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Fuzzy Preferences in Conflict Resolution

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***Abstract*—A systematic study of recent developments in preferences based on fuzzy logic is carried out in order to identify an effective design for fuzzy preference models in conflict resolution. Fuzzy preference, defined via a fuzzy relation over the alternatives or states, attempts to represent a decision maker's preferences more realistically. It generalizes the usual preference structures in the sense that it provides a uniform description of both certain (crisp) and uncertain preferences. The potential applicability of fuzzy preferences to the modeling, analysis, and understanding of strategic conflicts is investigated and connected to a literature survey.**

***Keywords*— Fuzzy preference relation, conflict resolution.**

I. Introduction

Decision making, in the sense of choosing a course of action from various states or alternatives, is a very common activity [33]. Here we treat this choice as based on the pairwise comparison of states, captured in a (binary) preference relation. In the standard conception, one of two states is preferred to the other, or the decision maker is indifferent between them, or there is no relation between the states, perhaps reflecting lack of information, or genuine incomparability. In most cases, the possibility of incomparability is excluded, making the preference relation complete. Most decision analysis methodologies in politics, engineering, management, business, economics, etc. use the preferences of decision makers as a fundamental input.

There are other representations of preference between two states x_i and x_j. One uses an index d_{ij} to distinguish the three cases of preference or indifference [14]:

$$d_{ij} = \begin{cases} 1, & \text{if } x_i \text{ is preferred to } x_j \\ 0, & \text{if } x_i \text{ is indifferent to } x_j \\ -1, & \text{if } x_j \text{ is preferred to } x_i \end{cases}$$

This index can be normalized to take values in the unit interval as follows:

$$r_{ij} = \frac{d_{ij}+1}{2} = \begin{cases} 1, & \text{if } x_i \text{ is preferred to } x_j \\ 0.5, & \text{if } x_i \text{ is indifferent to } x_j \\ 0, & \text{if } x_j \text{ is preferred to } x_i \end{cases}$$

A preference relation represented using this system is called crisp, as it presumes that the decision maker is certain about preferences. Details on crisp preference can be found in [29] or any text on decision analysis.

However, in many real world decision problems, decision makers find it difficult to express their preferences in crisp form, perhaps reflecting cultural and educational background, personal habits, lack of information, and the inherent vagueness of human judgment. For this reason, a more flexible representation of the preference between two states is needed. Fuzzy preference seems well-suited to solve this problem. Fuzzy preference is a generalization of crisp preference obtained by allowing the index r_{ij} to be any point in the closed unit interval $[0, 1]$, instead of restricting it to the set $\{0, 0.5, 1\}$.

If $R = (r_{ij})$ is a fuzzy binary relation on the set of states X with membership function

$$\mu_R : X \times X \to [0, 1],$$

we identify $r_{ij} = \mu_R(x_i, x_j)$. The value r_{ij} is interpreted in the literature in two main ways (see, e.g., [13]). Some authors (e.g., [40-41]) understand r_{ij} as the degree of certainty or confidence in the (strict or weak) preference of x_i over x_j. But for others, r_{ij} denotes the intensity with which x_i is preferred to x_j (see, for instance, [3, 13, 36, 43]).

Preference relations have been an active area of research, and many variants have been developed, including multiplicative preferences [8, 15, 21, 42], incomplete multiplicative preferences [18, 39], interval multiplicative preferences [26, 54], incomplete interval multiplicative preferences [55], triangular fuzzy multiplicative preferences [4, 34], incomplete triangular fuzzy multiplicative preferences [55], the fuzzy preference relation [5, 6, 10, 22, 32, 35, 40, 43, 44, 57, 60], the incomplete fuzzy preference relation [23, 50-51], interval fuzzy preferences [27, 49], incomplete interval fuzzy preferences [55], triangular fuzzy preferences [47], incomplete triangular fuzzy preferences [55], linguistic preferences [19-20, 48, 52], and incomplete linguistic preferences [2, 53].

The main objective of this paper is to study the various fuzzy representations of preference and to investigate the prospect of applying them to the analysis and resolution of strategic conflicts. The organization of the rest of the paper is now described.

In Section II, a review of the literature on the development of fuzzy preference relations, and their applications in decision making, is presented. Section III focuses on representations of fuzzy preference relations, and their suitability to conflict resolution is investigated in Section IV, including arguments about their specific usefulness in the graph model for conflict resolution. Finally, some conclusions are drawn in Section V.

II. Literature Review

L. A. Zadeh [59] pioneered fuzzy logic and fuzzy sets as an effective tool for mathematically modeling uncertainty or vagueness. Following Zadeh's notion of fuzzy set, Orlovsky [40] proposed a new preference relation, called fuzzy preference, to generalize crisp preference in a decision making situation. This work attracted the attention of many researchers, and fuzzy preference has developed into a useful tool in modeling decision problems (both individual and group decision making), in various fields, especially engineering.

After introducing some fundamental properties of fuzzy binary relations and certain conditions of reasonable orderings of fuzzy utilities, Nakamura [35] proposed a method for constructing a fuzzy preference, given a set of fuzzy utilities, to permit rational decision making. Tanino [43] discussed the use of fuzzy preference orderings in group decision making. He defined a fuzzy preference ordering as a fuzzy binary relation satisfying reciprocity and max-min transitivity, and developed group fuzzy preference orderings applicable when individual preferences are represented by utility functions, developing a method for group decision processes analogous to the extended contributive rule. Induced ordered weighted averaging (IOWA) operators to aggregate fuzzy preference relations in group decision problems were introduced in [7], where the reciprocity and consistency properties of the collective fuzzy preference relations obtained using IOWA operators were also analyzed.

A general multipurpose decision making model useful for decision problems with preference orderings, utility functions, or fuzzy preference relations was introduced in [5]. In the decision process, the information was first made uniform using fuzzy preference relations, and then selection processes based on the concept of fuzzy majority [28] and on ordered weighted averaging operators [58] were introduced. A study integrating multiplicative preference relations into fuzzy multipurpose decision models using different preference representation structures (preference orderings, utility functions, or fuzzy preference relations) was carried out in [6], which together with [5] provided a more flexible framework to manage different preference structures. It constituted a decision model that approximated real decision situations involving experts from different knowledge areas very well.

The new relation for fuzzy ranking proposed in [30] was easy to compute and preserved the inherent uncertainty of fuzzy numbers during computations. A comparability property for fuzzy preference was introduced in [31]. It was found computationally less expensive to compare fuzzy numbers using fuzzy preference relations if the fuzzy preference relation satisfied the reciprocal, transitive and comparable properties. Aggregation of fuzzy preference relations was the subject of [45]. The authors proposed two optimization aggregation approaches to assess the relative weights of individual fuzzy preferences to permit "additively optimal" aggregation into a collective fuzzy preference relation.

Obviously, an uncertain preference relation can contain contradictory or inconsistent information, and information is much more useful if it is consistent. Traditionally, the consistency of fuzzy preference relations is measured in some way by transitivity. A characterization of consistency based on the additive transitivity of fuzzy preference relation was introduced in [22]. For example, this new characterization of consistency allowed for easy checking of the consistency of the experts' opinions. The authors of [22] also presented a method for constructing consistent preference relations by using a set of specific $n-1$ preference data obtained by comparing each alternative only with the next one. This method ensured better consistency of fuzzy preference provided by the decision makers and minimized inconsistent input to the decision making processes. A procedure to improve the consistency of a fuzzy preference matrix was presented in [57] in the form of an iterative algorithm to derive a modified fuzzy preference matrix with acceptable consistency. Ma et al. [32] proposed two methods for judging the weak transitivity and the inconsistency of a fuzzy preference relation, and developed an algorithm to repair any inconsistency.

Interval fuzzy preference is another way of expressing the decision makers' uncertain preferences. The concept of the degree of compatibility of two interval fuzzy preference relations was introduced in [49], where a theoretical basis for the application of interval fuzzy preference relation in group decision making was developed. A method for calculating interval weights from an interval fuzzy preference relation, by solving only one linear programming model, was proposed in [46]. A method for group decision making based on interval fuzzy preference relations was developed in [27]. In the process, the method first gave an index to measure the degree of similarity of two interval fuzzy preference relations, and then used it to check the degree of consistency of group opinion.

The concepts of incomplete fuzzy preference relation, additive consistent incomplete fuzzy preference relation and multiplicative consistent incomplete fuzzy preference relation were introduced in [50-51]. The two goal programming models proposed in [50] used an additive consistent incomplete fuzzy preference relation and a multiplicative consistent incomplete fuzzy preference relation to obtain the priority vector of the incomplete fuzzy preference relation. After presenting some properties of incomplete fuzzy preference relations, a system of equations for the priority vector of the incomplete fuzzy preference relation was formed; then the procedure for decision making developed in [51] could be applied to the incomplete fuzzy preference relation.

A significant amount of research on decisions based on linguistic assessments also appears in the literature. The consensus model of group decision making presented in [19] was based on the use of linguistic preferences to provide individuals' opinions, and on consensus achieved by a fuzzy majority. Several linguistic degrees of consensus and linguistic distances were also introduced; the degree of consensus indicated the distance from a group of individuals to the maximum consensus, and linguistic distances measured how far each individual was from current consensus labels over the preferences. In [20], the three steps to solve a linguistic decision problem of multicriteria decision making were set out as follows: (1) to choose the linguistic term set and its semantics, (2) to choose the operator to aggregate linguistic information, and (3) to choose the best state.

For group decision making with linguistic preferences, solution methodologies suggested in [48] were based on aggregation by linguistic geometric averaging operators or linguistic hybrid geometric averaging operators. A measure of the deviation of linguistic preferences in a group was discussed in [52]. Based on additive consistency, a complete procedure to estimate missing preference values in incomplete two-tuple fuzzy linguistic preference relations was proposed in [2]. The authors also presented a process whereby a group could select a state using incomplete fuzzy linguistic preference relations.

III. Representations of Fuzzy Preference Relations

Decision makers' preference information must be a crucial component of any decision analysis methodology. Preference information is found mainly in the form of a crisp preference relation, a multiplicative preference relation, a fuzzy preference relation, and a utility function. Fuzzy preference relation is regarded as the appropriate uniform representation tool for preference information [5, 9, 28, 40, 43].

Below we present the fuzzy preference relation, and the interval fuzzy preference variant, with some of their properties. We also present incomplete fuzzy preference relations and incomplete interval fuzzy preference relations.

A. *Fuzzy preference relation [6, 28, 32, 43, 56]:*

Let X represent the set of n states/alternatives: $x_1, x_2, \ldots x_n$. A *fuzzy preference relation* on X is represented by a complementary matrix $R=(r_{ij})_{n\times n}$ with membership function $\mu_R: X\times X \to [0, 1]$, where $\mu_R(x_i, x_j)=r_{ij}$ denotes the preference degree of state x_i over x_j, satisfying

$$r_{ij}+r_{ji}=1 \text{ and } r_{ii}=0.5\text{, for all } i, j=1, 2, \ldots, n.$$

Note that

(i). $r_{ij}=0.5$ indicates indifference between x_i and x_j;

(ii). $r_{ij}>0.5$ indicates that x_i is preferred to x_j; the larger r_{ij}, the stronger the preference of state x_i over x_j; $r_{ij}=1$ indicates that x_i is absolutely preferred to x_j;

(iii). $r_{ij}<0.5$ indicates that x_j is preferred to x_i; the smaller r_{ij}, the stronger the preference of state x_j over x_i; $r_{ij}=0$ indicates that x_j is absolutely preferred to x_i.

B. *Properties of fuzzy preference relations*

A fuzzy preference relation $R=(r_{ij})_{n\times n}$ is an *additive consistent* fuzzy preference relation, if it satisfies additive transitivity [22, 43, 56]:

$$r_{ij}=r_{ik}-r_{jk}+0.5,$$

for all $i, j, k=1, 2, \ldots, n$.

A fuzzy preference relation $R=(r_{ij})_{n\times n}$ is a *multiplicative consistent* fuzzy preference relation, if it satisfies multiplicative transitivity [22, 43, 56]:

$$r_{ij}r_{jk}r_{ki}=r_{ik}r_{kj}r_{ji},$$

for all $i, j, k=1, 2, \ldots, n$.

$R=(r_{ij})_{n\times n}$ satisfies *weak transitivity* [22, 44, 56], if

$$r_{ik}\geq 0.5 \text{ and } r_{kj}\geq 0.5 \text{ implies } r_{ij}\geq 0.5,$$

for all $i, j, k=1, 2, \ldots, n$.

$R=(r_{ij})_{n\times n}$ satisfies *max-min transitivity* [10, 22, 56, 60], if

$$r_{ij}\geq \min\{r_{ik}, r_{kj}\},$$

for all $i, j, k=1, 2, \ldots, n$.

$R=(r_{ij})_{n\times n}$ satisfies *max-max transitivity* [10, 22, 56, 60], if

$$r_{ij}\geq \max\{r_{ik}, r_{kj}\},$$

for all $i, j, k=1, 2, \ldots, n$.

C. *Interval fuzzy preference relations [49, 56]*

Sometimes a decision maker may only have vague knowledge of the degree of preference of one state over another, and cannot estimate this preference exactly, but may wish to do so using an interval of numbers. Formally,

An *interval fuzzy preference* relation $\tilde{R}$ on the set X of states is defined as a matrix $\tilde{R}=(\tilde{r}_{ij})_{n\times n}$ with $\tilde{r}_{ij}=[\tilde{r}_{ij}^L, \tilde{r}_{ij}^U]$ that satisfies

$$\tilde{r}_{ij}^U\geq \tilde{r}_{ij}^L\geq 0,\ \tilde{r}_{ij}^L+\tilde{r}_{ji}^U=\tilde{r}_{ij}^U+\tilde{r}_{ji}^L=1,\text{ and } \tilde{r}_{ii}^L=\tilde{r}_{ii}^U=0.5,$$

for all $i, j=1, 2, \ldots, n$; where $\tilde{r}_{ij}$ indicates the interval-valued preference degree of state x_i over x_j; $\tilde{r}_{ij}^L$ and $\tilde{r}_{ij}^U$ are the lower and upper limits of $\tilde{r}_{ij}$ respectively.

Note that an interval fuzzy preference relation $\tilde{R}=(\tilde{r}_{ij})_{n\times n}$ with $\tilde{r}_{ij}=[\tilde{r}_{ij}^L, \tilde{r}_{ij}^U]$ can be transformed into a fuzzy preference relation $R=(r_{ij})_{n\times n}$ by using the weighted arithmetic averaging operator:

$$r_{ij}=\alpha\tilde{r}_{ij}^L+(1-\alpha)\tilde{r}_{ij}^U,\quad 0\leq\alpha\leq 1,$$

for all $i, j=1, 2, \ldots, n$, where α is an index that represents the decision maker's risk attitude.

D. Incomplete fuzzy preference relations

A complete fuzzy preference relation contains all $\frac{n(n-1)}{2}$ entries (i.e., preference judgments) in its entire upper triangle. Sometimes a decision maker cannot provide all of these entries. If so, the result, $R = (r_{ij})_{n \times n}$ is an *incomplete fuzzy preference relation*, provided that all known entries satisfy the conditions [50, 56]:

$$r_{ij} \geq 0\,,\ r_{ij} + r_{ji} = 1\,,\text{ and } r_{ii} = 0.5\,.$$

In an incomplete fuzzy preference relation, entries not provided by the decision maker are usually denoted *x*.

An additive consistent incomplete fuzzy preference relation, a multiplicative consistent incomplete fuzzy preference relation, and an incomplete interval fuzzy preference relation are defined analogously.

IV. Applicability of Fuzzy Preferences in Conflict Resolution

Conflict may arise whenever human beings with inconsistent interests and objectives interact [24]; strategic conflict, where individuals interact through their decisions, is observed in many human activities including bargaining, group meetings, and even wars. A conflict model is a structure describing systematically the main characteristics of a conflict that is either currently taking place or occurred in the past. The major components of a conflict model are the decision makers, the possible states of the conflict, the movements between states that each decision maker controls, and each decision maker's preferences over the available states [11]. Often, a decision maker's choices are represented as options (any combination of which can be selected). Then a move is a change of options, and each feasible selection of options by all decision makers constitutes a state.

Among various solution methodologies, the graph model for conflict resolution has attracted the attention of conflict resolution researchers due to its simplicity and flexibility [11]. To account for the diversity of decision styles, a range of stability definitions has been put forward, the simplest of which are Nash stability [37-38], general metarationality (GMR) [25], symmetric metarationality (SMR) [25], and sequential stability (SEQ) [12]. A state is *stable* for a decision maker if that decision maker would not choose to move away from it (under the stability definition appropriate for that decision maker). A state that is stable for all decision makers is called an *equilibrium* of the model; if it forms, it is predicted to persist.

Preference information is crucial to the determination of which states are stable for any decision maker. The graph model for conflict resolution utilizes only relative preferences, expressed using the binary relations "is (strictly) preferred to" and "is indifferent to". Thus, preference input is assumed to be crisp. In real world problems, decision makers do not always have crisp relative preferences over the states, and indeed may be unable to do so. However, preference information may be expressed as the degree of strength of the preference of one state over another. In other situations, decision makers may not be able to provide a crisp cardinal utility of a state, but may be able to express the utilities in the form of fuzzy numbers (*fuzzy utilities*). By pairwise comparison of these fuzzy utilities it is possible to obtain, for each pair of states, a degree of preference of one state over another. As discussed earlier, the formal representation of these degrees of preference is a fuzzy preference relation. From the definition of fuzzy preference relation, it is also clear that crisp preference is a special case.

The analysis of a graph model involves the identification of equilibrium states under appropriate stability definition, which generally assume crisp preferences. To date, the only approach to fuzzy preferences in the graph model is found in [1], where the authors divided the fuzzy domain of preferences into five regions with linguistic labels: *much more*, *more*, *indifferent*, *less*, and *much less*. Based on these divisions, and adapting the concepts of strong and weak stability proposed by Hamouda et al. (see [16-17]), they introduced an analogous strong and weak stability, and hence strong and weak equilibrium, to suggest possible resolutions of the conflict.

There are opportunities to apply the graph model for conflict resolution to models of strategic conflict in which decision makers' preferences over the states are fuzzy. It should be possible to generalize stability definitions that capture the sense of the original definition yet incorporate all uncertain relative preferences. These new stability definitions may be termed α *-fuzzy stability*, where α is a number in [0, 1] indicating a decision maker's willingness to accept risk. The definition of an equilibrium state can be generalized accordingly. Hence, it appears that a strategic conflict can be better modeled using fuzzy preference, and that the graph model for conflict resolution can be restructured to fit this model. The authors are in the process of formalizing these generalizations.

V. Conclusions

We have studied fuzzy preference as a generalized representation of preference in a decision situation. A literature review has been carried out in order to understand and compare systematic developments of fuzzy preference relations, and to identify application techniques to solve decision problems in various fields. The formal representations of fuzzy preference, and its variant, interval fuzzy preference, are given together with some of their properties. Finally, the applicability of a fuzzy preference relation in the modeling and resolution of strategic conflict is investigated. It is concluded that decision making in strategic conflicts is an important area of application for fuzzy preference, and that appropriate generalizations of existing methods would be an important contribution.

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