in a single step, or in multiple steps.

B. Disaggregate Control

By aggregation, we can simplify the system. After the control solution for this simplified system is obtained, the problem is how to apply this control to the original system. Assume we know the original system structure and have the operation command u_{agg} for the aggregated system, we need to find out how to calculate the corresponding operation u for the original system. This is referred to as disaggregate control.

For the two subsystem structures that we have examined for aggregation, we present the following results for the disaggregate control:

Theorem 4 (Disaggregate control of nodes with fixed production ratio)**.** *Let $\{N_1, N_2, ..., N_m\}$ be a set of nodes defined by Definition 1, and N_{agg} be the aggregated node obtained from Equation (9). Assume the aggregated node has the operation command of $u_{agg}(k)$. In addition, denote the original subsystem operation command as $u(k) = [u_1(k) \;\; u_2(k) \;\; ... \;\; u_m(k)]$. Let*

$$\begin{aligned} &[u_1(k) \;\; u_2(k) \;\; ... \;\; u_m(k)] \\ &= [u_{agg}(k)\hat{u}_1 \;\; u_{agg}(k)\hat{u}_2 \;\; ... \;\; u_{agg}(k)\hat{u}_m], \end{aligned} \quad (14)$$

where $\hat{u}_i \forall i$ is defined as in Definition 1. Then $u(k)$ and $u_{agg}(k)$ correspond under aggregation function $Agg(\cdot)$.

Theorem 5 (Disaggregate control of OR nodes)**.** *Let $\{N_1, N_2, ..., N_m\}$ be a set of notes defined by Definition 2. N_{agg} is the aggregated node obtained from Equation (12). Assume the aggregated node has the operation command of $u_{agg}(k)$. In addition, denote the original subsystem operation command as $u(k)$. Let*

$$u_i(k) = \begin{cases} u_{agg}(k), & when \;\; i = j; \\ 0, & when \;\; i \neq j, \end{cases} \quad (15)$$

where j is defined as in Theorem 2 as the index of the node with the lowest weighted production cost. Then, $u(k)$ and $u_{agg}(k)$ correspond under aggregation function $Agg(\cdot)$.

Thus, for fixed production ratio nodes, the production of the aggregated node is assigned to the original nodes according to the production ratios. For OR nodes, the production of the aggregated node is assigned to the OR node that has the lowest weighted production cost.

V. Conclusion

In this paper, we present a mathematical model to represent enterprises and their production operations. An analysis of a model of a simple system is introduced to characterize the system under disruption. Using them as a building block for the study of resilience, we considered the problem of aggregation, in order to simplify a set of nodes to a single node with equivalent behavior. A control strategy can be calculated for the aggregated system, and then be mapped back to the individual nodes of the original system.

References

[1] Y. Hu, J. Li and L.E. Holloway, "Towards Modeling of Resilience Dynamics in Manufacturing Enterprises: Literature Review and Problem Formulation," *Proc. of IEEE Int. Conf. on Automation Science and Engineering*, pp. 279-284, Washington DC, 2008.

[2] Y. Sheffi, *The Resilient Enterprise: Overcoming Vulnerability for Competitive Advantage*, MIT Press, 2005.

[3] J. Blackhurst, C.W. Craighead, D. Elkins and R.B. Handfield, "An Empirically Derived Agenda of Critical Research Issues for Managing Supply-Chain Disruptions," *Int. J. of Production Research*, vol. 43, pp. 4067-4081, 2005.

[4] R. Starr, J. Newfrock and M. Delurey, "Enterprise Resilience: Managing Risk in the Networked Economy," *Strategy and Business*, vol. 30, no. 1, pp. 110, 2003.

[5] O. Helferich and R. Cook, *Securing the Supply Chain*, Council of Logistics Management, 2002.

[6] Y. Sheffi, "Supply Chain Management under the Threat of International Terrorism," *Int. J. of Logistics*, vol. 12, no. 2, pp. 1-11, 2001.

[7] U. Juttner, "Supply Chain Risk Management: Understanding the Business Requirements from a Practitioner Perspective," *The Int. J. of Logistics Management*, vol. 16, pp. 120-141, 2005.

[8] B. Wang, H. Tang, C. Guo, Z. Xiu and T. Zhou, "Optimization of Network Structure to Random Failures," *Physica A*, vol. 268, pp. 607-614, 2006.

[9] A. Konak and M.R. Bartolacci, "Designing Survivable Resilient Networks: A Stochastic Hybrid Genetic Algorithm Approach," *Omega*, vol. 35, pp. 645-658, 2007.

[10] J. Ash, D. Newth, "Optimizing Complex Networks for Resilience Against Cascading Failure," *Physica A*, vol. 280, pp. 673-683, 2007.

[11] G. Brown, M. Carlyle, J. Salmern and K. Wood, "Defending Critical Infrastructure," *Interface*, vol. 36, pp. 530-544, 2006.

[12] Y. Sheffi and J.B. Rice, Jr., "A Supply Chain View of the Resilient Enterprise," *MIT Sloan Management Review*, vol. 47, pp. 41-48, Fall 2005.

[13] V. Babich, "Vulnerable Options in Supply Chains: Effects of Supplier Competition," *Naval Research Logistics*, vol. 53, pp. 656-673, 2006.

[14] S. Chopra, G. Reinhardt and U. Mohan, "The Importance of Decoupling Recurrent and Disruption Risks in a Supply Chain," *Naval Research Logistics*, vol. 54, pp. 544-555, 2007.

[15] Y. Hu, J. Li, and L. E. Holloway "The Modeling, Analysis and Control of Resilient Manufacturing Enterprises," Working Paper, Dept. of Electrical and Computer Engineering, University of Kentucky, 2009.