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An Evolutionary Model of Brand Competition

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***Abstract*— We study the evolutionary dynamics of brand competition in a market where two firms are competing against each other. A brand's strategy at each period could be either to innovate on its own or to copy the rival or maintain the same position as before. Consumers are heterogenous, they interact with each other, and under bounded rationality choose one of the products every period, based on their characteristics and price. A multi-agent simulation has been designed under three specifications – no network, a random network and a 2-level network. The cases of no networks, random networks and 2-level networks of different densities give very different results in terms of attainment of equilibrium. Moreover, convergence is always more frequent and faster in case of dense 2-level networks and in the case of sparse random networks. It was also noticed that a skew in the distribution of consumers in the characteristics space leads to more variation in equilibrium values as well as in the likelihood of convergence.**

I. Introduction

Competition amongst brands is an established phenomenon in today's marketplace. Some recent studies like [2], [5] have asserted that brand names are the most valuable and marketable assets for most organisations. And like any other kind of asset, these too depreciate over time unless regular investments are made in order to keep them intact as well as make them grow – and this is precisely what building brand equity is all about. This paper aims to build an agent based simulation which models the evolution of brand building strategies of rival firms over time. The focus is on the quality and price competition, within an evolutionary game theoretic framework, between *existing* brands. A discrete choice model is also constructed which provides the behavioural rules for consumer agents in the virtual market within which the virtual firms compete.

It has generally been seen that qualitative differences amongst products of competing brands seem to be reducing substantially over time. At the same time, price differences among brands which were traditionally perceived as qualitatively different, have been reducing too but at a slower pace. This is quite apparent from a glance at the supermarket shelves across all categories – breakfast cereals, spreads, home and personal care products etc. are a few of them. It is even more apparent in industries like pharmaceutical, electronics, software, telecommunications etc., where innovation and adaptation traditionally play very important roles.

A market is an exceedingly complex entity. The traditional analytical and empirical techniques of analysis have not been very successful in handling this complexity appropriately. As mentioned above, our model is built on an agent based framework, which is able to tackle this complexity much more efficiently than traditional approaches. Every individual consumer agent is unique in his tastes and preferences, making him behave independently. Every firm is also an independent agent and follows its own strategies, which are of course conditioned on it's *beliefs* about its rival's strategies.

Our simulations reveal some interesting temporal behaviour patterns, both for the competing firms as well as individual consumers. In a vertically differentiated industry, firms initially go through a highly competitive stage, where they frequently try to outdo each other in terms of their brand characteristics, but with time settle down into a steady state where the brands closely mimic each other. This holds true *irrespective* of how heterogenous consumers are and firms only have a partial idea of this heterogeneity. A price war also ensues in most cases, resulting in a convergence of market prices. Consumers, who are designed to be *boundedly rational*, may exhibit frequent brand switching depending on their preferences and changing product characteristics, but are also seen to exhibit *loyalty* to a particular brand even though others might be more suitable given their preferences — in essence we are able to illustrate a *lock in effect*. Interestingly, this loyalty is brought about through *peer pressure* via a social network[1].

At this point, we would like to qualify an important point about the present analysis. Our aim here is to capture the key *insights* behind how firms shape their brands with time. In that, results emerging from a general model such as ours seem to fit very well with some stylized facts commonly seen in many markets. Ideally, the next step would be to focus the model more towards specific markets using real life data. Market specific validations are one topic of our ongoing research. Hence we restrict ourselves to a combination of analytical and simulation approach in order to get the key insights that we are aiming for.

Existing literature on this topic can be divided into two major groups — the first comprises of those which deal with consumer behaviour and the second which are concerned with modelling product differentiation and brand competition. The study of how individuals make a choice when confronted with a finite number of discrete alternatives has been a subject of

[1]Ideally, psychological effects of cognition and memory should be a part of an agent based consumer behaviour analysis. However, for the purpose of illustrating brand competition per se, we aim to capture the major aspects of choice behaviour with the present model.

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study for a considerable amount of time. The roots of such models lie both in economics as well as in psychology, which traditionally had been inconsistent with each other. In order to economize on space, we do not discuss the two approaches in detail here but refer the interested reader to an excellent discussion in [4].

The important point to note is that this conflict has largely been resolved. Following the foundations laid by [27] and [21], [4] has shown that identical choice models can be derived using both approaches. And the key to deriving a consistent model lies in the Multinomial Logit (MNL) approach to modeling choice among discrete alternatives. The origins of the MNL can be traced back to the axiomatic analysis of choice probabilities in [19] and later in [20]. As we shall see subsequently, the MNL is the key to describing choice behaviour among consumers facing competing brands.

The literature on competition in differentiated products is quite extensive. Product differentiation models have typically been classified as being either horizontal or vertical differentiation. The difference in these two classes arise due to differences in the assumptions regarding consumer preferences. In horizontal differentiation, consumers' preferences are distributed over a range of products, and they generally tend to buy the one they prefer the most [6], [3], [7]. In such a case, firms typically offer a range of products of varying qualities and introduction on new varieties does not necessarily mean the withdrawal of old ones. Foods, health and personal care products, insurance markets, automobiles etc. are a few industries typically characterised by horizontal differentiation.

Vertical differentiation, on the other hand, is where all consumers have the same ranking over quality but differ in income. In this case, firms usually produce a single product and whenever any product enhancement takes place, they withdraw the earlier product. Typical examples are seen in high technology industries like electronics, software, heavy engineering etc. Some well known papers in this regard are: [10], [11], [14], [24], [22].

The literature on brand competition *per se* can be looked into separately. There have been a few publications on this topic which have built both theoretically and empirically on different kinds of product differentiation models. The situation with two firms who differentiate products both by quality and brand name[2] is examined in [12]. The issue of how firms respond to threats of entry by using strategies like introduction of a "fighting brand" or "product pruning" is looked at in [15]. The former refers to introduction of a lower quality product in order to expand product range and the latter refers to removing products from the market. The questions of whether and when do pricing practices on base products differ from those of premium products is addressed in [28].

This paper differs from the ones mentioned above in some crucial ways. First of all, it focusses on strategic investment choices only in existing brands or products. The aim is to examine how firms shape or re-invent their established brands over time – to examine the level of horizontal *and* vertical differentiation when firms have a choice of investing in multiple "quality" characteristics. Secondly, it is a dynamic model in which an evolutionary algorithm is used to examine the adaptation of strategies dynamically over multiple periods. Thirdly, it makes use of agent based techniques to create a model of the market bottom up where independent agents representing the firms and consumers behave strategically. As far as our knowledge goes, we are unaware of any prior attempt at explaining the dynamics of brand competition using a similar approach.

The rest of this paper is organized as follows. In Section II we design the basic model – first as a static game played by the firms and then modifying it to reveal its dynamic properties. Section III details the behavioural model we associate with the consumer agents. In Section IV, we set up the agent based simulation and analyze the results. Finally, we conclude the paper in Section V with a discussion on the results and directions for future research.

II. The Model of Brand Competition

A. The static game

Consider two firms, X and Y operating in an industry, each producing one product[3]. Each firm can be considered as representing a particular brand. The main assumption is that these products/brands are substitutable in the minds of the consumers. However, they need not be identical and this is where product differentiation enters the picture. Each firm can invest in modifying $K > 0$ product characteristics in order to distinguish it from that of its rival. Quality, packaging, advertising, taste, smell, composition etc. are some examples of the characteristics that each firm is able to control. This set is identical and exogenously given for both firms.[4]

Consider firm X. Define $\mathbf{x} = \{x^1, x^2, x^3, \ldots, x^K\} \in \mathbb{R}^K$ as a vector of independent[5] controllable characteristics in X's product and where $x^k \in [0, \infty)$. We can define a corresponding vector $\mathbf{y} \in \mathbb{R}^K$ for the rival Y. We make the following simplifying assumptions about the producers in this market.

Assumption 1: Both brands in the market are taken as given and there is no introduction of new or withdrawal of existing brands.

Assumption 2: Cost information of both firms is common knowledge.[6]

Define $q(\mathbf{x}, \mathbf{y}, p^x, p^y)$ as the market share of firm X, where $\mathbf{x}$ and p^x are the characteristics/attribute vector and unit price of X, and $\mathbf{y}$ and p^y are those of the rival. The market share

[2]That is, both horizontally and vertically.

[3]This is a simplification, but adding more products to the portfolio is irrelevant for the present analysis.

[4]Firms cannot invent new characteristics for their product, they can only change existing ones.

[5]The independence of characteristics is a simplification. We anticipate that relaxing this will not create too many problems in the analysis, if we consider each set of dependant characteristics as a single controllable unit and redefine preferences over these sets.

[6]Each firm knows its own as well as its rival's costs of production. Also, each knows that the other knows and so on. In many markets, this is not really an unreasonable assumption.

of the rival Y is given by, $1 - q(\cdot)$. The exact nature of $q(\cdot)$ will be characterised later, once we look into the demand side of the market in more detail.

Firms have the option to invest in innovations which change the attribute vector of its products. However, such changes always come at a cost, which is proportional to the magnitude of change. Let $\tilde{\mathbf{x}}$ be the initial and $\mathbf{x}$ the final characteristic vector as chosen by firm X. Let $\mathbf{c^i} = \{c^i(1), \ldots, c^i(K)\}$ be the vector of costs incurred due to such innovations, where the k^{th} element in $\mathbf{c^i}$ represents the cost of an unit change in the k^{th} characteristic. The superscript i represents innovation – the fact that the firm in question does something new and is not just copying the rival.

Firms also have a choice of mimicking (copying) what the other firm does. Here too the firm has to bear a cost proportional to the degree by which a firm changes its brands' characteristics. We assume that it is always cheaper for a firm to mimic the rival than invest in new innovations. If $\mathbf{c^c}$ represents vector of cost of mimicking, then according to our assumption, $\mathbf{c^c} < \mathbf{c^i}$.

The third choice available to firms is to keep one or more of its characteristics unchanged, i.e. maintaining *status quo*. The cost of maintaining status quo is naturally fixed at zero, i.e. $\mathbf{c^o} = \mathbf{0}$. All costs associated with changing brand characteristics in any of its brands is labeled as Research and Development (R & D) costs from now on. We make one additional assumption.

Assumption 3: All R & D costs are identical for both firms in each of their brands.[7]

Define $S = \{i, c, o\}$ as the set of actions available for each characteristic as defined above. Hence the set of *pure* strategies available to a firm is the cartesian product S^K. Let $s^x, s^y \in S^K$ be the pure strategies chosen by firms X and Y respectively. As per our definition, $s^x = \Big\{s^x(k)\Big\}_{k=1}^{K}$ and $s^y = \Big\{s^y(k)\Big\}_{k=1}^{K}$, where $s^x(k), s^y(k) \in S$.

Let,

$$\Delta x^k = x^k - \tilde{x}^k \qquad \forall\, k = 1 \ldots K.$$

Define $I_x \subset K$, such that if firm X invests in an innovation in characteristic $k \in K$, then $k \in I_x$. $K \backslash I_x$ represents the set of characteristics in which the firm either mimics the rival or maintains status quo. Similarly define I_y for firm Y. Then the *total* R & D cost of firms X and Y respectively are,

$$C(s^x) = \sum_{k \in I_x} c^i(k)\Delta x^k + \sum_{k \in K \backslash I_x} c^c(k)\Delta x^k \qquad \text{(1a)}$$

$$C(s^y) = \sum_{k \in I_y} c^i(k)\Delta y^k + \sum_{k \in K \backslash I_y} c^c(k)\Delta y^k \qquad \text{(1b)}$$

Let $c > 0$ be the per unit *production* cost for both firms and N, the total population size. The realized (*ex post*) profits for X and Y are,

$$\pi^X(s^x, s^y) = N(p^x - c)q(\mathbf{x}, \mathbf{y}, p^x, p^y) - C(s^x) \qquad \text{(2a)}$$

[7]Not an unreasonable assumption given that both firms have same access to technology. However, this can be modified very easily.

$$\pi^Y(s^x, s^y) = N(p^y - c)(1 - q(\cdot)) - C(s^y) \qquad \text{(2b)}$$

Consider firm X's problem when it has decided to innovate (carry out action i) in $I_x \subset K$ characteristics. For every s^y, X solves the following optimization problem:

$$\max_{x^k} \pi^X(x^k; k \in I_x)$$

Note that the above optimization is a sub problem which arises only if a firm decides to carry out new innovation in any of its characteristics. Also, the arguments of $\pi^X(\cdot)$ are different than (2a) as the function is now conditional on the strategic actions already decided upon.

Consider the following three stage interaction between both the firms. In the first stage, each observes the current state of the market, which is given by the characteristics and prices of its own brand as well as its rival's. In the second stage, both firms *simultaneously* decide on whether to innovate (i), mimic (c) or maintain status quo (o) in every product characteristic. Firms also set the price for its brand once the characteristics have been decided upon. In the third stage, production is carried out, final outputs are sold in the market and profits realized.

The three stage interaction described above, the payoffs (2a) and (2b), the set of pure strategies S^K and the firms X and Y constitute a one-shot simultaneous game G. As can be easily seen, G is a non-cooperative game and finite in the number of players and available strategies. We can establish the following Theorem using Nash [23]. For a generic proof, kindly see the reference.

Theorem 1: Game G has at least one Nash equilibrium either in mixed or pure strategies.

B. The dynamic game

We now consider the repeated version of game G to analyze the dynamics of interaction between players over time. The current setup involves an *infinitely* repeated game with no fixed time horizon. The interaction between the two firms happen in the following manner. Each period is divided into three stages. In stage one, both firms get to observe the outcome in the market of the *previous* period, which include its own and the rivals choice of characteristics and prices. In the second stage, each of them simultaneously decide on whether to innovate, mimic or maintain status quo in each of the K characteristic. Prices are also set in this stage. In the last stage, production is carried out, final outputs sold in the market and profits realized. This ends the current period and begins the next one, with the above interaction being repeated.

The complete interaction defined above can be represented by $\{G_t\}_{t=1}^{\infty}$, where G_t is the stage game in period t. The question that naturally arises is, what kind of dynamic behaviour can we expect from players in such a game. We develop the "replicator dynamics" – which is a set of difference equations governing the adjustment in the probabilities with which players play each strategy at every iteration of the game – a well known adjustment procedure for evolutionary games, first proposed by Taylor and Jonker in [26].

While Result 1 mentioned the existence of a mixed strategy equilibrium, we did not define the concept of mixed strategies formally. Since the mechanism of replicator dynamics necessitates the use of mixed strategies, we do so now.

Define $\Omega(S)$ as the set of possible discrete probability distributions over the set S. Essentially, $\Omega(S)$ is nothing but a two dimensional simplex. Correspondingly, the mixed strategy space for any firm is given by $\Omega(S^K)$, which is a simplex of $3^K - 1$ dimensions. Any element $\sigma \in \Omega(S^K)$ is a mixed strategy, i.e. a probability distribution over S^K.

To ease up the notational complexity and given the fact that the firms are symmetric, we drop X and Y identifiers from all expressions for this section. Let $\mathbf{x}_t = \{x_t^1, \ldots, x_t^K\} \in \mathbb{R}^K$ represent the vector of K characteristics at period t of any one of the firm's product. The corresponding vector for the rival is represented by $\mathbf{x}'_t \in \mathbb{R}^K$.

Define π_t as the payoff *matrix* for the firm we are examining, where each element $\pi_t(s, s')$, represents the *ex post* profit of the firm in period t when this firm chooses pure strategy s and the rival chooses s', where $s, s' \in S^K$. As in the earlier section, a firm's *ex post* profit as function of its own and its rival's choice of *pure* strategies is given by:

$$\pi_t(s_t, s'_t) = N(p_t - c)q(\mathbf{x}_t, \mathbf{x}'_t, p_t, p'_t) - C(s_t) \quad (3)$$

where

$$\begin{aligned} C(s_t) &= \sum_{k \in I^t} c^i(k)\Delta x_t^k + \sum_{k \in K \setminus I^t} c^c(k)\Delta x_t^k \\ \Delta x_t^k &= x_t^k - x_{t-1}^k \end{aligned}$$

and where x_0^k is exogenously fixed for all $k = 1, \ldots, K$. Note that the cost of changing brand characteristics is proportional to the difference between x_t^k and x_{t-1}^k, where x_{t-1}^k is the level of brand characteristic k in period $t-1$. We now turn our attention to the *evolution* of these strategies over time.

Let $\sigma_t(s) \in \Omega$ be a mixed strategy (a vector with 3^K elements) defined over the pure strategy $s = \left\{s_k\right\}_{k=1}^K \in S^K$ at a given period t for firm X.[8] $\sigma'_t(s')$ is the corresponding mixed strategy vector of the rival defined over *his* pure strategy $s' \in S^K$.

Let $\pi_t(s)$ be the *expected* (or *ex ante*) profit of the firm from choosing action s. In the nomenclature of evolutionary game theory, π_t is the fitness of the pure strategy s against σ'_t. Hence,

$$\begin{aligned} \pi_t(s) &= \sum_{s' \in S^K} \sigma'_t(s')\pi_t(s, s') \\ &= \left(\sigma'_t\right)^T \cdot \pi_t \end{aligned} \quad (4)$$

for all $s \in S$ and where a superscript T represents a transpose. The *overall* fitness of strategy σ_t against σ'_t or in other words, the expected profit from σ_t against σ'_t is,

$$\overline{\pi}_t = \left(\sigma'_t\right)^T \cdot \pi_t \cdot \sigma_t. \quad (5)$$

Let $\sigma_{t+1}(s)$ be the probability with which a firm plays s in period $t+1$. Using Taylor and Jonker's formulation, the inter-period adjustment in the probabilities are given by,

$$\sigma_{t+1}(s) - \sigma_t(s) = \frac{\pi_t(s) - \overline{\pi}_t}{\overline{\pi}_t}\,\sigma_t(s) \quad (6)$$

for every $s \in S^K$.

III. The Consumers

So far we had been looking at the way a market behaves as a whole in response to the product characteristics chosen by the firms. In this section we outline the individual consumer choice model which leads to this aggregate behaviour.

As mentioned before, each firm produces one product and there are two firms in the industry. There are $N > 0$ consumers, each wishing to buy *one* unit of the product every period. The consumers face prices p^x and p^y. The vectors of brand characteristics, once again are, $\mathbf{x}_t$ and $\mathbf{y}_t$.

In order to define the preferences of consumers in such a framework, we borrow heavily from the discrete choice literature where each commodity is consumed, not for its own sake, but for the attributes it embodies. Hence, given a vector of characteristics in a product, we are able to place it in a *characteristic space*. This vector is then called the *address* of this product. Each consumer's preference is defined using a complementary *ideal point*, which is a vector of characteristics that he would ideally like to see in a product. The closer this ideal point is to the actual mix of characteristics of a commodity, the higher the subjective utility of the consumer from purchasing it. First proposed by [25] and developed further by [16], [8], [9], this approach has become quite established in discrete choice literature. We adapt this ideal point framework into a linear random utility (LRU) characterisation, which gives rise to the well know Multinomial Logit Model (MNL) of consumer choice.

Consider a population of possible consumers, each of them contemplating a purchase of any one of the alternative products. Consider the choice made by a single individual at period t. Let B_t be *this* consumer's budget for the current product in the current period . According to the LRU approach, let his subjective utility derived from the purchase of $\mathbf{j}$ be,

$$V_t^j = B_t - p_t^j + v(\mathbf{j_t}) + \epsilon_j \qquad \mathbf{j} = \mathbf{x}, \mathbf{y}. \quad (7)$$

[8] To be more precise mathematically, let $\mathbb{H}$ be a matrix of dimensions $3 \times K$, such that the k^{th} column represents a probability distribution over S in the k^{th} characteristic. Let h_{jk} be the element of $\mathbb{H}$ from the j^{th} row and k^{th} column. Given $s = \left\{s_k\right\}_{k=1}^K$, we can define,

$$\sigma_t(s) = \prod_{k=1}^K h_{jk} \qquad \text{where } j = \begin{cases} 1, & \text{if } s_k = i \\ 2, & \text{if } s_k = c \\ 3, & \text{if } s_k = o \end{cases}$$

The function $v(\mathbf{j_t})$ captures the *known*[9] preference function over different attributes of the brand. Price, as an attribute enters utility separately as p_t^j. We characterize $v(\mathbf{j_t})$ in the following manner. Each consumer has an ideal point represented by, $\mathbf{I} = \{I^1, \ldots, I^K\} \in \mathbb{R}^K$. Consider product X and the corresponding characteristic vector $\mathbf{x_t}$. Let,

$$v(\mathbf{x_t}) = -\sum_{k=1}^{K} \omega_k (x_t^k - I^k)^2, \quad (8)$$

where ω_k is a weight attached to attribute k by the consumer. A similar expression exists for product Y. According to (8), $v(\mathbf{x_t})$ is the negative of a weighted Euclidean distance between product X and the ideal point of the consumer. Hence, *all else remaining same*, a consumer chooses the brand which is the closest (using weighted distance) to his ideal point.

A crucial component of V_t^j is ϵ_j, which is a random variable drawn from a distribution $F(\cdot)$, capturing the idiosyncratic taste and budgetary differences among individuals of a sub-population who buy the variant J and which are not observable to the modeler. In other word, the random variable captures the *unobserved heterogeneity* within the population. Additionally, we can qualify ϵ_j as representing the bounded rationality component of a consumer's decision making process.

Assumption 4: Let ϵ_j be drawn independently and identically from a double exponential distribution.
We make use of the following well known result, the proof of which can be found in [4] and [20].

Theorem 2: If ϵ_j are independently and identically distributed according to the double exponential distribution

$$F(z) = Pr(\epsilon_j \leq z) = \exp - \Big[\exp - \Big(\frac{z}{\mu} + \gamma \Big) \Big]$$

where γ is the Euler's constant ($\gamma \approx 0.5772$) and μ is a positive constant. Then the resulting probabilities of variants X and Y being chosen are given by,

$$Pr(X|t) = \frac{\exp(u_t^x/\mu)}{\sum_{j \in \{X,Y\}} \exp(u_t^j/\mu)}$$

$$\text{and} \quad Pr(Y|t) = \frac{\exp(u_t^y/\mu)}{\sum_{j \in \{X,Y\}} \exp(u_t^j/\mu)}$$

where $u_t^j = B_t - p_t^j + v(\mathbf{j_t}) + \psi_t^j$ for $j = x, y$.

What this means is that, a consumer chosen at random at time t from the population is expected to choose X with probability $Pr(X|t)$ and Y with probability $1 - Pr(X|t)$ as defined above. And given a large enough N, the *expected*[10] demand for variant X at period t is then given by, $NPr(X|t)$ and for variant Y, is $NPr(Y|t) = N(1 - Pr(X|t))$. To make it consistent with the characterization in Section 2, we define $q(\mathbf{x}_t, \mathbf{y}_t, p_t^x, p_t^y) = Pr(X|t)$, which represents the market share of brand X. The market share of brand Y is similarly defined. As can be seen, the choice probabilities are multinomial logit expressions.

We are finally left with the characterization of the social network amongst consumers. Each consumer $i \in N$ is connected to $n_i \geq 0$ other consumers, who may affect his choice of a brand. Such a network is represented by a directed graph $\Psi = \{N, L\}$ where $L = (u, v) \subset N \times N$ is a collection of ordered couples of customers with (u, v) representing the information flow from consumer u to consumer v.

We try to capture a peer effect amongst the consumers in the following manner. For any individual i, let $n_t(x) \leq n_i$ be the number of *neighbours* of i that have purchased brand X in period t. Assume that there exists $\alpha \in [0, 1]$ such that we can re-formulate the utility from purchasing X in the following manner.

$$V_t^j = \begin{cases} B_t - p_t^j + v(\mathbf{j_t}) + \epsilon_j, & \text{if } \dfrac{n_t(x)}{n_i} < \alpha \\ \infty, & \text{if } \dfrac{n_t(x)}{n_i} \geq \alpha \end{cases} \quad (9)$$

Equation (9) implies that a consumer is swayed by his neighbours if *enough* of them are buying a certain product. The proportion of his neighbours buying, say X, have to be higher than a threshold $0 < \alpha < 1$ for him to purchase X over Y, *even though Y is preferred over X in terms of subjective utility.*

This way of modeling peer pressure is similar to the threshold models used extensively in analysing spread of various kinds of social phenomena over networks. Introduced by [13], these models are used in situations where agents possess some kind of resistance to change, but may choose to change provided there is enough motivation.

IV. Simulation Results

We have developed a multi-agent simulation of our model in order to explore how consumers' heterogeneity and their information exchange influences competitive strategies of brands. We have modeled the brands and individual consumers as distinct and independent[11] decision making agents. The model is implemented and simulations run using the RePastJ 3.0 [1] Java environment. The model parameters and simulations settings are described in the following subsection while in subsection IV-B, simulations results are given and findings further discussed.

A. Settings

We have run simulations with two brands X and Y which are respectively the (only) products of two firms *X* and *Y*. The consumers are separated into two segments – Group 0 and Group 1, with the number of consumers NoConsumersGroup0 and NoConsumersGroup1 in each group respectively. The consumers in each group are randomly distributed in a circle within the characteristics space, where the centers of the circles are IGroup0 and IGroup1 and the diameters are δ_0 and δ_1 for Group 0 and Group 1 respectively. The values for δ_0

[9] Known to the modeler and the firm.

[10] The firms only have partial knowledge demand, represented by the known components of the utility function. For the rest, they only know the distribution from which it is drawn.

[11] The term "independent" indicates that each agent is a distinct decision making unit, although each may be *influenced* by others.

Run	NoConsGroup0	NoConsGroup1
1	100	100
2	500	500
3	200	800
4	800	200
5	1000	1000

TABLE I
THE NUMBER OF CONSUMERS IN EACH SEGMENT

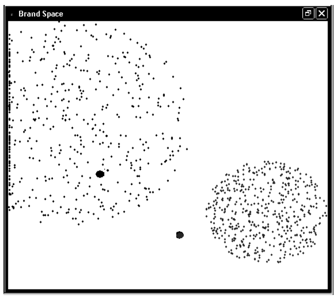

Fig. 1. A possible distribution of the ideal points for two consumer segments. The large grey dots are the current positions of the two brands. The origin is at the north-west corner and Group 0 is the south-east segment.

are set to $\{20, 40, 60\}$ and for δ_1 are $\{60, 40, 20\}$.As discussed above, the centers IGroup0 and IGroup1 form the *known* part of a consumer's preference function ($v(\cdot)$ in the model) and is the same for every member within a group, while their *exact* positions are unknown to the firms (captured by ϵ_j). The parameters NoConsumersGroup0 and NoConsumersGroup1 are also known to both firms. The set of values for the number of consumers in both groups used in simulations are given in the Table I. Fig 1 gives an example of one such arrangement where the two circular segments show the distribution of consumers in the characteristics space. Each dot within each cloud represents an individual consumer.

Each brand occupies one position in the two dimensional characteristics space. The two dimensions are labeled 0 (Char0) and 1 (Char1) respectively, where Char0, Char1 $\in \mathbb{R}_+$. For our simulations, IGroup0 was fixed at $\{170.0, 150.0\}$ and IGroup2 at $\{40.0, 80.0\}$. We initialized characteristics of two brands as BrandXChar0: 170.0, BrandXChar1: 160.0, BrandXPrice: 200.0 for the brand X and BrandYChar0: 50.0, BrandYChar1: 40.0, and BrandYPrice: 250.0 for brand Y.

We used two types of networks, a *random network* and a *two-level network* in the analysis. Both networks are directed. Probability of a link in a random network is given by a parameter ConnectionRate. In a random network, pairs of consumers, say (A,B), are chosen at random, and with a ConnectionRate probability, a directed link is assigned from A to B. In a 2-level network, a small set S of consumers ($\|S\| =$ ConnectionRate $*$ No. of consumers) is chosen uniformly at random. Each consumer in S is connected to 5 randomly chosen individuals (neighbours) and then each neighbor is further is connected to 2 random individuals. Using the above settings, we have tried to capture some characteristics of recommendations networks reported in [18], [17]. For both networks we have assumed a threshold level of 50%, (i.e. $\alpha = 0.5$, see Sec. III for the definition of α), and ConnectionRate has been set to $\{0.00003, 0.0003, 0.003, 0.03\}$. All experiments have been run sequentially, once without any networks, then once with a random network, and finally with a 2-level network. Each set of experiments have been marked as NN for No Network, RN for random network, and 2LN for 2-level network).

B. Results

Simulations were run for all combinations of parameters given in Tables I for the three network settings (NN, RN and 2LN), and for each combination, an average is taken over 500 runs. Each run of the simulation was limited to 200 time-steps. We recorded the values at which each of the major decision variables, price, characteristic 0 and characteristic 1 showed convergence in X and Y. We refer to the state of the market where convergence has occurred, as *market equilibrium*[12].

For two of the scenarios – no networks (NN) and a sparse 2LN switched on – we noticed that the three measured variables (price, Char0 and Char1) reached equilibrium much faster in cases when both segments have the same number of consumers, i.e. when NoConsumersGroup0 = NoConsumersGroup1 (see Fig. 3). We also noticed that the case of (NoConsumersGroup0=200, NoConsumersGroup1=800) was the most difficult one, i.e. the convergence was slowest there. One possible explanation is that the evolving probability distribution of mixed strategies is more "uniform", as a result of which no single action stands out as the most likely one at any given period. Firms then tend to carve out a niche for themselves while in markets where tastes are distributed more symmetrically, firms tend to converge on product characteristics.

A look at the average equilibrium values (see Fig. 2) reveals that the NN and a sparse 2-LN scenarios match each other quite closely, while the random networks and denser 2-level networks are different. First of all, for the denser 2-level networks, the percentage of non-convergence (see Fig. 4, middle plot) goes down more or less uniformly in all the three variables and across all population distributions. Secondly, as far as equilibrium values are concerned, prices behave strangely at (200, 800). It seems that price competition alone is not able to bring down the prices at this distribution in the presence of dense networks. Thirdly, in case of equilibrium values of characteristics, we see an interesting pattern. In the presence of denser 2-level networks, characteristics 0 and 1 exhibit an apparent substitutability. While equilibrium levels of characteristic 0 tend to go higher with increasing network connectivity, the levels of characteristic 1 tend to go lower with the lowest connection rate acting as the benchmark. As far

[12]The term equilibrium should not be interpreted too rigorously in terms of economics or game theory. It has some characteristics of a classical equilibrium concept, but a more correct way of referring to the same situation would be a *steady state*.

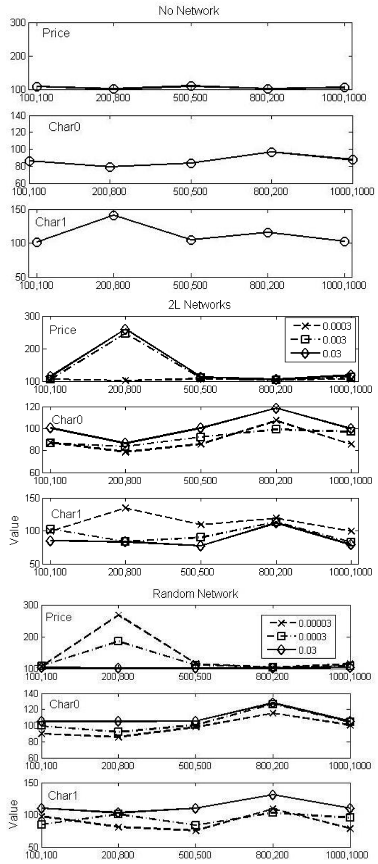

Fig. 2. Equilibrium values of price, char0 and char1.

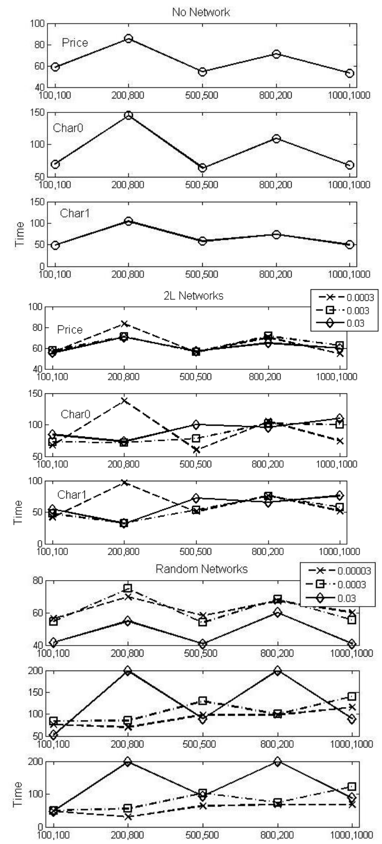

Fig. 3. Average time taken to reach equilibrium.

as timing is concerned, denser 2L networks definitely results in convergence happening sooner for the (200, 800) case. Generally speaking, timing of convergence becomes more uniform across all distributions with denser networks.

Random networks seem to behave very differently as compared to 2-level networks. Consider percentage of non-convergence in Figure 4. Unlike the 2LN, here a low level of connectivity is *not* consistent with a no network situation. There is a significant difference when compared with NN, especially at asymmetric population distributions. Increasing the connection rate seems to make convergence in prices more frequent, although the convergence in characteristics become *less* frequent. One can also see this in Figure 3, where for RN, average time of convergence in price goes down with increasing connectivity but in the characteristics, it seems to go up substantially for asymmetric distributions. In terms of equilibrium values, the characteristics do not show too much variation either across population distributions or across connectivity, and the substitutability seen in the 2LN case is also absent. For price, the convergence properties change completely at the (200, 800) point. Whereas in this case, the highest connection rate results in a lower equilibrium price, in the 2LN, the lowest connection rate resulted in the lowest price.

V. Conclusions

We studied the dynamic competition between brands where each brand developed its strategy taking into account competitors strategies and estimated response from consumers. We designed an evolutionary brand model where each brand can either innovate or copy the competitor's product or maintain the same position. Consumers are heterogenous, can interact with each other and choose one of the products every period under bounded rationality.

We implemented this repeated dynamic game as a multi-agent simulation the brands compete over a segmented of consumer base. In order to implement the social interactions we have implemented two types of networks – a random network, and a two-level network which is based on empirical findings on large online recommendation networks. We also consider the no network case as a benchmark. We measured the time and values when the convergence of price and products' characteristics of two brands occurred. A number of interesting observations stand out from the experiments. Firstly, firms seem to have greater difficulty in coordinating into a symmetric equilibrium in which they produce identical products when the population distribution is asymmetric – with or without networks. Secondly, the presence of 2 level networks mitigate the coordination problem to a large extent across all distributions, with higher connection rates being more efficient. Thirdly, random networks behave very

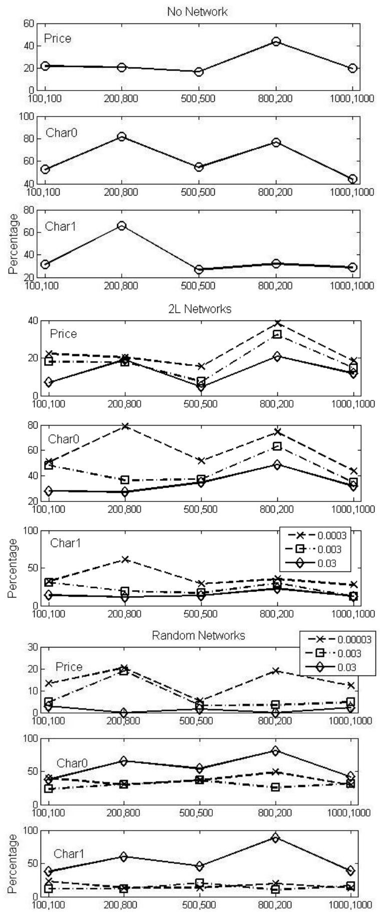

Fig. 4. Percentage of time that equilibrium is not reached.

differently from 2 level networks and in fact increase the coordination problem in characteristics but not in price.

We are confident that our findings are one step forward to a holistic model of brand competition which takes into account heterogeneity and complexity of modern consumers – as well as the strategic considerations of brand management. We hope to validate our model with a real-life data, without network settings in specific markets.

Another direction to proceed is to observe the model's behaviour with different network settings. We are aware that with the current state of the art it is still impossible to capture the relevant "influence" patterns between consumers. It might also be the case that the underlying networks are very different for the different products. However, analysis of large social networks obtained from different online sources gives us hope that we could maintain our research efforts in this direction.

Additionally, we hope to continue modifications on the current model of firm and consumer behaviour. One aspect of firm behaviour that was missing from the current analysis was budget constrained strategic decision making. We hope to rectify this in our ongoing research. We would also like to address the issue of bounded rationality of consumers in some more detail in the future.

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