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Signal Processing Using Fuzzy Fractal Dimension and Grade of Fractality –Application to Fluctuations in Seawater Temperature–

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***Abstract*— Discrete signal processing using fuzzy fractal dimension and grade of fractality is proposed based on the novel concept of merging fuzzy theory and fractal theory. The fuzzy concept of fractality, or self-similarity, in discrete time series can be reconstructed as a fuzzy-attribution, i.e., a kind of fuzzy set. The objective short time series can be interpreted as an objective vector, which can be used by a newly proposed membership function. Sliding measurement using the local fuzzy fractal dimension (LFFD) and the local grade of fractality (LGF) is proposed and applied to fluctuations in seawater temperature around the Izu peninsula of Japan. Several remarkable characteristics are revealed through "fuzzy signal processing" using LFFD and LGF.**

I. Introduction

Fuzzy fractal dimension and grade of fractality are proposed based on the novel concept of merging fuzzy theory [1], [2] and fractal theory [3]-[8]. Fractals and fractal dimensions have been investigated extensively by Mandelbrot [3]-[8] and have been applied to a wide range of scientific fields [9]-[12]. Various examples in image processing as an artificial intelligence for medicine have been reported by Zhou and Lu [13], and several examples in plant monitoring systems have been reported by O. Castillo and P. Melin [14], [15] based on the hybrid fuzzy-fractal approach for time series analysis. In addition, complex systems observed in various phenomena, often treated as "signal processing" in discrete dynamical systems, have been investigated by Waldrop [16]. The present paper proposes the application of sliding measurement using local fuzzy fractal dimension (LFFD) and local grade of fractality (LGF). The LFFD and LGF for such systems are calculated for the short time series inside the processing unit. Secular changes in both indices can then be obtained by successively sliding the unit and repeating the procedure.

Although numerous methods for analyzing discrete time series have already been developed, the present study searched for fuzzy fractal structure hidden in the form of self-similarity in a dynamical system. In other words, the authors attempted analysis and application from a different approach with respect to the orbit of a dynamical system for "fuzzy signal processing".

As an application in the present paper, we describe a new fuzzy fractal approach for a system to monitor seawater temperature around Japan. Local fuzzy fractal dimension is used to measure the complexity of a time series of observed seawater temperature.

II. Fuzzy Fractal Structure

Generally, when the fractal dimension is calculated formally for a phenomenon that is not guaranteed to be fractal in nature, the fractal dimension depends on the observation scale [17]. In this case, the dimension is referred to as a scale-dependent fractal dimension. However, if the "fuzzy fractal dimension" can be treated as a function of the observation scale, then it may be possible to extend and apply the fuzzy fractal dimension to phenomena that are not originally fractal in nature [17]. That is to say, for example, characteristics in various time series can be treated as a "fuzzy fractal phenomenon" or a "fuzzy concept" from the viewpoint of fuzzy system.

III. Definition of Local Fuzzy Fractal Dimension (LFFD)

For a discrete time series, which can be considered as an objective vector,

$$\boldsymbol{x}_k = \{x_k, x_{k+1}, x_{k+2}, \cdots, x_{k+L-1}\}, \quad (1)$$

the accumulated change $N(r, k, L)$ can be defined as [18], [19]:

$$N(r,k,L) = \frac{1}{r}\sum_{i=1}^{r}\sum_{j=0}^{\left[\frac{L}{r}\right]-2}\left|x_{k+jr+r+i-1} - x_{k+jr+i-1}\right| \quad (2)$$

where

L: length of the discrete time series,
r: sampling interval.

The accumulated change $N(r, k, L)$ can also be redefined as:

$$N(r,k,L) = A\, r^{-D_k} \qquad (3)$$

where

D_k: *k-th* local fuzzy fractal dimension,
A: proportion constant.

Therefore,

$$\log N(r,k,L) = -D_k \log r + \log A \qquad (4)$$

Then, based on a regressive analysis, D_k obtained above is defined as the local fuzzy fractal dimension, and in this paper we will refer to $LFFD_k$ instead of D_k, as the *k-th* local fuzzy fractal dimension.

That is,

$$LFFD_k \equiv D_k \qquad (5)$$

IV. Fuzzy Concept or Attribution of Fractality

The local fuzzy fractal dimension itself is thought to express the extent to which the time series pattern is complex in a processing unit on the long time series. In contrast, it might be necessary to derive some new kinds of scales to express the degree of strength of the fractal performance. Therefore, in the present work, we have adopted the "local grade of fractality", LGF, to satisfy the above-mentioned necessity. The LGF consists of the degree-of-freedom-adjusted contribution ratio, which is popular and often used in the fields of the multivariate analysis or statistics, to show the smoothness of fit of the regressive line. The reason for this is because the use of only six measurement points increases the influence of observation errors, which necessitates that observers be aware that the influence of errors must be taken into account. Accordingly, it can be understood from this condition that the greater the LGF, the greater the fractality in the time series of the change pattern or fluctuation.

V. Local Grade of Fractality (LGF)

A method for obtaining the LFFD from the information on six plots has already been proposed as the Six-Point Evaluation Method [19], and the degree-of-freedom-adjusted contribution ratio that indicates how well the regression line fits is called the local grade of fractality (LGF). This LGF is related to the fractality associated with the "manner of change" of the said time series, and indicates the extent of fractal strength. In the case of a perfect mathematical fractal, the LGF is 1, while, alternately, the LGF becomes a value close to 0 when there is absolutely no fractality. In other words, this may be considered as being equivalent to "grade" in so-called fuzzy logic, and is equivalent to the "extent of fuzziness of fractality" being quantized within the range (0,1).

Further, as described later, changes in LGF can be investigated by a sliding measurement, and so the association between the strength of fractality in the "manner of change" for the time series of the said physical quantity and the original physical phenomenon can be discussed.

The LGF is defined as follows using each variance (mean square: MS) in a variance table for regressive analysis, such as TABLE I described later.

$$LGF = 1 - \frac{V_e}{V_T} \qquad (6)$$

where
V_e: error variance,
V_T: total variance,

Then we obtain

$$0 \le LGF \le 1 . \qquad (7)$$

Therefore it is evident that the LGF possesses enough qualifications for the so-called "grade" in fuzzy theory.

Moreover, as the regression analysis in this instance is performed on six measurement plots, it is well known that regression analysis is extremely susceptible to the influence of measurement error. To avoid the over-evaluation of the regression-based contribution ratio, the contribution ratio must be evaluated after the error variance possessing a one degree-of-freedom has been subtracted from the sum of squares (SS) caused by regression.

VI. Calculation Principles for Fuzzy Signal Processing (Numerical Value Example)

This section explains in concrete terms the series of processes leading up to the preceding paragraph. For example, chaotic fluctuation (discrete dynamical system orbit) caused by logistic mapping on the assumption that control parameter a=3.95 with suitable initial value is adopted to calculate each LFFD and LGF for a time series of length 120.

Generally, when observations are performed by changing the observation scale (sampling interval; r=1, ... , 6), six time series, such as those shown in Fig. 1, are obtained.

Next, Fig. 2 displays the relationship between r and the accumulated change (actual working distance) N on which the power function is fitted.

From Fig. 2, the LFFD and LGF are found to be 1.205 and 0.9817, respectively.

Next, the graph is replotted using the natural logarithm of both axes in Fig. 2, and the results are presented in Fig. 3. The negative gradient of the regression line in this figure reconfirms that the LFFD is 1.205.

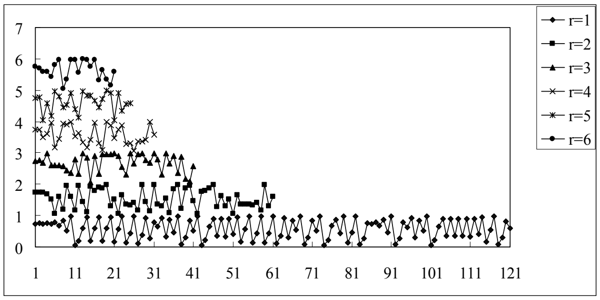

Fig. 1. Time series generated by six kinds of sampling interval

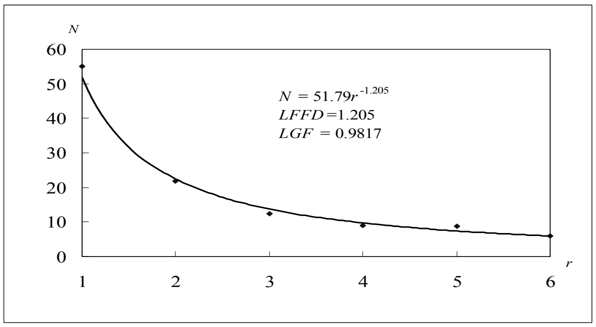

Fig. 2. Curve fitting by power function, and calculation for LFFD and LGF

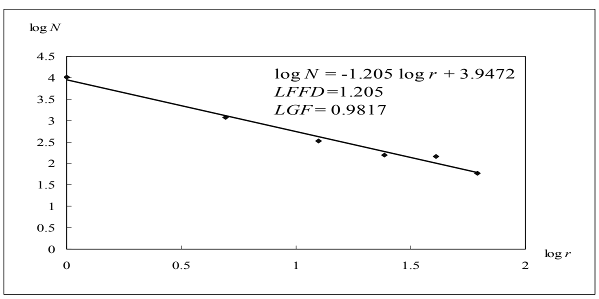

Fig. 3. Regressive line fitting on log-log plane

Also, TABLE I shows the variance analysis for this regression procedure.

Incidentally, the "graph shape" of the power function is self-similar. In other words, as N is usually k^{-D} times if r is generally k times, the graph need not be rewritten anew; only the scale of the graph should be redefined.

TABLE I
VARIANCE ANALYSIS FOR REGRESSION BY SIX-POINT EVALUATION METHOD

SV	SS	DF	MS	Fo
R	3.187982	1	3.187982	269.2283
e	0.047365	4	0.011841	
T	3.235347	5	0.647069	

where
SV: Source of Variation,
SS: Sum of Squares,
DF: Degree of Freedom,
MS: Mean Square,
F_o: Observed F value,
R: Regression,
e: Error,
T: Total.

VII. How to Obtain Local Grade of Fractality (Numerical Value Example)

LGF becomes the following from the results of TABLE I if definition equation (6) is applied:

$$LGF = 1 - \frac{0.011841}{0.647069} = 0.9817 \quad (8)$$

Furthermore, the same result is obtained using the following equation if the approach of subtracting one error variance from the sum of squares caused by regression is applied:

$$LGF = \frac{3.187982 - 0.011841}{3.235347} = 0.9817 \quad (9)$$

(Proof)

$$LGF = \frac{S_R - V_e}{S_T} = \frac{S_T - S_e - V_e}{S_T} \quad (10)$$

$$= \frac{S_T - (\varphi_e V_e + V_e)}{S_T} = \frac{S_T - V_e(\varphi_e + 1)}{S_T} \quad (11)$$

$$= 1 - \frac{V_e}{\dfrac{S_T}{\varphi_e + 1}} = 1 - \frac{V_e}{V_T} \quad (12)$$

where
S_T: sum of squares for total,
S_R: sum of squares by regression,
S_e: sum of squares for error
V_e: error variance,
V_T: total variance,
φ_e : degree of freedom for error.

VIII. Mathematical Properties of Local Fuzzy Fractal Dimension

This section considers the mathematical properties of the local fuzzy fractal dimension proposed in this paper.

[Property 1]

The values of the local fuzzy fractal dimension are invariable even if a constant value is added to each value in the discrete time series.

(Proof 1)

Property 1 is self-evident even if a constant value is added as the accumulated change is ultimately obtained using the differences between each value owing to the properties of definition equation (2).

[Property 2]

The values of the local fuzzy fractal dimension are invariable even if each value in the discrete time series is multiplied by a constant value.

(Proof 2)

The local fuzzy fractal dimension as the gradient is invariable and Property 2 is self-evident as the regression line can be shifted in parallel translation when the accumulated change is obtained using the product of each value owing to the properties of definition equation (2) with logarithmic conversion.

Moreover, from the above two properties, it is evident that even if each value in the original time series undergoes a so-called standardization, where the mean value of the time series is subtracted and divided by the standard deviation, the values of the local fuzzy fractal dimension remain invariable.

[Property 3]

The values of the local fuzzy fractal dimension can be expected to be almost constant without depending on L, if unit width L is sufficiently large when the same properties in the time series are considered as being retained in the unit.

(Proof 3)

When unit width L (length of observed time series) in which local fuzzy fractal dimension *LFFD* (*i.e.* here D instead of *LFFD*) is measured is extremely long, the accumulated change N_r' that is equivalent to the actual working distance can also be expected to be as follows, if L is increased s times:

$$N_r' = sN_r \qquad (13)$$

$$\log N_r' = \log s + \log N_r \qquad (14)$$

From equation (4), using N_r instead of $N(r, k, L)$,

$$\log N_r = -D \log r + \log A \qquad (15)$$

Then,

$$\log N_r' = (-D \log r + \log A) + \log s \qquad (16)$$

Due to this fact, the graph of log N_r' is formed by shifting the graph of log N_r in parallel translation by log s in the forward direction of the vertical axis, and its gradient $-D$ is invariable.

Accordingly, when unit width L is extremely long, and the same properties of the time series (e.g. the control parameter in this term is invariable in case of logistic time development) are considered as being retained in that term, then it can be fully predicted that the local fuzzy fractal dimension obtained from the said time series will not be depend on unit width L (length of time series).

IX. Calculation for Local Fuzzy Fractal Dimension

We assume that a unit having width L is installed on the long time series

$$\{x_0, x_1, x_2, \cdots x_k, x_{k+1}, x_{k+2}, \cdots, x_{k+L-1} \cdots\}$$

related to some physical quantity targeted for observation, and that this unit is progressively slid to the right in increments of a single epoch for each successive calculation.

Here, the series of processes (*i.e.* the procedure for calculating the regression coefficient) after logarithmic conversion in the processing unit is expressed by functions f and F. If the Six-Point Evaluation Method (*i.e.* evaluation of six plots) is adopted for regression analysis, then $LFFD_k$ can be expressed as:

$$LFFD_k = f(\{[r, N(r,k,L)] \mid r = 1:6\}) \qquad (17)$$

$$= F(\boldsymbol{x}_k) \qquad (18)$$

In other words, equation (17) can also be called the mean local fuzzy fractal dimension in the observation scale (*i.e.* sampling interval) r = 1 to 6 in the processing unit.

And LFFD corresponds to a kind of "property" or "manner of change" in the time series.

X. Membership Function for Local Grade of Fractality

Likewise, if the process for calculating the local grade of fractality, LGF, that indicates how well the regression line fits is expressed by function g, which possesses the prescribed computational procedure and the Six-Point Evaluation Method is adopted, then LGF_k can be expressed as:

$$LGF_k = g(\{[r, N(r,k,L)] \mid r = 1:6\}) \qquad (19)$$

$$= \boldsymbol{\mu}_{\text{Fractal}}(\boldsymbol{x}_k) \qquad (20)$$

where $\boldsymbol{\mu}_{\text{Fractal}}$ is a kind of complex and particular "membership function" on the fuzzy concept "fractal".

Then we obtain

$$0 \le LGF \le 1 \qquad (21)$$

Therefore the LGF corresponds to "fuzziness of fractality" in the objective time series changes, and possesses enough qualifications as the so-called "grade" in fuzzy theory.

XI. SLIDING MEASUREMENT

The unit having width L is slid successively on the long time series of the physical quantity targeted for observation according to the above procedure, and the specified regression analysis is repeated.

That is, since each of the individual *LFFD*s and *LGF*s can be calculated for the *k-th* unit (observed time series or objective vector)

$$\boldsymbol{x}_k = \{x_k, x_{k+1}, x_{k+2}, \cdots, x_{k+L-1}\}, \qquad (22)$$

changes in the LFFD and LGF time series can be observed if k is increased successively. Ultimately, if these three time series (*i.e.* the time series of the original physical quantity targeted for observation, the LFFD time series, and LGF time series) are judged from an entirely analytical standpoint, then various characteristic properties can be obtained. Fig. 4 shows the concept of sliding measurement using LFFD and LGF based on the above procedure [19].

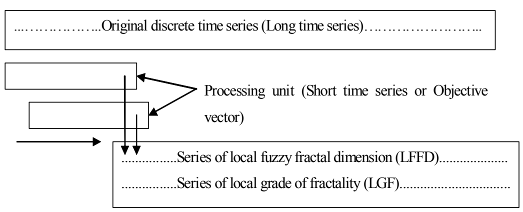

Fig. 4. Sliding measurement using LFFD and LGF

XII. APPLICATION TO FLUCTUATIONS IN SEAWATER TEMPERATURE

A. Outline

We describe in this section a new fuzzy fractal approach for monitoring system in seawater temperature. We use the concept of the local fuzzy fractal dimension to measure the complexity of a time series of observed seawater temperature. Continuous observation of the fluctuations in seawater temperature is an important aspect of studying changes in weather patterns. The sea surface constitutes the boundary between the atmosphere and the sea, with heat exchange constantly occurring between these two bodies. As a result, fluctuations in seawater temperature can be employed as an index, with daily, seasonal and secular changes being most apparent in the surface layer (0 to 15 m). In the present study, the characteristics of fluctuations in seawater temperature in the near-surface layer (0-15 m) were observed and these properties were used to propose a method for estimating the seasonal boundary. In order to achieve these aims, we decided to use LFFD and LGF as a measure of the complexity from the viewpoint of "fuzzy signal processing".

B. Research Methods

The meteorological characteristics of the near-surface layer were obtained at a point off the Izu Peninsula (Usami, Ito City, Japan) by placing temperature data loggers at three depths: 5 m (upper layer), 10 m (middle layer) and 15 m (lower layer). The loggers were set to continuously measure seawater temperatures at 1-hour intervals. The discrete time series data was averaged on a daily basis and the differences in time series data for seawater temperatures between levels were compiled (between water depths 10 m and 5 m, and 15 m and 5 m), and then LFFD and LGF were calculated according to the previously specified method. Time series data was also subjected to moving average analysis (with sample count set to 30 epochs to account for the influence of the tides) to assess the characteristics of changes in the time series.

C. Analytical Results and Consideration for LFFD

Fig. 5 shows an example of the difference time series in seawater temperature, LFFD change, and the moving difference average between water depth 15 m and 5 m from 2003/7/4 through 2004/7/4. The following characteristics were obtained from an analysis of the data obtained from the near-surface layer:

1) Upon creating a difference time series between seawater temperatures at each depth with respect to the temperature at 5 m, and having observed the results obtained from the moving average (sample count set to 30 epochs), it was found that negative values were generally observed in the summer and positive values in the winter, suggesting the existence of a seasonal boundary.
2) If the seasonal boundary is defined as the day on which the plus/minus values of the moving average are reversed for the first time, then seasonal boundaries in this analysis are estimated to be October 17 and April 11. In the summer, the differences between seawater temperatures at each depth can be represented as a negative value because the seawater temperature near the sea surface is warmed by insolation and is thus warmer than the seawater in the lower layers. Conversely, in the winter, the difference between seawater temperatures at each depth assumes a positive value because the phenomenon is reversed.
3) The unusually cold summer of 2003 in Japan caused a reversion in seawater temperature, with the difference between temperatures becoming temporarily positive. And an increase in LFFD was observed, most notably during the period of August 15-19, the near-surface layer could be said to have temporarily entered a mode typical of winter.
4) Although the decrease in LFFD probably arose at the summer/fall boundary as a consequence of a phase transition to a regular mode from the cold summer, further comparisons and observations should be undertaken for a typical year so as to better understand the influence of the cold summer. An increase in LFFD is seen at the winter/spring boundary, because it is likely that the difference time series in seawater temperature became unstable at the onset of the following season and probably increased.

5) Despite similarities between LFFD values, the corresponding time series patterns were always found to differ.

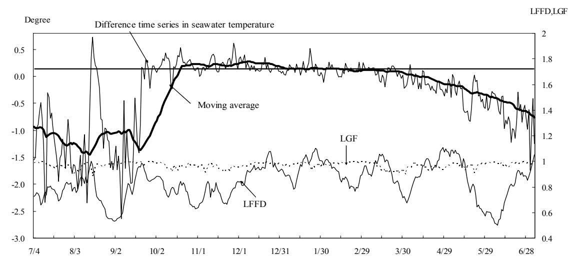

Fig. 5. Difference time series in seawater temperature, LFFD change and moving difference average between water depth 15 m and 5 m from 2003/7/4 through 2004/7/4

XIII. Characteristics of LGF Change before Seasonal Boundary

This section presents an example of an LGF-related time series. In the preceding section, various phenomenal analyses were attempted with the emphasis placed on LFFD as opposed to LGF. It has been decided, however, to attempt to study the relationship between changes in LGF and the said physical quantity, too.

Fig. 6 shows a 30-point moving difference average between water depth 15 m and 5 m, and the time series of the LGF (*i.e.* extent of strength of fractality) in a seawater temperature difference series (5 to 15 m). The following property can be read if the point of intersection between the 0° C reference line and the 30-point moving average is assumed to be the seasonal boundary [19].

(Property)

"As a premonitory phenomenon of a seasonal boundary, there is a temporary peaking of the LGF, and this is followed by the seasonal boundary when the LGF changes to a decreasing state."

The "decrease in LGF" refers to the place where the fractality in the "manner of change" of the said time series weakens. However, as this mechanism is completely unknown, observations must be continued and a careful investigation into whether or not this kind of property occurs at all times must be conducted.

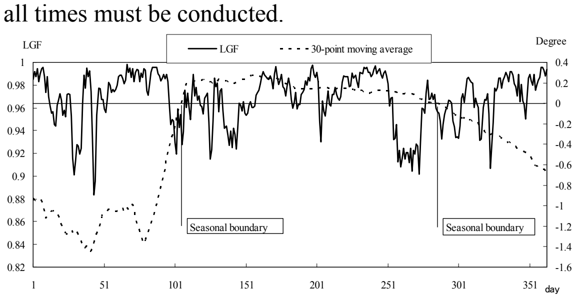

Fig. 6. 30-point moving difference average of seawater temperature between water depth 15 m and 5 m from 2003/7/4 through 2004/7/4, and LGF change with seasonal boundary

XIV. Conclusions

In the present paper, fuzzy fractal dimension and grade of fractality were proposed based on the novel concept of merging fuzzy logic and fractal theory. In addition, sliding measurement for "fuzzy signal processing" using a local fuzzy fractal dimension (LFFD) and local grade of fractality (LGF) were proposed. The authors were able to quantize an index related to the characteristics of orbits in a short-term discrete time series. The method was then applied to the characterization of the changes in seawater temperature around Izu Peninsula, Japan. The difference time series for seawater temperatures between water depths of 5 m and 15 m was compiled, and the LFFD and LGF were calculated according to the prescribed method. The LFFD was found to decrease at the summer/fall boundary and increase at the winter/spring boundary. A method was proposed for estimating the seasonal boundary based on the temperature reversion phenomenon observed in the near-surface layers of seawater. In addition, the LGF will be useful in monitoring seawater temperature using "fuzzy signal processing". This phenomenon may provide a highly sensitive sensor with LFFD and LGF in a weather monitoring system.

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