Proceedings of the 2009 IEEE International Conference on Systems, Man, and Cybernetics
San Antonio, TX, USA - October 2009

Adaptive Control of Partially Known Continuous-Time Systems

Rasul Mohammadi
Control and Robotics Group
Department of Electrical and Computer Engineering,
Concordia University
Montréal, QC, Canada, H3G 1M8
E-mail: a_mohamm@ece.concordia.ca

***Abstract*—An approach is developed for the adaptive model-reference control of the continuous-time systems consisting of known and unknown subsystems. It is assumed that the plant is composed of the cascade, parallel or feedback interconnection of known and unknown subsystems. Many physical control systems can be modeled by such interconnections. In our approach, by using the prior knowledge about the plant, the adaptive law is driven only by the states of unknown subsystems. Moreover, we demonstrate by simulations that incorporating the partial knowledge about the system improves the response of the system in terms of tracking error and response time.**

***Index Terms*—Adaptive control; model-reference control; partially known systems.**

I. Introduction

In most of the work on adaptive controller design, the plant under control is modeled as being completely unknown (see for example, [1], [3]). In this paper, we study systems that are partially known; there is some prior information about the system. Using the prior knowledge about the plant to make the adaptive algorithm simpler and more efficient has been the subject of research (see e.g., [2], [4], [5], [6], [7], [8]).

As an early work, in [2], some schemes are presented for discrete-time adaptive control of a linear time-invariant (LTI) system which is partially known. The plant is modeled by a single transfer function, and partial plant knowledge is simulated by assuming that some of the parameters of the transfer function are known. In fact, the prior information about the system is incorporated to reduce the number of parameters to be identified in a single transfer function.

In [4], a block-diagram representation is used for realistic characterization of partial plant knowledge. It is assumed that the plant is composed of the interconnection of some basic subsystems, each represented by a discrete-time model. The prior information about the system is used to distinguish between known and unknown subsystems. It is also assumed that the system is composed of the interconnection of two basic blocks each of which consisting of the cascade of a known and an unknown subsystem.

In [8], the prior knowledge about the system is employed for simplifying the adaptive algorithm in continuous-time systems. The partial knowledge of the stable zero or pole dynamics in linear plants are used to reduce the order of the closed-loop systems and therefore, simplifying the adaptive algorithm.

In this work, we study adaptive control design in partially known *continuous-time* systems. Similar to the approach in [4], we use a block-diagram representation for characterization of partial plant knowledge, and extend the results of [4] to continuous-time systems. We assume that the plant is consisting of subsystems (basic blocks) each modeled by a continuous-time state space. We also assume that the plant is composed of the cascade, parallel and feedback interconnection of basic blocks. It can be shown that many systems fall into one of these interconnection structures.

First, we consider a case where the plant is composed of one known basic block and one unknown basic block. We show that in all the three configurations of the blocks, the update law of the control algorithm is only driven by the states of the unknown subsystem; however the error of all states is still needed. This results in a simpler control algorithm which can be implemented easier.

Moreover, we generalize the results to the cases where the plant is modeled by a block diagram consisting of multiple known and unknown subsystems. We also provide a sufficient condition for decoupling the update law such that the updating algorithm is only driven by the errors corresponding to the states of unknown subsystems. This implies that no measurement of known states is needed for driving the adaptive algorithm.

We show by simulations that the response of the system for model-reference tracking is improved in terms of tracking error and response time compared with the case that no prior knowledge about the plant is used.

The remainder of the paper is organized as follows. Sec. II presents the problem formulation. The continuous-time adaptive method employing partial plant knowledge is explained in section III. In section IV, we propose a sufficient condition under which the adaptive update law is only driven by the errors of the states of unknown subsystems. Section V shows the simulation results. We present the conclusion in section VI.

II. Problem Formulation

The system studied in this paper is composed of the series, parallel and feedback interconnection of simpler subsystems. It can be shown that many systems fall into one of these interconnection structures. Each subsystem is represented by

978-1-4244-2794-9/09/$25.00 ©2009 IEEE

a single-input single-output (SISO) state space model and is referred to as a basic block. Assume that there are $K \geq 1$ basic blocks in the plant. The following state space model is assumed for subsystem i ($1 \leq i \leq K$).

$$S_i := \begin{cases} \dot{x}_i = A_i x_i + B_i u_i \\ y_i = C_i x_i + D_i u_i \end{cases},$$

where $A_i \in \mathbb{R}^{n_i \times n_i}, B_i \in \mathbb{R}^{n_i}, C_i \in \mathbb{R}^{1 \times n_i}$ and $D_i \in \mathbb{R}$.

We make the following assumption.

$\mathbf{A0}$: The matrices B_i and D_i of all basic blocks are known.

A basic block S_i is called *unknown* if the matrices A_i and C_i are unknown; otherwise, it is called a *known* basic block.

Let $n = \sum_{i=1}^{K} n_i$. The plant is represented as follows.

$$T := \begin{cases} \dot{x_p} = A_p x_p + B_p u \\ y_p = C_p x_s + D_p u \end{cases}, \tag{1}$$

where u and y are the input and output of the plant, respectively, $A_p \in \mathbb{R}^{n \times n}, B_p \in \mathbb{R}^n, C_p \in \mathbb{R}^{1 \times n}$ and $D_p \in \mathbb{R}$.

We further, assume that the plant is stable, no zero-pole cancelation occurs in the system, the pair (A_p, B_p) is controllable, and all of the states of the plant are measurable. The reference model is represented by:

$$M := \begin{cases} \dot{x_m} = A_m x_m + B_m r \\ y_m = C_m x_m + D_m r \end{cases}, \tag{2}$$

where $A_m \in \mathbb{R}^{n \times n}$ is assumed Hurwitz, and r is the reference signal and is assumed bounded. $B_m \in \mathbb{R}^n, C_m \in \mathbb{R}^{1 \times n}$ and $D_m \in \mathbb{R}$.

We hold the following two assumptions.

$\mathbf{A1}$: There exists a Q^* such that $B_m = B_p Q^*$.

$\mathbf{A2}$: There exists a $\theta^* \in \mathbb{R}^n$ such that $A_p + B_p \theta^{*T} = A_m$.

Let the state of the plant be $x_p = \begin{bmatrix} x_{p,un} \\ x_{p,kn} \end{bmatrix}$, where $x_{p,un}$ is the states of the unknown subsystems and $x_{p,kn}$ is the states of the known subsystems. Given the assumptions $\mathbf{A1}$ and $\mathbf{A2}$, the objective is to find an adaptive state feedback control for the plant (1) such that the adaptive update law is only driven by the states of the unknown subsystems, i.e., $x_{p,un}$, all the closed loop signals remain bounded, and the plant states track the states of the reference model, i.e., $\lim_{t \to \infty} (x_p - x_m) = 0$.

III. Continuous-Time Adaptive Control with Partial Plant Knowledge

In the following, initially, we assume that the plant consists of the interconnection of one known basic block and one unknown basic block, i.e., $K = 2$. We study adaptive control with prior plant knowledge in three different configurations of the two basic blocks. Generalization of the results to the case that $K > 2$ and combination of interconnections will follow.

For the case that $K = 2$, θ^{*T} is in the form $\theta^{*T} = \begin{bmatrix} \theta_1^{*T} & \theta_2^{*T} \end{bmatrix}$, where $\theta_i^* \in R^{n_i}$ for $1 \leq i \leq 2$. We also have $A_m = \begin{bmatrix} A_{1m} & A_{2m} \\ A_{3m} & A_{4m} \end{bmatrix}$, where $A_{1m} \in \mathbb{R}^{n_1 \times n_1}$, $A_{2m} \in \mathbb{R}^{n_1 \times n_2}$, $A_{3m} \in \mathbb{R}^{n_2 \times n_1}$ and $A_{4m} \in \mathbb{R}^{n_2 \times n_2}$.

Let the control be

$$u = \theta^T x_p + Q^* r. \tag{3}$$

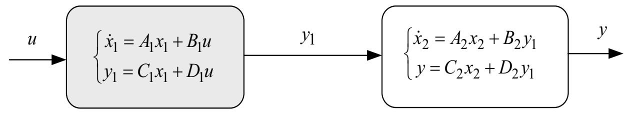

Fig. 1. A continuous-time plant with the series interconnection of basic blocks.

Since $K = 2$, θ has the form $\theta^T = \begin{bmatrix} \theta_1^T & \theta_2^T \end{bmatrix}$, where $\theta_i \in R^{n_i}$ for $1 \leq i \leq 2$. Defining the error as $e = x_p - x_m$, we have:

$$\dot{e} = A_m e + B_p \phi^T x_p, \tag{4}$$

where $\phi^T = \begin{bmatrix} \phi_1^T & \phi_2^T \end{bmatrix} = \begin{bmatrix} \theta_1^T - \theta_1^{*T} & \theta_2^T - \theta_2^{*T} \end{bmatrix}$.

A. Series Interconnection of Basic Blocks

Fig. 1 shows the series interconnection of the basic blocks. Suppose the basic block 1 is the unknown basic block. The unknown basic block is shown by gray. In this configuration, the plant matrices (A_p, B_p, C_p, D_p) can be obtained as

$$x_p = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix},$$

$$A_p = \begin{bmatrix} A_1 & 0 \\ B_2 C_1 & A_2 \end{bmatrix},$$

$$B_p = \begin{bmatrix} B_1 \\ B_2 D_1 \end{bmatrix},$$

$$C_p = \begin{bmatrix} D_2 C_1 & C_2 \end{bmatrix},$$

and

$$D_p = D_2 D_1.$$

A_i, B_i, C_i, D_i are the matrices of basic block i ($1 \leq i \leq 2$).

According to the assumption $\mathbf{A2}$, there exists a θ^{*T} such that $A_p + B_p \theta^{*T} = A_m$. Substituting A_p and B_p in the assumption, it yields:

$$\begin{bmatrix} A_1 & 0 \\ B_2 C_1 & A_2 \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 D_1 \end{bmatrix} \begin{bmatrix} \theta_1^{*T} & \theta_2^{*T} \end{bmatrix} = A_m.$$

Therefore, we obtain:

1) $A_1 + B_1 \theta_1^{*T} = A_{1m}$;
2) $B_1 \theta_2^{*T} = A_{2m}$;
3) $B_2 C_1 + B_2 D_1 \theta_1^{*T} = A_{3m}$;
4) $A_2 + B_2 D_1 \theta_2^{*T} = A_{4m}$.

We have assumed that B_1 and D_1 are known. Thus, from the above equations, θ_2^{*T} can be calculated using the known parameters, and we can take $\theta_2 = \theta_2^*$. Therefore, equation (4) can be written as

$$\dot{e} = A_m e + B_p \phi_1^T x_1. \tag{5}$$

Consider the following Lyapunov function:

$$V(e, \phi_1) = e^T P e + tr(\phi_1^T \phi_1), \tag{6}$$

where the symmetric positive-definite matrix $P \in \mathbb{R}^{n \times n}$ is the solution of the following Lyapunov equation:

$$A_m^T P + P A_m = -Q.$$

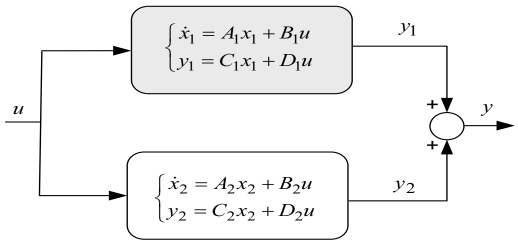

Fig. 2. A continuous-time plant with the parallel interconnection of basic blocks.

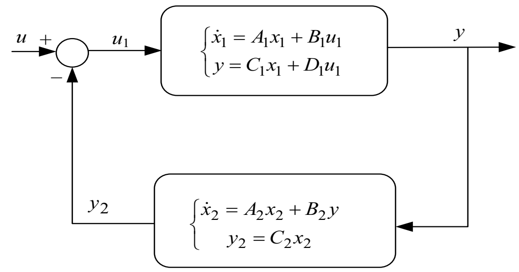

Fig. 3. A continuous-time plant with the feedback interconnection of basic blocks.

$Q \in \mathbb{R}^{n\times n}$ is a symmetric positive-definite matrix.

Theorem 1: Given the assumptions $\mathbf{A0}$, $\mathbf{A1}$ and $\mathbf{A2}$, if the adaptive control as in (3) with update law

$$\dot{\phi}_1 = -B_P^T Pex_1 \tag{7}$$

is applied to the system (1) with the series interconnection of one unknown basic block and one known basic block, then all the closed loop signals remain bounded and $\lim_{t\to\infty} x_p - x_m = 0$.

Proof: Using equation (5), the derivative of the Lyapunov function is calculated to be:

$$\dot{V} = -e^TQe + 2tr(\phi_1^T(B_P^TPex_1 + \dot{\phi}_1)). \tag{8}$$

Choosing $\dot{\phi}_1$ as in (7), we can verify that $\dot{V} = -e^TQe \le 0$. We can conclude that e and ϕ_1 remain bounded. From the assumption of the stability of the plant, it can be inferred that all closed-loop signals are bounded. Furthermore, using the Barbalat's Lemma, we can show that $\lim_{t\to\infty} e = 0$. ■

The update law as in (7) is only driven by the states of the unknown subsystems. However, the total error (error corresponding to all states) are used.

B. Parallel Interconnection of Basic Blocks

Fig. 2 shows the parallel interconnection of the basic blocks. Suppose the basic block 1 shown by gray is the unknown basic block. In this configuration, the plant matrices can be obtained as

$$A_p = \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix},$$

$$B_p = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix},$$

$$C_p = \begin{bmatrix} C_1 & C_2 \end{bmatrix},$$

and

$$D_p = D_1 + D_2.$$

Based on the assumption $\mathbf{A2}$, there exists a θ^{*T} such that $A_p + B_p\theta^{*T} = A_m$. Therefore, we have:

$$\begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} \begin{bmatrix} \theta_1^{*T} & \theta_2^{*T} \end{bmatrix} = A_m.$$

We obtain the following equations:

1) $A_1 + B_1\theta_1^{*T} = A_{1m}$;
2) $B_1\theta_2^{*T} = A_{2m}$;
3) $B_2\theta_1^{*T} = A_{3m}$;
4) $A_2 + B_2\theta_2^{*T} = A_{4m}$.

We have assumed that B_1 and D_1 are known. Thus, from the above equations, both θ_1^{*T} and θ_2^{*T} can be calculated using the known parameters, and we can take $\theta_1 = \theta_1^*$ and $\theta_2 = \theta_2^*$. Therefore, equation (4) can be written as

$$\dot{e} = A_m e. \tag{9}$$

It can be seen that in this case, the optimal values of both θ_1^{*T} and θ_2^{*T} can be calculated using the known parameters of the system. Therefore, no adaptive algorithm is required.

C. Feedback Interconnection of Basic Blocks

Fig. 3 shows feedback interconnections of the basic blocks. In feedback configuration, we assume that the subsystem in the feedback path is strictly proper. Thus, the plant matrices are:

$$A_p = \begin{bmatrix} A_1 & -B_1C_2 \\ B_2C_1 & A_2 - B_2D_1C_2 \end{bmatrix},$$

$$B_p = \begin{bmatrix} B_1 \\ B_2D_1 \end{bmatrix},$$

$$C_p = \begin{bmatrix} C_1 & -D_1C_2 \end{bmatrix},$$

and

$$D_p = D_1.$$

Based on the assumption $\mathbf{A2}$, there exists a θ^{*T} such that $A_p + B_p\theta^{*T} = A_m$. Therefore, we have:

$$\begin{bmatrix} A_1 & -B_1C_2 \\ B_2C_1 & A_2 - B_2D_1C_2 \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2D_1 \end{bmatrix} \begin{bmatrix} \theta_1^{*T} & \theta_2^{*T} \end{bmatrix} = A_m.$$

We obtain the following equations:

1) $A_1 + B_1\theta_1^{*T} = A_{1m}$;
2) $-B_1C_2 + B_1\theta_2^{*T} = A_{2m}$;
3) $B_2C_1 + B_2D_1\theta_1^{*T} = A_{3m}$;
4) $A_2 - B_2D_1C_2 + B_2D_1\theta_2^{*T} = A_{4m}$.

First suppose the unknown basic block is the basic block in the forward path. We have assumed that B_1 and D_1 are known. Thus, from the above equations, θ_2^{*T} can be calculated using the known parameters, and we can take $\theta_2 = \theta_2^*$. Therefore, equation (4) can be written as

$$\dot{e} = A_m e + B_p\phi_1^T x_1. \tag{10}$$

Consider the following Lyapunov function:

$$V(e, \phi_1) = e^T P e + tr(\phi_1^T \phi_1), \tag{11}$$

where the symmetric positive-definite matrix $P \in \mathbb{R}^{n\times n}$ is the solution of the following Lyapunov equation:

$$A_m^T P + P A_m = -Q.$$

$Q \in \mathbb{R}^{n\times n}$ is a symmetric positive-definite matrix. Using equation (10), the derivative of the Lyapunov function is calculated to be:

$$\dot{V} = -e^T Q e + 2tr(\phi_1^T (B_P^T P e x_1 + \dot{\phi}_1)). \tag{12}$$

Choose the update law

$$\dot{\phi}_1 = -B_P^T P e x_1. \tag{13}$$

We can verify that $\dot{V} = -e^T Q e \le 0$. Therefore, e and ϕ_1 remain bounded. Furthermore, using the Barbalat's Lemma, we can show that $\lim_{t\to\infty} e = 0$.

Now suppose, the basic block in the feedback path is the unknown basic block. Matrices B_2 and D_2 are assumed to be known. Thus, θ_1^{*T} can be calculated using the known parameters, and we can take $\theta_1 = \theta_1^*$. Therefore, equation (4) can be written as

$$\dot{e} = A_m e + B_p \phi_2^T x_2. \tag{14}$$

Consider the following Lyapunov function:

$$V(e, \phi_2) = e^T P e + tr(\phi_2^T \phi_2), \tag{15}$$

where the symmetric positive-definite matrix $P \in \mathbb{R}^{n\times n}$ is the solution of the following Lyapunov equation:

$$A_m^T P + P A_m = -Q.$$

$Q \in \mathbb{R}^{n\times n}$ is a symmetric positive-definite matrix. Using equation (14), the derivative of the Lyapunov function is calculated to be:

$$\dot{V} = -e^T Q e + 2tr(\phi_2^T (B_P^T P e x_2 + \dot{\phi}_2)). \tag{16}$$

Choose the update law

$$\dot{\phi}_2 = -B_P^T P e x_2. \tag{17}$$

We can verify that $\dot{V} = -e^T Q e \le 0$. Therefore, e and ϕ_2 remain bounded. Furthermore, using the Barbalat's Lemma, we can show that $\lim_{t\to\infty} e = 0$.

Let x_{un} be the states of the unknown subsystem, and ϕ_{un} be a vector consisting of the difference between the unknown control parameter and the corresponding optimal control parameter.

Theorem 2: Given the assumptions $\mathbf{A0}$, $\mathbf{A1}$ and $\mathbf{A2}$, if the adaptive control as in (3) with update law

$$\dot{\phi_{un}} = -B_P^T P e x_{un} \tag{18}$$

is applied to the system (1) with the feedback interconnection of one unknown basic block and one known basic block, then all the closed loop signals remain bounded and

$$\lim_{t\to\infty} x_p - x_m = 0.$$

■

The update laws as in (13) and (17) are only driven by the states of the unknown subsystems. However, the total error (error corresponding to all states) are used.

D. Generalization to Any Block Diagram Structure

Suppose the plant is consisting of multiple blocks with different interconnections. It can be shown that the adaptive algorithm developed for the case of $K = 2$ can be generalized to the systems consisting of $K \ge 3$ subsystems with different interconnections. In other words, assuming the B matrices of all subsystems to be known, the control parameters corresponding to the known subsystems of the plant can be calculated using assumption $\mathbf{A2}$. Then, using the Lyapunov function

$$V(e, \phi_{un}) = e^T P e + tr(\phi_{un}^T \phi_{un}),$$

where ϕ_{un} is a vector consisting of the difference between the unknown control parameters and the corresponding optimal control parameters, the following update law can be developed.

$$\dot{\phi_{un}} = -B_P^T P e x_{un}, \tag{19}$$

Let x_{un} be the vector of augmented states of the unknown subsystems.

Theorem 3: Given the assumptions $\mathbf{A0}$, $\mathbf{A1}$ and $\mathbf{A2}$, if the adaptive control as in (3) with update law

$$\dot{\phi_{un}} = -B_P^T P e x_{un} \tag{20}$$

is applied to the system (1) with the interconnection of known and unknown basic blocks, then all the closed loop signals remain bounded and

$$\lim_{t\to\infty} x_p - x_m = 0.$$

■

IV. DECOUPLING THE UPDATE LAWS

So far, we have shown that using prior knowledge about the system, a control algorithm can be developed such that the update law is only driven by the states of the unknown subsystems. However, the error of all states is still needed. It is easier for implementation if the error of only unknown subsystems is used for the adaptive algorithm. In other words, the unknown subsystems become decoupled from the known subsystems for the update law. Here, we study the conditions that lead us to such decoupled adaptive schemes.

A sufficient condition for the update law to become only driven by the error of unknown subsystems is that in (20), we have:

$$B_p^T P = Z,$$

where Z is a matrix whose columns corresponding to the states of the known subsystems are zero. In other words, this condition puts a *constraint* on the structure of the solution of the Lyapunov equation (P).

In the following, we present our simulation results.

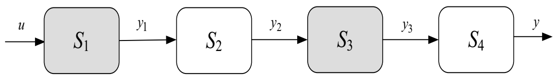

Fig. 4. Block Configuration 1.

V. Simulation Results

In this section, by MATLAB simulations, we show that by using the update laws driven only by the states of unknown subsystems for adaptive control, the reference-model tracking is improved; i.e., the overshoot is smaller than the case that no prior knowledge of the plant is employed in the adaptive control algorithm, and the response time is also smaller.

We consider an unknown 4th order SISO plant, and assume that the plant is made up of the interconnection of four subsystems S_1, S_2, S_3 and S_4. Moreover, we assume that the subsystems S_1 and S_3 are unknown and the subsystems S_2 and S_4 are known. The state space models of the plant's subsystems are

$$S_1 = \begin{cases} \dot{x_1} = k_1 x_1 + 4u_1 \\ y_1 = k_2 x_1 \end{cases},$$

$$S_2 = \begin{cases} \dot{x_2} = -4x_2 + 2u_2 \\ y_2 = -3x_2 + 2u_2 \end{cases},$$

$$S_3 = \begin{cases} \dot{x_3} = k_3 x_3 + 2u_3 \\ y_3 = k_4 x_3 \end{cases}$$

and

$$S_4 = \begin{cases} \dot{x_4} = -1.5x_4 + 0.5u_4 \\ y_4 = -x_4 + u_4 \end{cases},$$

where k_1, k_2, k_3 and k_4 are unknown parameters.

The plant has four states, each of which corresponding to one of the subsystems, i.e, $x_p = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$.

We present the simulation results for the series, the parallel and the feedback block configurations. For each block configuration, we consider a reference model such that assumptions **A1** and **A2** are satisfied. We take a step signal as the reference input to the model for the three block configurations.

A. Block Configuration 1

Fig. 4 shows a block configuration in which all subsystems S_1, S_2, S_3 and S_4 are in series. In this case, we have $u_1 = u$, $u_2 = y_1$, $u_3 = y_2$, $u_4 = y_3$ and $y = y_4$.

Consider the following reference model.

$$M = \begin{cases} \dot{x}_m = \begin{bmatrix} -2 & 0 & 0 & 0 \\ 5 & -4 & 0 & 0 \\ 10 & -6 & -3 & 0 \\ 0 & 0 & 0.75 & -1.5 \end{bmatrix} x_m + \begin{bmatrix} 4 \\ 0 \\ 0 \\ 0 \end{bmatrix} r \\ y_m = \begin{bmatrix} 0 & 0 & 1.5 & -1 \end{bmatrix} x_m \end{cases}.$$

Fig. 5 shows the response of the model and the response of the plant for the two cases of using prior knowledge about the known subsystems and not using prior knowledge in the adaptive control algorithm. It can be seen that the system response has a lower overshoot and smaller settling time when the prior knowledge about the known subsystems is used.

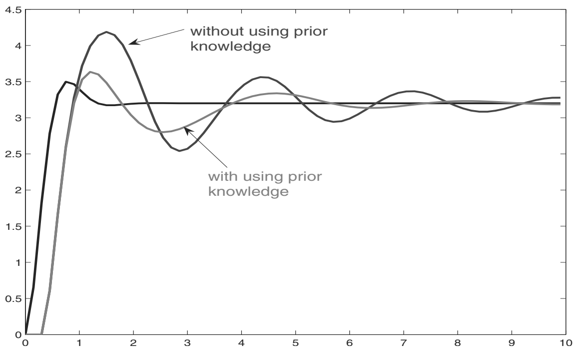

Fig. 5. Block Configuration 1: The response of the model and the response of the plant with different control algorithms.

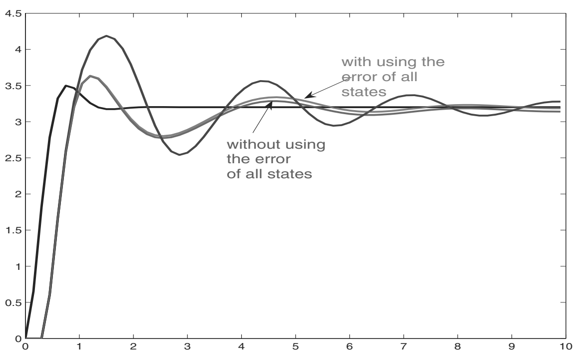

Fig. 6. Block Configuration 1: Comparison between the response of the plant for the case where the error of all states is used in the update law, and for the case where only the error of the unknown states is used.

Fig. 6 shows the response of the plant using an adaptive algorithm with decoupled update law. Here, matrix P is the solution of the constrained Lyapunov equation. It can be seen that the response of the plant with this update law is very similar to the case where the error of all states is used in the update law.

B. Block Configuration 2

Fig. 7 shows a block configuration in which the series of the subsystems S_1 and S_2 is in parallel with the series of the subsystems S_3 and S_4. In this case, we have $u_1 = u$, $u_2 = y_1$,

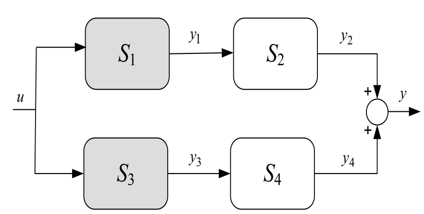

Fig. 7. Block Configuration 2.

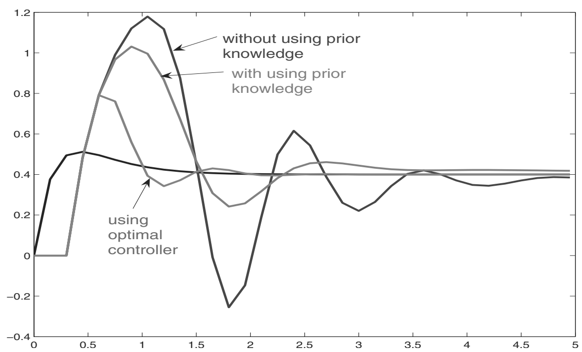

Fig. 8. Block Configuration 2: The response of the model and the response of the plant with different control algorithms.

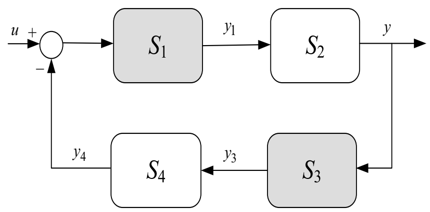

Fig. 9. Block Configuration 3.

$u_3 = u$, $u_4 = y_3$ and $y = y_2 + y_4$.

Consider the following reference model.

$$M = \begin{cases} \dot{x}_m = \begin{bmatrix} -2 & 0 & 0 & 0 \\ 5 & -4 & 0 & 0 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & 0.75 & -1.5 \end{bmatrix} x_m + \begin{bmatrix} 4 \\ 0 \\ 2 \\ 0 \end{bmatrix} r \\ y_m = \begin{bmatrix} 5 & -3 & 1.5 & -1 \end{bmatrix} x_m \end{cases}.$$

Fig. 8 shows the response of the model and the response of the plant for the two cases of using prior knowledge and not using prior knowledge in the adaptive control algorithm. The response of the plant for the case that the optimal controller is used (the plant is assumed to be completely known) has also been shown.

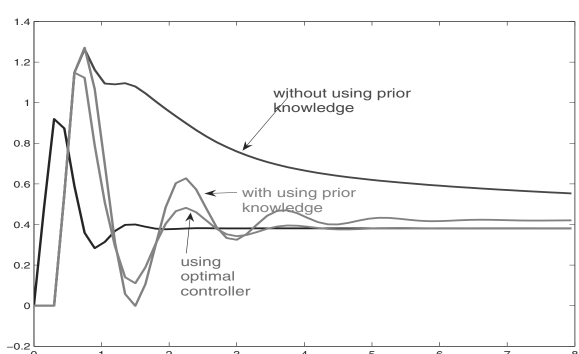

Fig. 10. Block Configuration 3: The response of the model and the response of the plant with different control algorithms.

C. Block Configuration 3

Fig. 9 shows a block configuration in which the series of the subsystems S_3 and S_4 is in the feedback path of the series of the subsystems S_1 and S_2. In this case, we have $u_1 = u - y_4$, $u_2 = y_1$, $u_3 = y_2$, $u_4 = y_3$ and $y = y_2$.

Consider the following reference model.

$$M = \begin{cases} \dot{x}_m = \begin{bmatrix} -2 & 0 & -6 & -4 \\ 5 & -4 & 0 & 0 \\ 10 & -6 & -3 & 0 \\ 0 & 0 & 0.75 & -1.5 \end{bmatrix} x_m + \begin{bmatrix} 4 \\ 0 \\ 0 \\ 0 \end{bmatrix} r \\ y_m = \begin{bmatrix} 5 & -3 & 0 & 0 \end{bmatrix} x_m \end{cases}.$$

Fig. 10 shows the response of the model and the response of the plant for the two cases of using prior knowledge and not using prior knowledge in the adaptive control algorithm. The response of the plant for the case that the optimal controller is used has also been shown.

VI. Conclusion

An approach has been developed for adaptive model-reference control in continuous-time systems consisting of known and unknown subsystems. A block diagram representation has been assumed for the system with a state space model representing each block. In this approach, using the prior knowledge about the system, the update law is driven only by the states of unknown subsystems. A sufficient condition has been proposed for making the adaptive update law only dependent on the errors of the states of unknown subsystem.

In this paper, we have shown by simulations that by incorporating the prior knowledge about the subsystems of the plant, the model reference tracking improves in terms of overshoot and settling time. The formal verification of this improvement in our approach is the subject of an ongoing research. Here, we considered LTI systems. As a future research, the method can be extended to nonlinear systems. Moreover, studying the problem using transfer functions of the subsystems may be an interesting topic.

References

[1] K. Astrom, and B. Wittenmark, “Adaptive Control,” Pearson Education, 1995.

[2] E-B. Bay and S.S. Sastry, “Discrete-time adaptive control utilizing prior information,” *IEEE Transactions on Automatic Control*, Vol. AC-31, No. 8, pp. 779-782, Aug. 1986.

[3] G.C. Goodwin, and K.S. Sin, “Adaptive filtering, prediction and control,” Englewood Cliffs, NJ: Printice-Hall, 1984.

[4] Z. Li, “Discrete-time adaptive control of systems consisting of known and unknown subsystems,” *IEEE Transactions on Automatic Control*, Vol. 34, No. 3, pp. 375-379, March 1989.

[5] R. B. McLain, M. A. Henson, and M. Pottmann, “Direct adaptive control of partially known nonlinear systems,” *IEEE Transactions on Neural Networks*, Vol. 10, No. 3, pp. 714-721, May 1999.

[6] R. Ortega, L. Hsu, and A. Astolfi, “Adaptive control of multivariable systems with reduced prior knowledge,” *Proc. of the 40th IEEE Conf. on Decision and Control*, pp. 4198-4203, Orlando, FL, USA, Dec. 2001.

[7] Z. Qu, “Adaptive and robust controls of uncertain systems with nonlinear parameterization,” *IEEE Transactions on Automatic Control*, Vol. 48, No. 10, pp. 1817-1823, Oct. 2003.

[8] G. Tao, “Adaptive control of partially known systems,” *IEEE Transactions on Automatic Control*, Vol. 40, No. 10, pp. 1813-1818, Oct. 1995.