

IDENTIFICATION OF DYNAMIC NONLINEAR SYSTEMS USING COMPUTATIONAL INTELLIGENCE TECHNIQUES

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ABSTRACT

A novel approach based on computational intelligence techniques for the identification of nonlinear dynamic systems is presented in this paper. The technique encompasses both the properties of the Karhunen-Loève Transform in representing stochastic processes and the approximation capabilities of multi-layer neural networks. Experimental results on nonlinear systems governed by difference equations demonstrate the effectiveness of the proposed approach that is based on a real-time learning algorithm. Exhaustive experimentation on specific case studies was performed and some experimental results were compared with other existing techniques such as the Lee-Schetzen method, Least Mean Square (LMS), Recursive Least Square (RLS) and Normalized Least Mean Square (NLMS) algorithms. A better identification-accuracy was also achieved, and a reduction of some orders of magnitude in training-times compared with the well-known Lee-Schetzen method was obtained, thus making the proposed methodology one of the current best practices in this field.

Index Terms – System Identification, Multi-Layer Neural Networks, Karhunen-Loève Transform, Nonlinear Dynamic Systems.

1. INTRODUCTION

Nonlinear system identification (NSI) is an important topic that has been extensively studied for decades. The reason for this great interest, is related to the intrinsic nonlinear nature of physical phenomena occurring in the real world, making the linear hypothesis just an approximation of the real behavior.

NSI has applications not only in traditional fields such as automatic control [1, 2], and communications [3, 4], but also in image processing [5] and signal processing in general [6].

Several different methods have been suggested for nonlinear system modelling, which those based on the Volterra

series and Neural Networks are the most commonly used.

In the former the system is modelled as a Taylor series expansion the generic term of which is a multiple integral depending on a function called a Volterra kernel [7, 8]. These identification techniques are aimed at determining the kernels by stimulating the system with suitable signals. The Lee-Schetzen method [7], based on the Wiener theory of nonlinear systems and using white and non-white Gaussian inputs, is the most successful technique. The main drawback of this approach is the computational effort required in Kernel estimation, which limits the method to the identification of low-order nonlinearities.

More recently identification techniques using recurrent neural networks have been suggested [9, 10]. This approach is based on the properties of these networks which can be viewed as nonlinear dynamic networks whose nonlinearity in the feedback loop is implemented with a multilayer static network. Identification of a specific nonlinear system corresponds to a learning problem: the need to determine an adaptive algorithm or rule which adjusts the parameters of the network on the basis of a given set of input-output pairs. This method has proved to be very powerful for the identification of some classes of systems represented by differential and difference equations. The main limitation of this technique is the need for an a priori knowledge of the equation to be modeled.

In this paper an approach¹ to nonlinear system identification that does not have the previously mentioned restrictions is suggested. The technique is based on the Karhunen-Loève Transform (KLT) [12, 13] and uses a non-recurrent neural network for the approximation of the no-memory nonlinearity resulting from KLT expansion. The analyzed nonlinear systems are stochastic nonlinear difference equations that generate complex output signals from several initial conditions and random parameters of the input signals. Exhaustive experimentation on specific case studies showed

¹A patent application of the proposed algorithm has been deposited by C. Turchetti and F. Gianfelici [11] according to the copyright laws of the Italian Government.

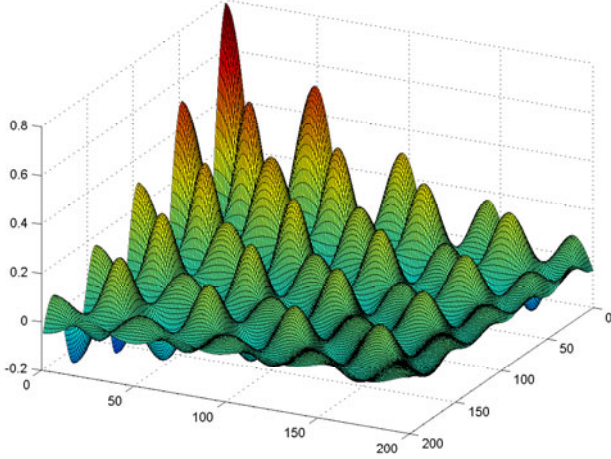


Fig. 1. Autocorrelation Matrix as achieved from $3 \cdot 10^4$ realizations of the Duffing SP.

high identification performance with a limited number of signals used in the training phase. Several comparisons with Least Mean Square (LMS), Recursive Least Square (RLS) and Normalized Least Mean Square (NLMS) algorithms showed higher identification performance. Moreover, a reduction of some orders of magnitude on the processing-times of the Lee-Schetzen method were achieved.

The paper is organized as follows. In Sect. 2 the representation of nonlinear system with the Karhunen-Loève Transform is proposed. In Sect. 3 the learning algorithm is presented. In Sect. 4 experimental results are shown. Sect. V gives some exhaustive comparisons with the state-of-the-art. Finally, Sect. 6 concludes the work.

2. REPRESENTATION OF NONLINEAR SYSTEMS WITH THE KARHUNEN-LOÈVE TRANSFORM

A system is defined, in a broad sense, as a rule \mathcal{T} that associates an input \mathbf{u} to an output \mathbf{y} , both \mathbf{u} and \mathbf{y} being stochastic processes (SPs) (here we use a *boldface letter* for random variables and SPs and a *letter in normal font* for the deterministic variables and the SP realizations).

Since the output and the input are, in general, functions of an independent variable t -time, position or any other quantity - the rule is expressed symbolically as

$$\mathbf{y}(t) = \mathcal{T}[\mathbf{u}(t)]. \quad (1)$$

The *identification problem* can be stated as follows. Let us denote with \mathcal{U}_{t_0} the set of all functions $\mathbf{u}(t)$ with $t \geq t_0$ and with \mathcal{Y}_{t_0} the corresponding set of functions $\mathbf{y}(t)$ obtained by transformation (1). We want to establish a mathematical relationship that is able to describe all the input-output pairs so defined by varying the time instant t_0 . This

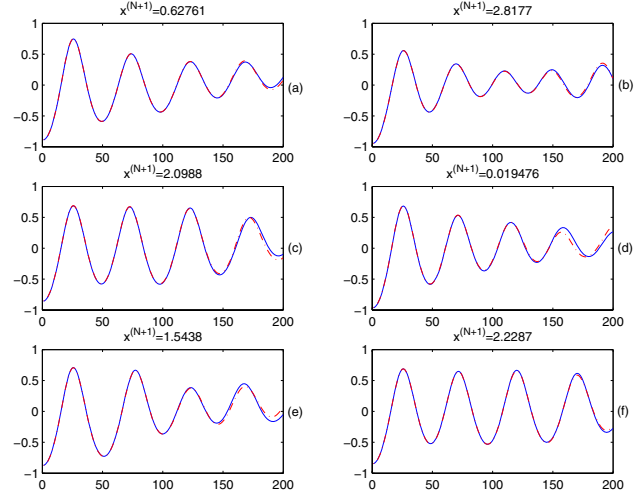


Fig. 2. Identification of the Duffing system $N=50$, $L=200$, Poly. Order ($n = 10$).

definition implies that in order to achieve a complete description the system has to be stimulated by all the functions belonging to the space \mathcal{U}_{t_0} . In the Wiener method, the use of a white noise as the input signal ensures an exhaustive exploration of the input space.

However, in most application problems a complete description of the system is not required. In fact, in the identification stage the input signal space could be restricted to the set of signals actually occurring in the problem under observation.

Let us suppose \mathbf{u} is an SP depending on a random parameter \mathbf{x} , (for the sake of notational simplicity and without loss of generality we assume \mathbf{x} is a scalar) with known statistics. Since \mathbf{x} varies, $\mathbf{u}(t, \mathbf{x})$ belongs to a subset of \mathcal{U}_{t_0} and $\mathbf{y}(t)$ is also an element of a subset of \mathcal{Y}_{t_0} .

By referring to discrete-time finite-time signals, $\mathbf{u} = \{\mathbf{u}[n], n = 0, 1, 2, \dots, L-1\}$ and $\mathbf{y} = \{\mathbf{y}[n], n = 0, 1, 2, \dots, L-1\}$ eq. (1) become:

$$\mathbf{y}[n, \mathbf{x}] = \mathcal{T}\{\mathbf{u}[n, \mathbf{x}]\}, \quad n = 0, \dots, L-1. \quad (2)$$

In this case \mathbf{y} can be represented by the Discrete Karhunen-Loève Transform (DKLT)

$$\mathbf{y}[n, \mathbf{x}] = \sum_{j=1}^L \mathbf{k}_j(\mathbf{x}) \phi_j[n], \quad n = 0, \dots, L-1 \quad (3)$$

where $\phi_j[n], j = 1, 2, \dots, L$ are orthonormal functions.

Eq. (3) and its inverse are written in matrix form as

$$\begin{cases} \mathbf{y} = \Phi \mathbf{k}(\mathbf{x}) \\ \mathbf{k}(\mathbf{x}) = \Phi^T \mathbf{y} \end{cases} \quad (4)$$

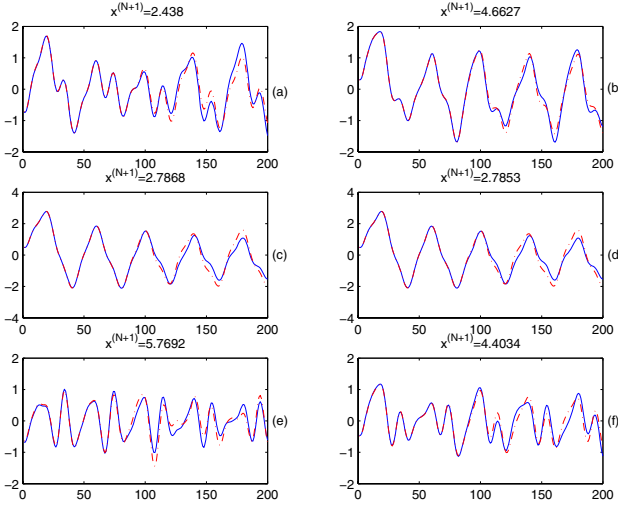


Fig. 3. Identification of the Narendra system $N=50$, $L=200$, Poly. Order ($n = 10$).

where \mathbf{y} and \mathbf{k} are both vectors $\mathbf{y} = [y_1, \dots, y_L]$, $\mathbf{y}_j = \mathbf{y}[j - 1]$, $\mathbf{k} = [k_1, \dots, k_L]$. Φ is the orthogonal matrix $\Phi = [\phi_1, \dots, \phi_L]$ whose columns are the eigenvectors $\phi_j[n]$, solutions of the eigenvalue equation

$$R_{yy}\Phi = \Phi\Gamma \quad (5)$$

where

$$R_{yy} = E\{\mathbf{y}\mathbf{y}^T\} \quad (6)$$

is the autocorrelation matrix of \mathbf{y} , and Γ is the matrix with eigenvalues on the main diagonal.

In order to approximate (4) a multilayer neural network can be used, thanks to its properties of universal approximator. By referring for simplicity to a single layer net, the output $\mathbf{z} = [z_1, \dots, z_L]^T$ is given by

$$\mathbf{z} = \mathbf{N}[\mathbf{x}; \mathbf{W}] \quad (7)$$

where $\mathbf{N}[\cdot]$ is a nonlinear operator and \mathbf{W} is the matrix of weights. The learning algorithm is particularly simple if $\mathbf{N}[\mathbf{x}; \mathbf{W}]$ is a linear function of \mathbf{W} , so that each component z_j is a linear combination of n functions $\{N_1(\mathbf{x}), \dots, N_n(\mathbf{x})\}$, namely

$$\mathbf{N}[\mathbf{x}; \mathbf{W}] = \mathbf{W}\mathbf{N}[\mathbf{x}] \quad (8)$$

with $\mathbf{N}[\mathbf{x}] = [N_1(\mathbf{x}), \dots, N_n(\mathbf{x})]^T$ and $\mathbf{W} = [w_1, \dots, w_L]^T$. Due to the approximating properties of the neural network we have

$$\mathbf{k}(\mathbf{x}) \approx \mathbf{W}\mathbf{N}[\mathbf{x}] \quad (9)$$

and also

$$\mathbf{y} \approx \Phi\mathbf{W}\mathbf{N}[\mathbf{x}] \quad (10)$$

The above equation represents a model for the system (2), with the matrix \mathbf{W} to be estimated.

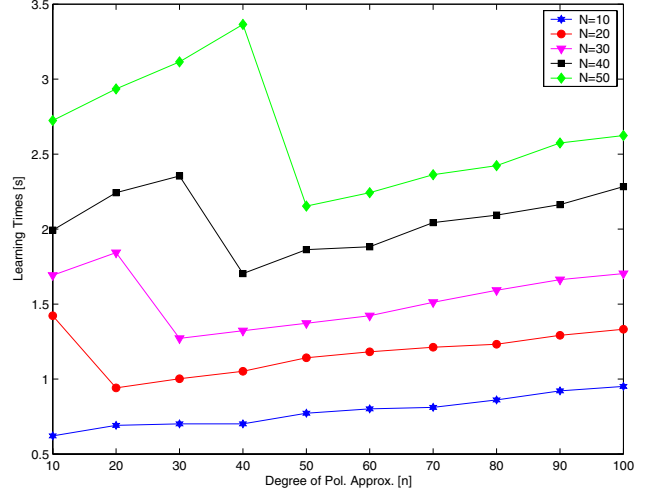


Fig. 4. Learning Times ($L=200$).

3. LEARNING ALGORITHM

Determining the matrix of weights is equivalent to the neural network learning from a collection of examples. The previous equations relate the SPs, while in order to derive a learning algorithm, we must refer to the realizations x_1, x_2, \dots, x_N of \mathbf{x} and the realizations $y^{(1)}, y^{(2)}, \dots, y^{(N)}$ of \mathbf{y} which can be put in matrix form as $X = [x_1, x_2, \dots, x_N]^T$ and $D = [y^{(1)}, y^{(2)}, \dots, y^{(N)}]$. A currently used estimation of the autocorrelation matrix is given by

$$R_{yy} \approx \hat{R}_{yy} = \frac{1}{N}DD^T \quad (11)$$

and its spectral representation is

$$\hat{R}_{yy}U = U\Lambda \quad (12)$$

where $U = [u_1, \dots, u_L]$ is the matrix of eigenvectors and Λ the matrix of eigenvalues. By projecting all the N realizations onto the basis U we obtain the KLT representation

$$\begin{cases} y^{(i)} = Ua^{(i)} \\ a^{(i)} = U^T y^{(i)} \end{cases} \quad i = 1, \dots, N \quad (13)$$

that is the equivalent of (4) for the realizations. By using (9), by virtue of the properties of multilayer networks, the following approximation holds

$$a^{(i)} \approx \mathbf{W}\mathbf{N}[x^{(i)}], \quad i = 1, \dots, N \quad (14)$$

Consequently

$$y^{(i)} \approx U\mathbf{W}\mathbf{N}[x^{(i)}], \quad i = 1, \dots, N \quad (15)$$

and for the single components we have

$$u_\mu^T y^{(i)} \approx w_\mu^T \mathbf{N}[x^{(i)}] \quad \begin{matrix} \mu = 1, \dots, L \\ i = 1, \dots, N \end{matrix} \quad (16)$$

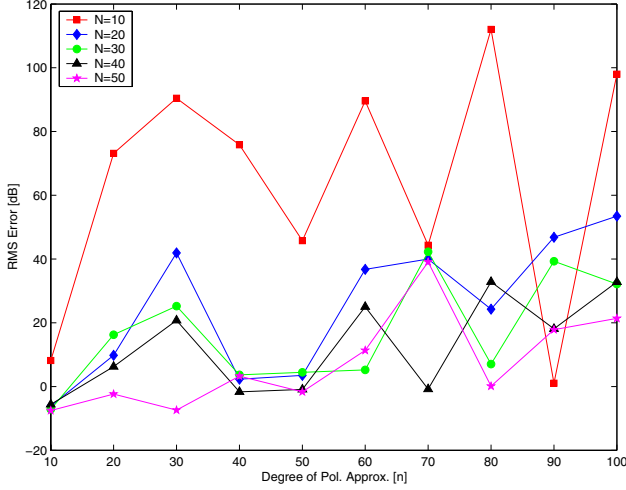


Fig. 5. RMS error as a function of the degree of the polynomial approximation. The RMS error has been estimated with $5 \cdot 10^3$ signals ($L=200$).

The matrix W can be easily determined by using the least-mean-square approach, that corresponds to the back-propagation algorithm. To that end minimizing the mean-square-error for the μ -th component

$$E_\mu = \sum_{i=1}^N \left[\varepsilon_\mu^{(i)} \right]^2 = \sum_{i=1}^N \left\{ w_\mu^T y^{(i)} - w_\mu^T N \left[x^{(i)} \right] \right\}^2 \quad (17)$$

yields the following equation

$$\frac{\partial E_\mu}{\partial w_\mu^T} = 0 \quad (18)$$

which reduces to

$$D^T w_\mu = N^T [X] w_\mu, \quad \mu = 1, \dots, L. \quad (19)$$

Finally the weights w_μ are estimated by solving the L linear matrix equations (19).

4. EXPERIMENTAL RESULTS

In this section some experimental results of nonlinear system identification using the previously described approach are presented. Several examples with *noiseless* and *noise* output signals are considered and presented in the three parts into which this section is methodologically divided.

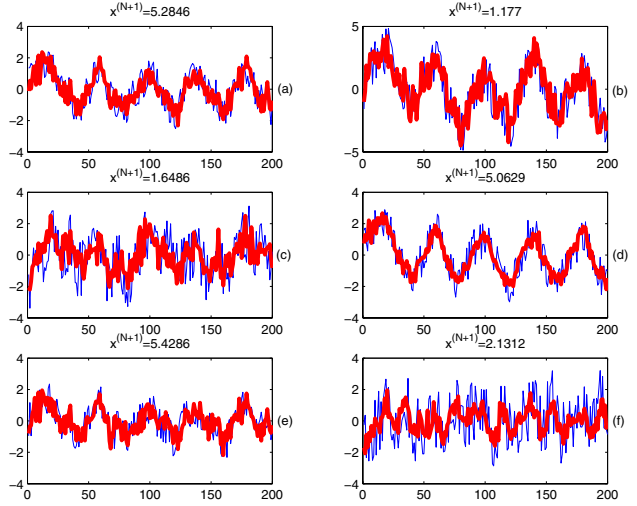


Fig. 6. Identification of the Narendra system (with superimposed additive Gaussian noise), $N=50$, $L=200$, Poly. Order ($n = 10$).

4.1. Example 1

The first example of system to be identified is described by the difference equation:

$$y(t+1) = \frac{\Delta t^2 [\Gamma \cos(\omega t) - \beta y^3(t)] - y(t-1)}{(1+k\Delta t)} + \frac{y(t)[k\Delta t - \alpha \Delta t^2 + 2]}{(1+k\Delta t)} \quad (20)$$

where Δt , k , α , β , Γ are constant parameters and ω is a random variable (r.v.). Equation (20) is the discrete-time version of the well-known Duffing equation with $\Delta t = 0.05$, $k = 0.3$, $\alpha = -4$, $\beta = 1$, $\Gamma = 0.5$, ω being an r.v. uniformly distributed in the interval $[0, 1]$ with mean $E\{\omega\} = 3.5$.

The autocorrelation matrix as estimated by eq. (11), is depicted in Fig. 1 for $3 \cdot 10^4$ realizations.

Choosing a polynomial for the nonlinearity in (8), that is

$$N[\mathbf{x}] = [1, \mathbf{x}, \dots, \mathbf{x}^n]^T, \quad (21)$$

corresponds to using a polynomial neural network [14, 15] for approximating the function. As these networks are particularly suitable due to the well-known approximating properties of polynomials in function representation, they were used in the identification of the experimental examples reported here.

The ability of polynomial networks in representing nonlinear input-output mappings is proved by the results in Fig. 2. As can be seen for $N=50$ realizations of length $L=200$, and a polynomial order = 10 the approximation is very good.

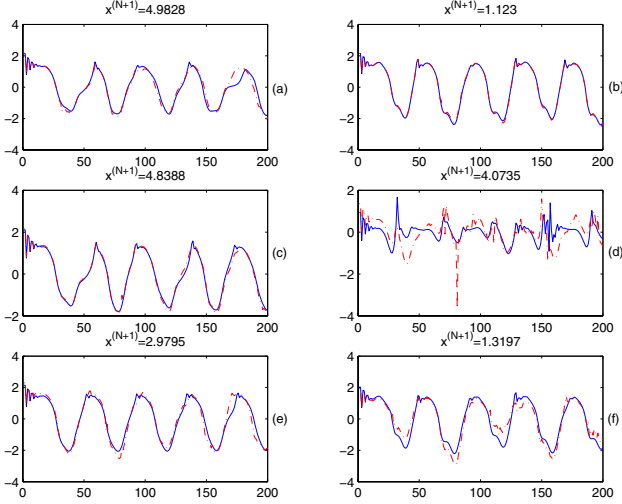


Fig. 7. Identification of the third system $N=50$, $L=200$, Poly. Order ($n = 10$).

4.2. Example 2

The second example is more complex. The system to be identified is governed by a modified version of Narendra's difference equation [9]:

$$\begin{cases} u(t) = 0.6 \sin^\alpha(\pi t) + 0.3 \sin(3\pi t) + 0.1 \sin(\alpha t) \\ \tilde{y}(t+1) = 0.3 \tilde{y}(t) + 0.6 \tilde{y}(t-1) + u(t) \\ y(t+1) = \Re[\tilde{y}(t+1)] \end{cases} \quad (22)$$

where $\Re[\cdot]$ denotes the real part, and α is an r.v. uniformly distributed in the interval $[1, 2.5]$ with $E\{\alpha\} = 3.5$. Figure 3 shows the identification capabilities of the proposed approach for this system, with $N=50$, $L=200$, and Poly. Order ($n = 10$) comparing several trajectories of eq. (22) with those of the approximating model. The time required in the learning times as a function of N and n is reported in Fig. 4, clearly illustrating: (i) the well-known behavior of Vandermonde-optimization for $N \leq n$, and (ii) the low computational complexity of the learning stage. Figure 5 shows the Root-Mean-Square (RMS) error in system identification as a function of the polynomial degree. In order to guarantee accurate measurements, $5 \cdot 10^3$ signals were considered. The experimental results clearly show that the RMS error decreases as N increases. Finally, identification with additive gaussian noise superimposed on the Narendra solutions, was considered. Figure 6 reports six cases (randomly chosen) which show that the dynamics of the approximating signals are very-close to those of the system to be identified, even when the signals are corrupted by noise.

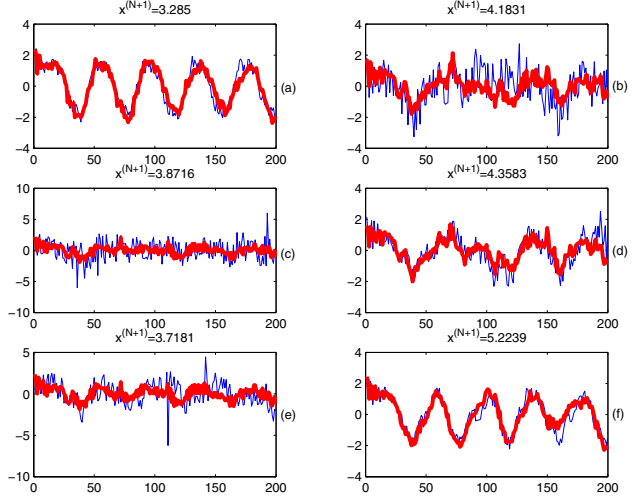


Fig. 8. Identification of the third system (with superimposed additive Gaussian noise) $N=50$, $L=200$, Poly. Order ($n = 10$).

4.3. Example 3

The third example of system to be identified is described by the difference equation:

$$\begin{cases} u(t) = 0.6 \sin^\alpha(\pi t) + 0.3 \sin(3\pi t) + 0.1 \sin(\alpha t) \\ f[\tilde{y}(t), \tilde{y}(t-1)] = \tilde{y}(t)\tilde{y}(t-1)[\tilde{y}(t) - 2.5] \\ g[\tilde{y}(t), \tilde{y}(t-1)] = 1 + \tilde{y}^2(t) + \tilde{y}^2(t-1) \\ \tilde{y}(t+1) = \frac{f[\tilde{y}(t), \tilde{y}(t-1)]}{g[\tilde{y}(t), \tilde{y}(t-1)]} + u(t) \\ y(t+1) = \Re[\tilde{y}(t+1)] \end{cases} \quad (23)$$

where $\Re[\cdot]$ denotes the real part, and α is an r.v. uniformly distributed in the interval $[1, 2.5]$ with $E\{\alpha\} = 3.5$. Figure 7 shows the identification results with $N=50$, $L=200$, and Poly. Order ($n = 10$). Except for the case depicted in Fig. 7-(d) where the abrupt variations in the dynamics generate several spikes (due to the low values of N and n), in the other cases the approximated dynamics accurately follow the signals generated by eq. (23). Moreover, it is worth noting that the initial transitions, and the micro and macro dynamics of signals are generally captured by this method. Finally the case of additive Gaussian noise superimposed on (23) was considered. Figure 8 reports the signals of the given system corrupted by noise, and those from modelling in bold lines. The dynamics are excellently identified in cases (a), (d), and (f). Good results are obtained in cases (c) and (d) where the strong spikes are filtered. Finally case (b) has a good result only in two intervals of the signals:

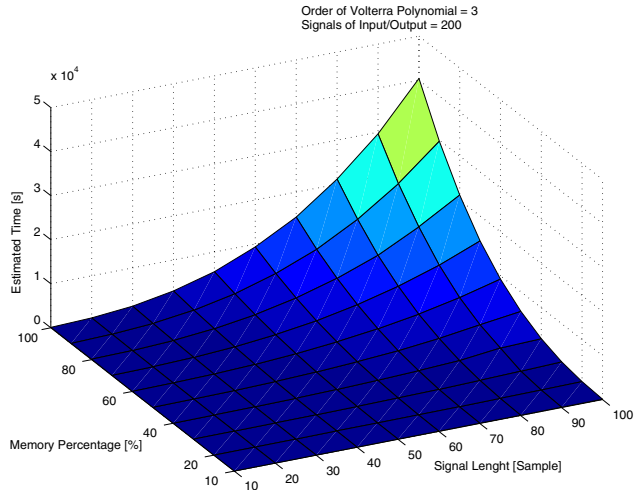


Fig. 9. Estimated processing-times of the Lee-Schetzen method as a function of the signal and memory lengths. The order of Wiener-Volterra approximation is 3 and the Input/Output signals (N) are 200.

$0 \leq t \leq 70$ and $130 \leq t \leq 200$. Nevertheless, an excellent improvement in results can easily be obtained by increasing the low values of N and n without a strong deterioration in the learning times.

5. COMPARISON WITH THE STATE-OF-THE-ART

In order to clearly show the effectiveness of our approach, several comparisons with current best practices were considered. In this analysis the Lee-Schetzen method [1, 7], based on the cross-correlation technique for the identification of Volterra-Wiener kernels, and the adaptive filtering based approaches [16], such as the Least Mean Square (LMS), the Recursive Least Square (RLS) and the Normalized Least Mean Square (NLMS) algorithms, were taken into account. Methodologically, the performance evaluation was divided into two parts: processing times and identification accuracy. Figure 9 shows the estimated processing times of the Lee-Schetzen method for the example 1. As a reference implementation for this technique the algorithm by Orcioni *et al.* [17, 18] was used, which is, to the best of the authors' knowledge, one of the best optimized implementations available. The simulation results were obtained with several signal and memory lengths, whose the second ones are mathematically expressed as the percentage between the signal length of each simulation and the used memory. The order of Wiener-Volterra approximation is 3 and the Input/Output signals are 200. It is worth noting that a reduction of several orders of magnitude in training-times was obtained using our approach (Fig. 4) compared with the above-mentioned Lee-Schetzen method. Additional com-

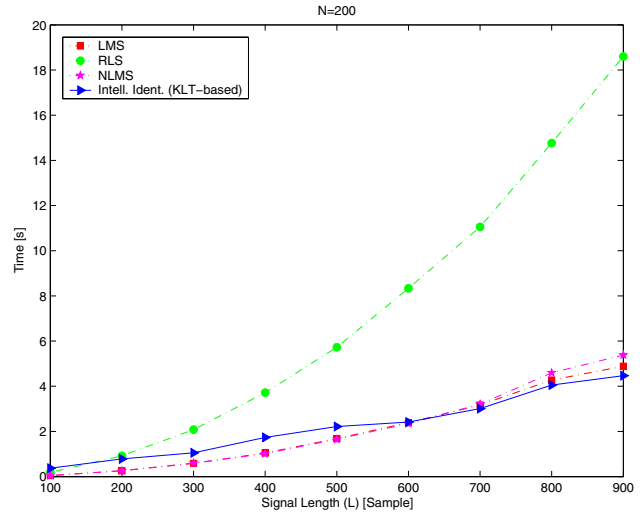


Fig. 10. The processing-times of: (a) the proposed approach (Intelligent Identification based on KLT), (b) LMS, (c) RLS, and (d) NLMS algorithms. The Poly. Order is $n=5$.

parisons with adaptive filter-based approaches are shown in Fig. 10. In this case, the considered techniques are: (a) the proposed approach (Intelligent Identification based on KLT), (b) LMS, (c) RLS, and (d) NLMS algorithm. The comparison results show that the identification-times of our approach are similar to the LMS and NLMS algorithms, and lower than the RLS algorithm. Moreover, it is worth noting that our approach provides a better identification-accuracy than the other techniques with no limitations in terms of training-set dimensions and the signal length (Fig. 11).

6. CONCLUSIONS

A novel approach based on the approximation capabilities of multi-layer neural networks for system identification has been proposed. Experimental results on several non-linear dynamical systems with random parameters, clearly show the effectiveness of the algorithm. Excellent learning-times and identification-performance both with *noiseless* and *noise* signals were achieved. Some comparisons with the state-of-the art showed a better identification accuracy than current best practices based on adaptive-filtering. Moreover a reduction of some orders of magnitude in training times compared with the well-known Lee-Schetzen method was achieved, thus making the proposed methodology one of the current best practices in this field.

7. ACKNOWLEDGMENTS

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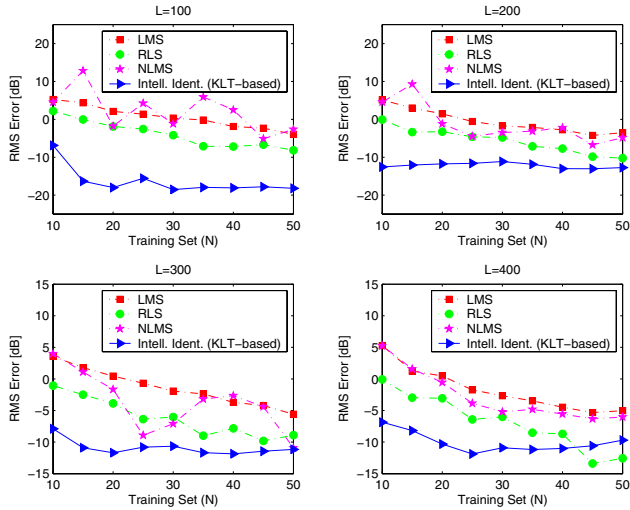


Fig. 11. The Root-Mean-Square (RMS) Error of: (a) the proposed approach (Intelligent Identification based on KLT), (b) LMS, (c) RLS, and (d) NLMS algorithms. The Poly. Order is $n=5$.

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