The Emergence of Social Consensus in Boolean Networks

David G. Green, Tania G. Leishman and Suzanne Sadedin
Clayton School of Information Technology
Monash University, Clayton 3800, Australia
david.green@infotech.monash.edu.au

Abstract - Social order and unity require consensus among individuals about cooperation and other issues. Boolean network models (BN) help to explain the role played by peer interactions in the emergence of consensus. BN models represent a society as a network in which individuals are the nodes (with two states, e.g., agree/disagree) and social relationships are the edges. BN models highlight the influence of peer interactions on social cooperation, in contrast to models, such as prisoner's dilemma, that focus on individual strategies. In BN models, the behavior that emerges from peer interactions differs in subtle, but important ways from equivalent mathematical models (e.g. Markov, dynamic systems). Despite their simplicity, BN models provide potentially important insights about many social issues. They confirm that there is an upper limit to the size of groups within which peer interactions can create and maintain consensus. In large social groups, a combination of peer interaction and enforcement is needed to achieve consensus. Social consensus is brittle in the face of global influences, such as mass media, with the peer network at first impeding the spread of alternative views, then accelerating them once a critical point is passed. BN models are sensitive both to the network topology, and to the degrees of influence associated with peer-peer connections.

I. INTRODUCTION

In the tribal societies in which early humans evolved, social groups usually consisted of mere handfuls of people. One of the unsung triumphs of civilization is that we have managed to create communities in which literally millions of people can live together in relative harmony. As societies continue to grow, to mix and to adopt new forms, it is important to understand how harmony is achieved, and why it breaks down so catastrophically from time to time.

Social order and unity require a level of agreement (consensus) between individuals. For a society to avoid splitting apart, its members need to agree on certain basic issues. They need to agree that they are all members of the same society. At the very least they need to agree to cooperate enough for society to function. Note that by consensus here we do not mean complete conformity, nor even that every individual will agree on every issue.

In tribal societies, consensus arises through interpersonal relationships. The anthropologist Robin Dunbar argues that whether it be apes or humans, there is a natural group size, and larger groups fragment [1]. Many apes maintain social relationships by grooming one another, but the number of individuals that one ape can groom regularly is limited. Consequently, for apes and monkeys, social grooming leads to a natural group size of about 40-60 individuals. For humans, Dunbar argues that the greater efficiency of speech leads to a natural group size of 100-150 individuals. This observation raises a crucial question: how is social cohesion maintained in the much larger human societies of today? How does a community with shared values emerge from a large group of self-interested individuals? One answer is that the structure of human social networks can influence the spread of information and attitudes.

However, many questions about consensus remain. How do personal relationships lead to consensus? Why is there a limit, a natural group size? How can consensus and cooperation be achieved in large societies? Conversely, what factors allow diverse opinions to coexist? What effect do different social structures have on consensus?

In this short account, we summarize some of our recent findings about social consensus that derive from the application of Boolean network (BN) models. We summarize these results as a series of Alife experiments that deal with three issues: (1) emergence of social consensus by peer interactions; (2) the influence of mass media on public opinion; and (3) the role of peer influence in maintaining law and order. In such a short account, we cannot present a comprehensive, detailed overview of all our past results. For this reason our accounts of experiments (1) and (2) are chiefly summaries of the main findings. However, the present study does include several new results, especially from our “law and order” model in experiment (3), which we do describe in more detail.

II. COMPLEXITY AND SOCIAL NETWORKS

A. From complex individuals to complex networks

Social groups consist of agents (people, often referred to as “actors”) linked by networks of interactions and relationships. The richness of these networks makes society complex. However, models of social order have usually omitted or simplified the pivotal role that inter-personal interactions play.

Game theoretic models focus on individual strategies, such as cooperation or defection. For instance, the Prisoner’s Dilemma model [2] identifies conditions under which reciprocity is favored within groups of individual agents. However, the patterns of interactions within a social network play a crucial role in the emergence of cooperation within such networks [3],[4],[5],[6],[7],[8]. Furthermore, the motivation for human behavior often extends beyond selfish opportunism. An indi-
individual’s beliefs and attitudes are often culturally mediated. They may be colored, for example, by the individual’s experiences and by interpersonal relationships and the influence of the person’s peer group.

In contrast to game theoretic models, our work stresses the way in which consensus emerges from interactions between individuals, rather than the strategies that individuals themselves adopt [9]. That is, we consider complex social networks, rather than complex individuals. In our models of social consensus, we have adopted an approach that is based on ideas and methods drawn from artificial life and complexity theory [10],[11]. Complexity can be understood in terms of networks, and networks make a “natural” way of representing patterns of social relationships (Fig. 1). So the most straightforward way of capturing social complexity, is to simulate a social group as a network in which the nodes are “actors” (agents) that represent individuals and the edges represent the relationships (interactions, communication links) between them.

### B. Social relevance of network topology

The pattern of connections between individuals in a social network is usually associated with particular kinds of social organization. Several kinds of networks are well-known to have social implications. Here we consider four kinds of networks: random, small worlds, scale-free, and hierarchies.

In random networks, the links are randomly assigned between pairs of actors. This is effectively a null model, which assumes no systematic patterns of social connections. It would be a valid representation for (say) newly formed groups of individuals. The most important parameter is the edge density γ (“connectivity”), which defines how many social connections there are and ranges from 0 (no connections) to 1 (every actor is linked to every other). In a random network of N nodes, a connectivity phase change occurs when γ = 1/N [12]. When γ<1/N, the network consists of small groups and isolated individuals.

Small world networks [13] are common in patterns of social connections. They fall between random networks at one extreme and regular networks at the other. In social terms, small worlds tend to arise where most connections are local, but are combined with some long-range connections (e.g. a traveller with friends in different cities).

In scale free networks the number of links per node (degree) follows an inverse power law. They form when a network grows with new nodes preferentially attaching themselves to highly connected nodes. This topology is common in some social and political networks as well as some in large communication networks, such as the World-Wide Web [14].

Hierarchies are common in certain social situations, including kinship relations (family trees) and power structures within organizations (e.g. military ranks). In network terms, hierarchies are trees: networks in which there are paths (sequences of links) between every pair of nodes, but without any cycles (a cycle is a sequence of links that form a loop). Hierarchies are trees that contain a “root node” (e.g. a common ancestor in a family tree).

### III. BOOLEAN MODELS OF SOCIAL NETWORKS

#### A. Boolean Networks

A Boolean Network (BN) is a network in which the nodes are simple processing elements, and the edges are communication links between pairs of processors. In effect they are switching networks: each node has a binary state (e.g. ON or OFF). Changes in the state of a node through time are governed by its programming, by its current state, and by the states of its neighbors (nodes directly linked to it by edges).

The appeal of Boolean Network models is their conceptual simplicity. The agents are reduced to the simplest possible representation—a binary switch—so the behavior that emerges in the system is dominated by interactions, not by individuals. This simplicity of the components makes BN models attractive for investigating the role of emergent properties.

For the above reasons, Boolean networks have formed the basis for simulations of many kinds of systems. Applications have included genetic regulatory networks [15] and spin glass models of molecular alignment in the formation of coherent media, such as glass [16]. Several recent studies have experimented with Boolean Networks as models of peer influence in social systems, e.g. [17],[18],[19].

#### B. Application to social networks

In BN models of social networks, the nodes of the network represent individuals (agents, people, or “actors”) and the edges represent social connections of some kind between pairs of individuals (e.g. family, friends, and neighbors). These social “ties” define the patterns of communication links be-

---

**Fig. 1.** Representation of peer-peer social interactions within a Boolean network model. The dots arranged in a circle represent members of the society; the two colors denote two different opinions (e.g. AGREE and DISAGREE) about some issue. The lines joining the dots indicate social relationships. The bold line denotes a social interaction in progress. Shown here is the user interface for the VLAB version of the model [11].

At left are controls; the model network is shown on the right.
tween the individuals.

In our models (Fig. 1) we have made several simplifying assumptions. We have considered only networks in which the pattern of connections between individuals is fixed. We also examined the effect on consensus of the four network topologies discussed earlier: random, scale-free, small-world and hierarchical networks.

All of our models simulate time as a series of time periods (e.g., days). During each time interval, social interactions take the form of encounters between pairs of linked individuals. These encounters occur within the social networks that form between friends, colleagues, family and casual acquaintances. In each event, two actors communicate and influence one another. For any interaction between a pair of individuals, A and B say, the possible outcomes are as follows:

1. If A and B initially agree, then neither changes opinion.
2. If A and B initially disagree, then either:
   - One of the pair (selected randomly) switches to share the other’s opinion. This outcome occurs with probability \( p_{\text{change}} \).
   - Neither changes opinion. This outcome occurs with probability \( 1 - p_{\text{change}} \).

For the models to provide a valid representation of social networks, it is essential that they mimic peer-peer interactions accurately. Our models therefore differ from a standard BN in one important respect: instead of updating the states of every node at once, they are updated in random asynchronous order [20]. Each time period consists of a fixed number of interactions between pairs of individuals, as described above. The pairs that interact are chosen by selecting an edge in the network at random. Each individual undergoes at least one interaction per time period.

IV. INSIGHTS FROM BOOLEAN NETWORK MODELS

A. Social interaction and consensus

Boolean network models (BN) help to explain the role played by peer interactions in the emergence of consensus. As a starting point, we carried out simulation experiments to test Dunbar’s hypothesis, mentioned in the Introduction. That is, in small social groups can consensus emerge from peer interactions? And is there an upper limit on the size of group for which it is possible?

We assigned initial states (0 or 1) at random to individuals in a social BN and ran the model to see whether consensus emerged. Both peer influence and network sizes were varied systematically to test their effects.

For networks with random topology, we found that the size of network in which consensus did emerge was highly sensitive to two parameters: peer influence and the density of social connections. In both cases, phase changes occurred: when either parameter fell below a critical level, only small networks would reach consensus [21].

In the absence of peer interactions, the equivalent model would be a Markov process in which an individual’s probability of switching opinion at any time would depend on the proportion of the population holding that opinion. In such a model, one opinion would ultimately take over through a process of random walk and positive feedback. The BN model differed in its behavior from the Markov case by changing in fits and starts: the proportions of each opinion would remain essentially constant for a time, and then shift rapidly as clusters of peers influenced each other.

This result supports Dunbar’s hypothesis [1], mentioned earlier, that increase in communication makes possible larger group sizes.

In our experiments, hierarchies, small worlds and scale-free networks tended to produce results similar to random networks [9]. That is, consensus rarely arose in large networks. The most revealing aspect was that networks with these topologies highlighted processes that make consensus unlikely. In hierarchies, for instance, large networks tended to produce entire branches that held different opinions. It then became a question of whether the individual at the junction of the two branches (i.e., their common leader) could influence one branch to convert, before the leader was itself converted.

Similar processes were at work in small worlds, where single individuals would often provide links between clusters with differing opinions. In scale-free networks, highly connected individuals played a similar role.

B. The role of leaders in forming opinions

The pivotal role of leaders in the above experiments raises the related question of whether alternative opinions can successfully invade a social group that has already reached consensus. Can a single individual (a leader) succeed in changing an opinion that is held by the entire social group?

The ability of a leader or some influential group of individuals to alter an established consensus depends on the structure of the social network.

In the previous consensus experiments, every individual had a 50% chance of changing the opinion of others. Here we assigned the leader higher influence and tested how frequently a network of 100 individuals ended up with the same opinion (state) as the leader.

In all cases, it proved difficult for a single individual (or even several) to convert the entire network, even if those individuals had greater influence than their peers.

Given the central nature of hierarchies in the earlier experiments, we tested this case more intensively. In this experiment, we performed 100 trials for each combination of parameter settings. The hierarchy was a simple binary tree with 100 agents in all. The experiment was a sensitivity analysis in which we varied the leaders’ influence over subordinates systematically. In all cases there was a 10% chance of subordinates in the hierarchy converting their superiors. Time here was expressed as number of peer-peer interactions and we ran the model for up to 1 million interactions or until consensus was reached. We plotted the number of trials in which the leader succeeded in converting the entire group as a function of the leader’s influence. In virtually all cases, failure to convert the group meant that the leader was converted to agree with the group’s existing opinion.

As expected, any leader with 100% influence always converted the group (Fig. 2), but the success rate fell away
sharply with even a small decrease in influence. An interesting aspect of this experiment was the “time” required to reach consensus (whether the leader or the group converted). The average time taken reached a maximum when the leader’s influence was slightly higher (0.2) than the influence of subordinates (0.1). This implies that the network underwent a lot of changes back and forth before settling into a consensus. When the leader’s influence was high, the subordinates were converted rapidly and reversions were much less likely.

**C. Influence of mass media**

We set out to test whether a social group could maintain its consensus when faced with a central medium exerting influence on every member. This question was inspired by debate over the extent to which media influences public opinion on social issues and political decisions [22], on television advertising (e.g. [23]) and the portrayal of violence in mass media [24].

In the model, we added to the network of peers an additional node, representing media, which has a one way link to every member of the society. This process represents a scenario in which people talk to each other during the day, then watch TV when they return home for the evening. At the start, every member of the social group is in the same state (say “0”), but the medium is pushing the alternative state (say “1”).

A mathematical approximation to what happens in this scenario is given by

$$\frac{dp}{dt} = \alpha(1-p) - \beta p(1-p), \quad (1)$$

where $p$ is the proportion at time $t$ of the population in the state advocated by the medium, $\alpha$ is the probability of medium changing the mind of any individual and $\beta$ is the probability of peer influence converting an individual back to the initial consensus view. If we set $k = \alpha - \beta$, then as $p$ approaches 1, the above model leads to the approximate relationship:

$$p(t) \approx 1 - e^{-kt}. \quad (2)$$

This model implies that the medium state will increase asymptotically towards total coverage.

BN simulations of the medium scenario show that except in totally connected networks, the influence of a central medium rapidly breaks down a pre-existing consensus and eventually leads to a complete reversal of opinion throughout a society [25]. As in the previous experiments, the peer network alters the course of events predicted by the equivalent mathematical model. It does this by changing the way in which the view promoted by the media spreads. Initially, it serves to slow the spread of the medium’s view by converting back some of the individuals who are converted. However, when the view promoted by the media reaches a critical prevalence, the peer network serves to accelerate its spread.

In Hebbian learning, the strength of the links between individuals (i.e. their influence on one another) changes through time [26]. Each interaction between individuals leads to a change in the peer-peer influence. If individuals are in the same state after an interaction (i.e. they agree), then the strength of their connection (i.e. their mutual influence) increases. But if they end up in different states (they “disagree”), then the strength of their connection decreases. If Hebbian learning occurs within a peer network that is subject to outside influence then the most common outcome is polarization of opinion and fragmentation of the network into factions with opposing views [21].

**D. Law and order in large social groups**

Given that there is an upper limit to the size of a social group in which peer-peer interactions can maintain consensus and cohesion, how can large societies persist? Our hypothesis was that in plausible human social networks, consensus is likely to emerge only with a combination of top-down control (law and punishment) and bottom-up influence (peer pressure) [27].

To test the above hypothesis, we formulated a simple Boo-
An lean network model to explore the possible dynamics of these interacting pressures in human societies (Fig. 3). In the model, we take the state of the individuals to be their attitude towards obeying the law. This attitude can take one of two values: HONEST or DISHONEST. If they are HONEST, then they will obey the law, except under the most extreme circumstances (see below). If they are DISHONEST, they will not hesitate to break the law.

We also assume that there is an incentive for individuals to commit a crime. This incentive is expressed as a probability that an HONEST individual will nonetheless commit a crime. During each time period, every actor has an opportunity to commit a crime, and there is a chance that if this happens, the individual may change state and become DISHONEST. The procedure is summarized as follows, where A is an individual (actor) whose current incentive to crime is \( I(A) \). We assume that honest actors do not break the law unless \( I(A) > F \), that is:

1. IF attitude\((A) = \text{DISHONEST}\), then \( A \) breaks the law.
2. IF attitude\((A) = \text{HONEST}\), then:
   - IF \( I(A) > \text{Random\_number} \), then \( A \) breaks the law;
   - ELSE \( A \) obeys the law.

Table 1 summarises the values used for the model parameters used in the law and order experiments.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of people in society</td>
<td>250</td>
</tr>
<tr>
<td>Initial percent of people who are HONEST</td>
<td>100</td>
</tr>
<tr>
<td>Peer-peer interactions per time period</td>
<td>10,000</td>
</tr>
<tr>
<td>Peer influence</td>
<td>0.5</td>
</tr>
<tr>
<td>Probability of corrupting HONEST people</td>
<td>0.1</td>
</tr>
<tr>
<td>Probability of DISHONEST reforming</td>
<td>0.0</td>
</tr>
<tr>
<td>Incentive to crime</td>
<td>0.1</td>
</tr>
</tbody>
</table>

As happened in the previous models, the peer network alters the way levels of honesty change over time (Fig. 4). In the absence of the of peer influence, the society responds as a Markov process. Assuming that the society starts out 100% honest, the course of change in the Markov case is a steady decline in honesty to an equilibrium level that is governed by the relative influences of economic stress and law enforcement. As before, the effect of the peer network is to resist change. Peer influence maintains honesty at near 100% most of the time (Fig. 4). However the network is subject to occasional outbreaks of dishonesty. In extreme cases, these outbreaks can flip the entire society from honest to dishonest.

At first sight, the above result seems unlikely. However, it is consistent with the abrupt onset of many riots and cases of anarchy, such as those observed during famous incidents such as the New York blackout of 1987 or widespread ethnic violence seen in France during the summer of 2005. There are even instance of widespread dishonesty in everyday life. In many countries, for example, drivers on the roads habitually exceed the speed limit, egged on by peer pressure exerted by other drivers.

Results from the model suggest that both enforcement and peer pressure are required to ensure social conformity. In general, law breaking increases as peer pressure decreases (Fig. 5). In the absence of enforcement, peer pressure can act either to reinforce law-abiding behavior, or to convert the entire society into outlaws. However, when connectivity in the social network is high, even a small incidence of punishment suffices to ensure conformity.

In the above scenario, the “crimes” committed by agents in the society were victimless: they had no impact on other agents. In real life, crimes are generally committed against another individual or group. When this happens the event is likely to affect the victim's opinion about honesty.
To test what effect this might have, we ran the above scenario again, but changed the model so that each crime involved a pair of agents: criminal and victim. In the case where victims automatically became honest, the change in behavior of the network was dramatic. Virtually all members of the society remained honest, even under extremely high economic stress (Fig. 5b).

V. CONCLUSION

Despite their simplicity, or perhaps because of it, BN models provide potentially important insights about many social issues. They confirm that there is an upper limit to the size of groups within which peer interactions can create and maintain consensus. In large social groups, a combination of peer interaction and enforcement is needed to produce consensus. Social consensus is brittle in the face of global influences, such as mass media, with the peer network at first impeding the spread of alternative views, then accelerating them once a critical point is passed. BN models are sensitive both to the network topology, and to the degrees of influence associated with social connections.

It is tempting, but dangerous, to draw general conclusions from the results of the experiments presented here. In the law and order model, for instance, we saw that a change in our assumptions about the nature of crimes led to major change in the outcome. So the results are strictly valid only where the assumptions match social conditions.

A common objection to these BN models is that humans are much more sophisticated than switching circuits. This, of course, is true. But there are many instances where human responses are akin to simple switches. Our argument is that before investigating models with complex agents, we need to understand what kinds of phenomena can be explained by simple agents embedded in complex networks of relationships. If we try to make our models more “realistic”, with complex and intelligent agents, then it could be very difficult to isolate exactly why and how particular features emerge.

Our models have implications for many kinds of “social” systems besides human social groups. In computing, for instance, many applications require coordination of many independent agents to achieve some goal. Examples include swarms of agents, nanotech devices, networks of sensors, and distributed processing via grids and clusters. Our results about the emergence of consensus imply that some kinds of coordination will be difficult to achieve in large networks of agents and that a degree of central coordination may be required. With appropriate modification, the models could also be applied to economic and commercial systems. For instance, simulations of interactions within networks of agents have been applied to the question of technology innovation [28].

ACKNOWLEDGMENT

This work was supported by the Australian Research Council and by the Australian Centre for Complex Systems.

REFERENCES

[9] D.G. Green, S. Sadedin and T. Leishman, “The emergence of social consensus in simulation studies with Boolean networks,” in Proceed-
ings of the First World Congress on Social Simulation, Kyoto. Vol. 2, S. Takahashi, D. Sallach and J. Rouchier, Eds. 2006, pp. 1-8...


