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Optimal Paths Design for a GMPLS Network using the Lagrangian Relaxation Method

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***Abstract*— We describe an optimal path design for a GMPLS network that uses the Lagrangian relaxation method, which can estimate the lower bounds of the solution to a problem. This feature helps the designer of the problem to take the accuracy of the solution obtained by the calculation into consideration when he makes a decision to assign the solution to a real network in critical situations. A formulation of the problem and how to solve it using the Lagrangian Relaxation method is described, and the results obtained by a prototype and considerations are shown in this paper.**

I. INTRODUCTION

REMARKABLE progress has been made on the Internet. The bandwidth of networks and the scale of networks have greatly increased. The core network of the Internet is evolving. The physical network architecture uses wavelength-routing switches at routing nodes, which enable the establishment of circuit-switched, all-optical, wavelength-division multiplexed (WDM) channels, called paths. The virtual topology consists of a set of such paths, and they may be used to carry packet-switched traffic through the network. Generalized multiprotocol label switching (GMPLS) is a technology that provides enhancements to multiprotocol label switching (MPLS) to support network switching for time, wavelength, and space switching as well as for packet switching [1][2]. In the network for the GMPLS, paths, which have to be set beforehand, are basically static. Techniques for designing the virtual topology of the paths are a significant problem for successful networking. The important goals of designing the paths for the GMPLS are to achieve the requested specifications that link one end-node to the other end-node of the network and to achieve a sufficient rest margin for the network capacity to enable alternative paths to be provided in case of emergency. Therefore, the problem of the paths design for the GMPLS can be formulated as an optimization problem that aims at efficient use of network resources [3][4][5]. However, this type of optimization problem is generally known as an NP-hard problem [6], and probabilistic search algorithms or heuristic algorithms are often applied to the problem to obtain practical approximate solutions [7][8][9], which is not usually guaranteed to be optimal. Furthermore, we have no means to measure how far the solution obtained by these algorithms is from the optimal one. Indeed, this is not a serious problem in case of typical situations to network management, because we have enough time to search for such a practical approximate solution. In emergencies, however, evaluating the gap between such a practical approximate solution and the optimal solution is meaningful in terms of penalty cost, which network carriers have to pay, because the penalty cost depends on the degree of actual damage to customers.

We present a formulation of such an optimization problem for the paths design of a GMPLS network and an algorithm that solves it and evaluates the gap between the practical approximate solution and the optimal solution using the Lagrangian relaxation method [10][11]. The Lagrangian relaxation method determines the lower bound of feasible solutions for the optimization problem using a heuristic search. We can therefore determine the gap between them even if we break a calculation of the optimization problem at any timing we wish before the optimal solution is found. We can hereby evaluate the gap to determine if there is still room for improving that solution, and we can judge whether or not to assign the paths obtained by the calculation to the actual network.

In the next section of this paper, we will describe the formulation of the optimization problem, which is the optimal paths design for the GMPLS network, and we present a way to solve the problem using the Lagrangian relaxation method in the 3rd section. After that, we show our results for sample cases and describe some considerations.

II. FORMULATION OF THE PROBLEM

Optical signals of traffic are switched by optical routers for the GMPLS network according to the wavelength of lights, and all of paths have to be set correspondingly from source nodes to terminal nodes. However, if we give them the shortest paths from source nodes to termination nodes without any intelligence, the use of network resources will be deflected, and adding new paths or providing alternative paths might be difficult in the event of network trouble. Therefore, we should set appropriate paths, not shortest paths.

In this section, we describe the formulation for this as an optimization problem, using principles from

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multi-commodity flow for physical routing of paths.

We use the following notation.

i denotes a node, which means a router of the network, and N denotes a set of the nodes. j denotes a branch between the nodes, which means an optical line in the network, and E denotes a set of the branches. r denotes a request to link a source node to a termination node, and R denotes a set of the requests. $s(r)$ and $t(r)$ denote source and termination nodes of the request r. x_j^r, which is a variable to be determined, is 1 if the request r uses the branch j, or 0 if the request r does not use the branch j. u_j denotes the capacity of the branch j. $A^-(i)$ denotes a set of branches, the start node of which is i, and $A^+(i)$ denotes a set of branches, the destination node of which is i.

The optimization problem of this paper can be described by using a flow conservation law and constraint conditions of the optical lines.

(P1)

$$\min_{x} \quad f(x) \tag{1}$$

$$\text{s.t.} \quad \sum_{j\in A^-(i)} x_j^{\,r} - \sum_{j\in A^+(i)} x_j^{\,r} = b_i^r \tag{2}$$

$$\sum_{r\in R} x_j^{\,r} \le u_j \tag{3}$$

$$x_j^{\,r} \in \{0,\ 1\} \tag{4}$$

Herein, b_i^r, which is a constant, is 1 in the case that i is equal to $s(r)$, -1 in the case that i is equal to $t(r)$, or 0 otherwise. $f(x)$ is an appropriate cost function of the problem that avoids an over concentration of paths to particular branches. We use the following function, which is equation (5), to equalize ratios of consumption of each branch.

$$f(x) = \sum_{j\in E} \left| \frac{\sum_{r\in R} x_j^{\,r}}{u_j} \right|^2 \tag{5}$$

III. Solution using the Lagrangian relaxation method

We apply the Lagrangian relaxation method to the optimal paths design problem described in section 2. It determines the lower bound of the problem. The Lagrangian relaxation method uses Lagrange multipliers to reduce a part of the constraint conditions by including the conditions in the cost function and divides the original problem, that is a primary problem, into sub problems independent of respective variables. The optimal solution is obtained by solving its dual problem. To divide the problem (P1), we introduce artificial variables, v_j, which indicates the number of remaining wavelengths that can be assigned in the line, and we transform problem (P1) into problem (P2).

(P2)

$$\min_{v} \quad g(x) = \sum_{j\in E} \left| \frac{u_j - v_j}{u_j} \right|^2 \tag{6}$$

$$\text{s.t.} \quad \sum_{j\in A^-(i)} x_j^{\,r} - \sum_{j\in A^+(i)} x_j^{\,r} = b_i^r \tag{7}$$

$$\sum_{r\in R} x_j^{\,r} + v_j \le u_j \tag{8}$$

$$x_j^{\,r} \in \{0,\ 1\} \tag{9}$$

$$0 \le v_j \le u_j \tag{10}$$

We then use Lagrange multipliers, $\lambda_j > 0 \ (j = 1,...m)$, to relax the equation (8), and obtain the Lagrangian relaxation problem (P3).

(P3)

$$\min_{v,x} \quad h_\lambda(v,x) \tag{11}$$

$$\text{s.t.} \quad \sum_{j\in A^-(i)} x_j^{\,r} - \sum_{j\in A^+(i)} x_j^{\,r} = b_i^r \tag{12}$$

$$x_j^{\,r} \in \{0,\ 1\} \tag{13}$$

$$0 \le v_j \le u_j \tag{14}$$

where function h is as follows.

$$h_\lambda(v,x) = p_\lambda(v) + q_\lambda(x) + c \tag{15}$$

$$p_\lambda(v) = \sum_{j\in E} \left[\left\{ z_j - \left(1 - \frac{\lambda_j u_j}{2}\right) \right\}^2 - \left(1 - \frac{\lambda_j u_j}{2}\right)^2 \right] \tag{16}$$

$$q_\lambda(x) = \sum_{r\in R} \sum_{j\in E} \lambda_j x_j^{\,r} = \sum_{r\in R} q_\lambda^{\,r}(x^r) \tag{17}$$

$$c = m - \sum_{j\in E} \lambda_j u_j = \text{const.} \tag{18}$$

$$z_j = \frac{v_j}{u_j} \tag{19}$$

We can hereby divide this Lagrangian relaxation problem (P3) into sub problems independent of either v_j or x_j, which are (P4) and (P5).

(P4)

$$\min_{v} \quad p_\lambda(v) \tag{20}$$

$$\text{s.t.} \quad 0 \le z_j \le 1 \quad (z_j = \frac{v_j}{u_j}) \tag{21}$$

(P5)

$$\min_{x^r} \quad q_\lambda^{\,r}(x) \tag{22}$$

$$\text{s.t.} \quad \sum_{j\in A^-(i)} x_j^{\,r} - \sum_{j\in A^+(i)} x_j^{\,r} = b_i^r \tag{23}$$

$$x_j^{\,r} \in \{0,\ 1\} \tag{24}$$

Sub problem (P4) is just a simple problem to calculate the minimum value of the quadratic function, and sub problem (P5) is a problem to search the minimum cost flow of a single commodity flow where the cost of branches is λ_j, which is just a shortest path problem. Thus we can easily solve sub problem (P5) by using Dijkstra's algorithm.

We can then obtain the solution to primary problem (P2) by solving Lagrangian dual problem (P6), which is a maximum problem for the Lagrange multipliers.

(P6)

$$\max_{\lambda \ge 0} \left\{ \begin{array}{l} \min_{v,x} h_\lambda(v,x) \\ \quad = \min_v p_\lambda(v) + \sum_{r \in R} \min_{x^r} q_\lambda{}^r(x^r) + c \\ \text{s.t.} \quad 0 \le z_j \le 1 \qquad (z_j = \dfrac{v_j}{u_j}) \\ \quad \sum_{j \in A^-(i)} x_j{}^r - \sum_{j \in A^+(i)} x_j{}^r - b_i^r \\ \quad x_j{}^r = \{0, 1\} \end{array} \right\} \tag{25}$$

$$\text{s.t.} \quad \lambda_j \ge 0 \tag{26}$$

The solution to primary problem (P2) is obtained by solving Lagrangian dual problem (P6), which is a maximum problem for the Lagrange multipliers. By using subgradient optimization to improve the Lagrange multipliers for equation (27), we can find a good solution to Lagrangian dual problem (P6).

$$\lambda^{n+1} = \lambda^n + T \frac{\partial h_{\lambda^n}(v,x)}{\partial \lambda} \tag{27}$$

where T is an adequate parameter to adjust the widths for improving the Lagrange multipliers. The improvement in the multipliers is repeated until the value of the cost function for Lagrangian dual problem (P6), which is known to a lower bound of the primary problem, is equal to the value of the cost function for problem (P2), or the difference in both is small enough.

We list the algorithm to solve the problem of the optimal paths design for the GMPLS Network below.

The algorithm
for the optimal paths design problem

INPUT:
Set of nodes and branches (*N, E*)
Set of requests, R
The capacity of branches, u

OUTPUT:
Paths for requests

ALGORITHM:
Step 1: Initialize the Lagrange multipliers, λ_j .
Step 2: For a given λ_j *, calculate the solution to Lagrangian relaxation problem (P4) and (P5), which is* $(v, x)_L$.
Step 3: Transform $(v, x)_L$ *to a feasible solution for primary problem (P2) using the heuristic algorithm later described.*
Step 4: Calculate the value of the cost function for $g(v)$ *in equation (6) and* $h(\lambda, v, x)$ *in equation (11). If the difference between g and h is small enough, or if the number of repetitions is big enough, then finish this process.*
Step 5: Improve the Lagrange multipliers λ *by the subgradient optimization and return to step 2.*

The heuristic algorithm used in step 3 is as follows.

Paths $p^r{}_L$, which correspond to the request *r*, obtained as a solution to the Lagrangian relaxation problem, are transformed so as to be accepted in order of their lengths according to the following rules.

(1) If there are remaining wavelengths to be assigned in all the branches for a path $p^r{}_L$*, let the path* $p^r{}_L$ *be accepted without transformation as the path* $p^r{}_E$*, which is a feasible path for the primary problem, and let the number of the remaining wavelengths of those branches be decreased.*
(2) In cases except for (1), search the shortest path, whose source and termination node are the same as $p^r{}_L$*, for the network. If the shortest path can be found, it is accepted as the feasible path* $p^r{}_L$.
(3) In cases except for (2), the network is divided into parts by the branches that have no remaining wavelength to be stored. First Pick a path already accepted and find its alternative path, which gives these branches remaining wavelength. Then, search the shortest path, whose source and termination node are the same as the path $p^r{}_L$.

Practically, in this search of the shortest path, we give a weighted cost to branches proportional to remaining wavelength to be stored in order to avoid concentrating an assignment of paths to particular branches.

IV. Results of Simulation and Considerations

We made a prototype for the algorithm described in section 3. In this section, we mention the results of our simulation for sample data, and our considerations for them.

Figure 1 is a sample network used in our simulation. There are 14 nodes and 21 branches. The capacities of the branches are all 5. We give this sample network 12 requests, all source nodes of which are node A, shown in the left side of the figure, and all termination nodes of which are node B, shown in the right side of the figure.

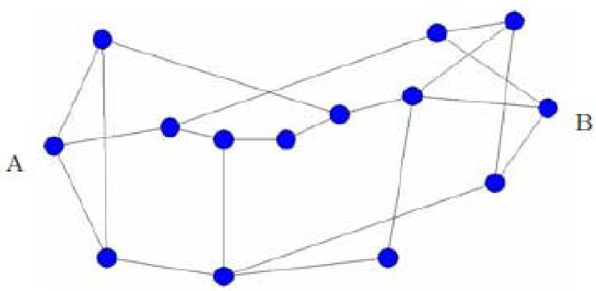

Fig. 1 Sample network

We solved the optimal paths design problem under these assumptions and obtained outputs that are shown in figure 2.

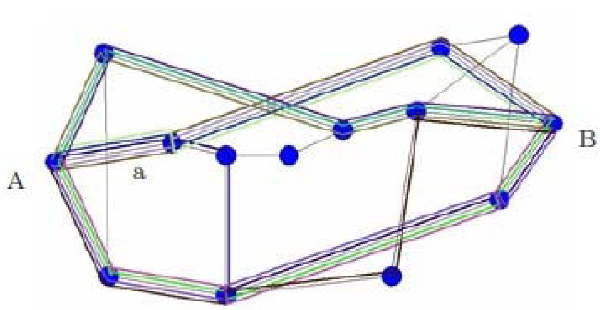

Fig. 2 Results of simulation with sample data

In figure 2, five requests use branch "a", that is linked to node A, and nothing remains of branch "a" to be stored. Four of those five requests achieve the shortest path from node A to node B, both of which have a lengths of three hops, and one of them takes a roundabout path through branch "a", whose length is five hops. The other seven requests also take roundabout paths. The lengths of six of their requests are four hops, and the length of one of their requests is five hops from node A to node B.

These results indicate that only 1 branch, that being branch "a", runs short of its capacity and that the other branches still have some remaining wavelengths to be stored.

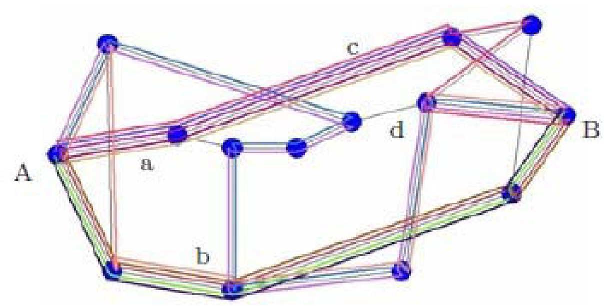

Fig. 3 Results when branch "d" is cut off

Figure 3 also shows the results of the simulation under the other assumptions that branch "d" is unusable in a situation such as down of line. In this simulation, four requests take the shortest paths from node A to node B, four requests take four hops, two requests take five hops, and two requests take eight hops. Three branches, labeled "a", "b", and "c" in figure 3, have no remains to be stored, but the other 18 branches have some remains.

We show the difference between the value of the cost function for the Lagrangian relaxation problem and the lower bound obtained from the Lagrangian dual problem in figure 4. The vertical axis indicates the differences scaled by the lower bound, and the horizontal axis indicates the number of iterations in our algorithm.

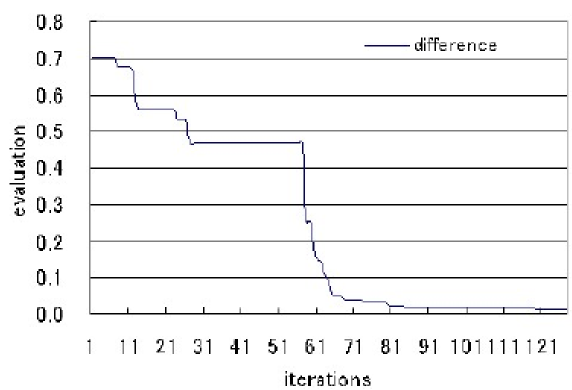

Fig. 4 Transition in differences between cost value for (P2) and lower bound of (P6)

The difference decreases to zero as the repetitions are done. In the end, the value of the cost function is equal to the lower bound. In fact, we arrived at the optimal solution early on during this repetition because of the heuristic algorithm used in step 3 that makes a feasible solution, and the remaining repetition improved the lower bound of this case.

Thus, we determined that the solution obtained in this simulation is optimal. However, obtaining the same results in a large-scale problem would likely be impossible. The solution obtained by our algorithm would not likely be optimal because we use the heuristic algorithm in the step 3. However, as shown in the figure 4, we can evaluate the accuracy of the obtained solution quantitatively by measuring the differences between the value of the cost function and the lower bound. For example, for figure 4, the difference becomes less than 0.1 after the 64 repetitions. We can obtain a provisional solution close enough to the optimal solution, that is probably applicable. Although we need a designer who assigns paths in an actual network to decide whether the evaluation is good or not, we can provide one of the judgment materials when he adopts the solution to the problem and applies it.

V. Conclusion

We presented a formulation of the paths design for a GMPLS network and an algorithm that uses the Lagrangian relaxation method, which measures the gap between a practical approximate solution and the optimal solution to help us determine the accuracy of the solution. Although we use a heuristic algorithm to obtain a feasible solution, albeit not the optimal solution, we can also obtain the lower bound of the problem. Therefore, we can evaluate the gap by comparing both of them, even if a calculation of the algorithm stops before an optimal solution is found.

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