Proceedings of the 2007 IEEE Symposium on Computational Intelligence in Image and Signal Processing (CIISP 2007)

# An Efficient RLS Algorithm For Output-Error Adaptive IIR Filtering And Its Application To Acoustic Echo Cancellation

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**Abstract- In this letter, an efficient recursive least squares (RLS) algorithm using infinite impulse response (IIR) filter for acoustic echo cancellation (AEC) is proposed. The RLS adaptive filter is naturally extended from the finite impulse response (FIR) structure to the IIR structure. One of the main advantages of an IIR RLS filter is that a long-delay echo can be synthesized by a relatively small number of filter coefficients leading to lesser computational complexity. In addition it is more suitable for modeling physical systems, due to its pole-zero structure, over their FIR counterparts. To investigate the tracking performance and the stability of the proposed IIR RLS filter in a practical implementation, real data for far end speech signal and its echo were used in car environment. The proposed algorithm is shown to have global convergence. To demonstrate the effectiveness of the proposed IIR RLS filter, it is compared to the usual FIR RLS filter. The good performance of the proposed IIR RLS algorithm for AEC have been verified via computer simulations.**

## I. INTRODUCTION

In modern hands-free communication systems such as hands-free mobile phones, audio and video conference systems, it is necessary to perform an acoustic echo cancellation of the far-end speaker signal. A number of efficient algorithms has been proposed for that purpose [1, 2]. In an acoustic echo cancellation algorithm, a model of the room impulse response is identified. Since the conditions in the room may vary continuously, the model needs to be updated continuously. This is done by means of adaptive filtering algorithms. The room acoustics can be modeled by a FIR filter or IIR filter. In practice, the length of the room acoustics and by consequence also the filter length of the adaptive canceller is typically 500-2000 filter taps. Long filters imply a large computational burden and slow convergence rate[3]. These problems have motivated a new approach: adaptive filtering in subbands with the double purpose of reducing the computational complexity and of improving the convergence speed of the algorithm. On the other hand, subband processing introduces transmission delays caused by the filters in the filter bank and signal degradations due to aliasing effects [4].

In this letter, an IIR RLS algorithm based on the output-error formulation for acoustic echo cancellation is investigated. One of the main advantages of an IIR RLS filter is that a long–delay echo can be synthesized by a relatively small number of filter coefficients leading to lesser computational complexity. In addition it is more suitable for modeling physical systems due to its pole-zero structure. Unfortunately, these good characteristics come along with some possible drawbacks inherent to IIR adaptive filters such as algorithm instability and local minimum solutions. Consequently, several algorithms for adaptive IIR filtering have been proposed to overcome these problems [2, 5]. Furthermore, the study of algorithms other than stochastic gradient-based algorithms has rendered it possible to guarantee that the stationary points of adaptive IIR algorithms are close to the global minimum of the least-squares output error performance surface [6]. In addition, this letter study the tracking performance of the proposed IIR RLS algorithm for time-varying system. Simulation results show that the proposed algorithm produces results that are significantly favorable than usual FIR RLS algorithm for AEC. Moreover the proposed algorithm has good ability to track the time-varying unknown system and remain stable.

This letter is organized as follows. Section II provides the theoretical formulation of the proposed algorithm. In Section III, simulation results and analysis will be presented to illustrate the improved performance. In Section IV, brief conclusions will be drawn.

## II. DESIGNING RLS ALGORITHM USING IIR FILTER

The block diagram in Fig. 1 depicts the proposed filter architecture. The RLS algorithm based on output error approach was used to derive the feed forward and feedback coefficients which are used to identify the time-varying unknown system. A similar technique has been adopted by Yeary and Griswold [7] to design adaptive IIR filter that employs a single input sensor and over sampling for speech enhancement.

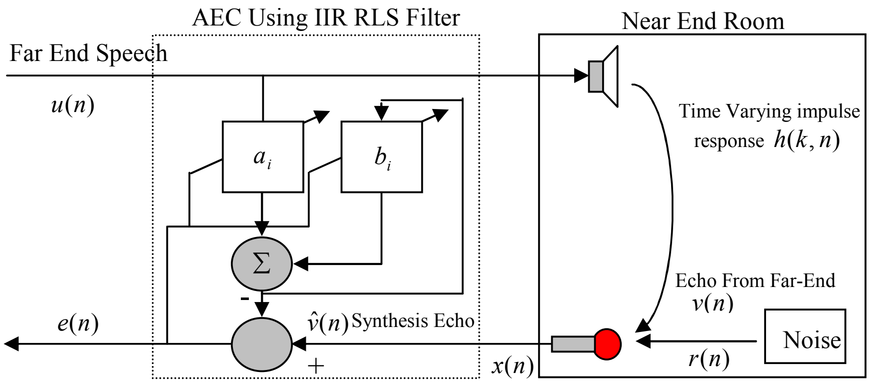


Fig. 1 Block Diagram Of Proposed IIR RLS Filter For AEC.

With reference to Fig. 1, the least square algorithm is designed to minimize the sum of squared errors (assuming here for simplicity no near end speech) as defined by

$$\varepsilon(n) = \sum_{i=0}^{n} e^2(i) \tag{1}$$

where $e(i) = x(i) - \hat{v}(i) = x(i) - \hat{\mathbf{h}}^{\mathbf{T}}\mathbf{z}(i)$ (2)

and $\hat{\mathbf{h}} = [a_0, a_1, .............a_N, b_1, b_2, ................, b_M]^T$ (3)

is the coefficient vector,

$\mathbf{z}(i) = [u(i), u(i-1), ....., u(i-N), \hat{v}(i-1), \hat{v}(i-2), .....\hat{v}(i-M)]^T$ (4)

is the signal vector containing the elements of far end speech vector $\mathbf{u}(n)$ and past samples of synthesis echo signal $\hat{v}(n)$. The vector $\mathbf{z}(i)$ has $N+M+1$ elements, indexed from $z(i)$ to $z(i-(N+M))$. The vector $\hat{\mathbf{h}}$ contains $a_0, a_1, .............a_N$ which are known as the feed forward coefficients, and $b_1, b_2, ................, b_M$ which are known as the feed back coefficients. This vector has $N+M+1$ elements, and these elements are indexed as $\hat{h}(j)$, where $j = 0,1,......., N+M$. To minimize the sum of the squared errors, the partial derivative of $\varepsilon(n)$ is evaluated as:

$$\frac{\partial\varepsilon(n)}{\partial\hat{h}(j)} = \frac{\partial}{\partial\hat{h}(j)}\left[\sum_{i=0}^{n} e^2(i)\right] = 2\sum_{i=0}^{n} e(i)\frac{\partial e(i)}{\partial\hat{h}(j)}$$
$$= -2\sum_{i=0}^{n} e(i)z(i-j), \textit{where} \quad j = 0,1..., N+M \tag{5}$$

To minimize this error function, the partial derivative is set to zero

$$\frac{\partial\varepsilon(n)}{\partial\hat{h}(j)} = -2\sum_{i=0}^{n} e(i)z(i-j) = 0 \quad , j = 0,1,..........., N+M \tag{6}$$

By combining (2) and (6), it follows

$$\sum_{i=0}^{n}\left[x(i) - \sum_{l=0}^{N+M}\hat{h}(l)z(i-l)\right]z(i-j) = 0 \quad , j = 0,1,........N+M \tag{7}$$

$$\sum_{l=0}^{N+M}\hat{h}(l)\sum_{i=0}^{n} z(i-l)z(i-j) = \sum_{i=0}^{n} x(i)z(i-j) \quad j = 0,1,................, N+M \tag{8}$$

Recognizing outer products, yields $\mathbf{r}_{zz}(n)\hat{\mathbf{h}} = \mathbf{r}_{xz}(n)$. Therefore, the optimum coefficients are

$$\hat{\mathbf{h}} = \mathbf{r}_{zz}^{-1}(n)\mathbf{r}_{xz}(n), \tag{9}$$

where $\mathbf{r}_{zz}(n) = \sum_{i=0}^{n}\mathbf{z}(i)\mathbf{z}^T(i)$, and $\mathbf{r}_{xz}(n) = \sum_{i=0}^{n} x(i)\mathbf{z}(i)$ (10)

Rather than solving equation (9) by computing the inverse of $\mathbf{r}_{zz}(n)$, the inverse will be recursively computed by making use of the matrix inversion lemma. The weight vector $\hat{\mathbf{h}}$ will become a function of discrete time, and will assume the notation $\hat{\mathbf{h}}(n)$. The following recursive equation is the first step towards determining a recursive formula that will allow the weight vector $\hat{\mathbf{h}}(n)$ to be updated at each $n$.

$$\mathbf{r}_{zz}(n) = \sum_{i=0}^{n}\mathbf{z}(i)\mathbf{z}^T(i) = \sum_{i=0}^{n-1}\mathbf{z}(i)\mathbf{z}^T(i) + \mathbf{z}(n)\mathbf{z}^T(n)$$
$$= \mathbf{r}_{zz}(n-1) + \mathbf{z}(n)\mathbf{z}^T(n) \tag{11}$$

Similarly, $\mathbf{r}_{xz}(n) = \mathbf{r}_{xz}(n-1) + x(n)\mathbf{z}(n)$ (12)

The inverse of $\mathbf{r}_{zz}(n)$ can be recursively computed using the matrix inversion lemma [7, 8]

$$(A + BCD)^{-1} = A^{-1} - A^{-1}B(C^{-1} + DA^{-1}B)^{-1}DA^{-1} \tag{13}$$

of allowing $A = \mathbf{r}_{zz}(n-1),\ B = \mathbf{z}(n),\ C = 1,\ and\ D = \mathbf{z}^T(n)$, equation (11) takes the form

$$\mathbf{r}_{zz}^{-1}(n)=\mathbf{r}_{zz}^{-1}(n-1)-\frac{\mathbf{r}_{zz}^{-1}(n-1)\mathbf{z}(n)\mathbf{z}^{T}(n)\mathbf{r}_{zz}^{-1}(n-1)}{1+\mathbf{z}^{T}(n)\mathbf{r}_{zz}^{-1}(n-1)\mathbf{z}(n)}. \tag{14}$$

At any instant of time, $\mathbf{r}_{zz}^{-1}(n)$ may also be determined by

$$\mathbf{r}_{zz}^{-1}(n)=\frac{\mathbf{r}_{zz}^{-1}(n-1)}{1+\mathbf{z}^{T}(n)\mathbf{r}_{zz}^{-1}(n-1)\mathbf{z}(n)} \tag{15}$$

To recursively update $\hat{\mathbf{h}}(n)$, the set of equations (9), (12), (13) may be used. Hence

$$\begin{aligned}\hat{\mathbf{h}}(n)&=\mathbf{r}_{zz}^{-1}(n)\left[\mathbf{r}_{xz}(n-1)+x(n)\mathbf{z}(n)\right]\\&=\mathbf{r}_{zz}^{-1}(n)\mathbf{r}_{xz}(n-1)+x(n)\mathbf{r}_{zz}^{-1}(n)\mathbf{z}(n)\end{aligned} \tag{16}$$

Therefore it follows that

$$\begin{aligned}\hat{\mathbf{h}}(n)=&\left[\mathbf{r}_{zz}^{-1}(n-1)-\frac{\mathbf{r}_{zz}^{-1}(n-1)\mathbf{z}(n)\mathbf{z}^{T}(n)\mathbf{r}_{zz}^{-1}(n-1)}{1+\mathbf{z}^{T}(n)\mathbf{r}_{zz}^{-1}(n-1)\mathbf{z}(n)}\right]\\&\cdot\mathbf{r}_{xz}(n-1)+x(n)\mathbf{r}_{zz}^{-1}(n)\mathbf{z}(n)\end{aligned} \tag{17}$$

An updating equation for IIR filter coefficients may be obtain as

$$\begin{aligned}\hat{\mathbf{h}}(n)=&\hat{\mathbf{h}}(n-1)-\left[\frac{\mathbf{r}_{zz}^{-1}(n-1)\mathbf{z}(n)}{1+\mathbf{z}^{T}(n)\mathbf{r}_{zz}^{-1}(n-1)\mathbf{z}(n)}\right]\\&\cdot\mathbf{z}^{T}(n)\hat{\mathbf{h}}(n-1)+x(n)\mathbf{r}_{zz}^{-1}(n)\mathbf{z}(n)\end{aligned} \tag{18}$$

By substituting $\mathbf{r}_{zz}^{-1}(n)$ from equation (15) in the third term on the right hand side of the equation (18), the following expression may be obtained

$$\begin{aligned}\hat{\mathbf{h}}(n)=&\hat{\mathbf{h}}(n-1)-\left[\frac{\mathbf{r}_{zz}^{-1}(n-1)\mathbf{z}(n)}{1+\mathbf{z}^{T}(n)\mathbf{r}_{zz}^{-1}(n-1)\mathbf{z}(n)}\right]\\&\cdot\mathbf{z}^{T}(n)\hat{\mathbf{h}}(n-1)+x(n)\left[\frac{\mathbf{r}_{zz}^{-1}(n-1)\mathbf{z}(n)}{1+\mathbf{z}^{T}(n)\mathbf{r}_{zz}^{-1}(n-1)\mathbf{z}(n)}\right]\end{aligned} \tag{19}$$

By recognizing the bracketed term in the above equation as a time varying gain term that modulates how much the error influences the magnitude of the update at each iteration, the following update equation is realized:

$$\hat{\mathbf{h}}(n)=\hat{\mathbf{h}}(n-1)+\mathbf{G}(n)\left[x(n)-\mathbf{z}^{T}(n)\hat{\mathbf{h}}(n-1)\right], \tag{20}$$

where $\mathbf{G}(n)=\dfrac{\mathbf{r}_{zz}^{-1}(n-1)\mathbf{z}(n)}{1+\mathbf{z}^{T}(n)\mathbf{r}_{zz}^{-1}(n-1)\mathbf{z}(n)}$. (21)

Since the IIR RLS filter is used for non stationary signals, the error term $e(n)$ that influences the weights may be modified so that only relatively recent values of $e(n)$ will be significant. Therefore, (1) is modified to reflect this change [7], as is typically done for an FIR RLS filter [8]

$$\varepsilon(n)=\sum_{i=0}^{n}\lambda^{n-i}e^{2}(i), \tag{22}$$

this consequently influences the time varying filter gain defined by (21), which becomes

$$\mathbf{G}(n)=\frac{\mathbf{r}_{zz}^{-1}(n-1)\mathbf{z}(n)}{\lambda+\mathbf{z}^{T}(n)\mathbf{r}_{zz}^{-1}(n-1)\mathbf{z}(n)}. \tag{23}$$

## III. SIMULATION RESULTS

To investigate the performance of the proposed IIR RLS filter consider the following two cases:

**Case 1**: The transfer function of the unknown linear system to be identified by proposed algorithm is given by [9]

$$H(z)=\frac{1+0.4z^{-2}-0.5z^{-3}+0.7z^{-4}}{1-1.1z^{-1}+0.8z^{-2}-0.88z^{-3}+0.64z^{-4}} \tag{24}$$

Thus N=5, M=5, and $L_{AEC}=N+M+1=9$. The system is excited by the colored input signal $y(t)$, which is generated by passing white noise through the second order IIR filter. The filter coefficients of second order IIR filter are {-1.126, 0.64}. The system output is corrupted by additive white Gaussian noise $n(t)$ with zero mean and unit variance, the $n(t)$ and $y(t)$ are mutually independent. In the simulation, the forgetting factor is chosen to be 0.98. In order to examine the tracking behavior of the proposed algorithm in a nonstationary environment, suppose that the second pole and first zero of the transfer function given by (24) are suddenly changed from value (0.8) to (0.4) and from (0) to (1.0) respectively, which are indicates the effect of time-varying unknown system. The evolution curves of the estimated parameters to identify the new transfer function of the time-varying system are plotted in Fig. 2. It can be seen that the proposed algorithm has good ability of tracking in nonstationary environment. Moreover, the stability of the IIR RLS algorithm has been tested by checking the pole-zero locations at each iteration of the coefficient vector update equation (20). Fig. 3 (a) shows the pole-zero locations of the transfer function given by (24), while Fig. 3 (b) shows the pole-zero locations of the new transfer function for time-varying unknown system.

**Case 2**: To verify these results in a physical environment, the far end speech signal and its real echo in car environment were used [10] at a sample rate 8kHz with 16 bit resolution. The far end speech signal and its real echo (assuming no near end speech) were fed into proposed IIR RLS filter and usual FIR RLS filter. For the given real test signals, the performance of the FIR RLS filter is shown in Fig. 4. The main disadvantage of this algorithm is high computational complexity as given in Table 1, which is on the order of $L_{AEC}^{2}$ Multiplications per sampling interval T (MUL's per T), where $L_{AEC}$ is length of the RLS filter. The performance of the designed IIR RLS filter, according to the algorithms developed in Section II, is shown in Fig. 5. To compare the effectiveness of the proposed IIR RLS and FIR RLS filters, the mean square error (MSE) of both filters for different filter length are depicted in Figs. 6, 7 and 8. Note that the MSE of proposed IIR RLS filter (see Fig. 8) with filter length equal to

115 taps (100 feedforward taps and 15 feedback taps) is approximately equal to the MSE of the usual FIR RLS filter (see Fig. 7) with filter length equal to 256 taps. In this case the computational complexity of the proposed IIR RLS filter is lesser when compared to usual FIR RLS filter as given in Table 1. In another case, if both adaptive filters have same filter length (see Figs. 6 and 8) and consequently same computational complexity. it is observed that the proposed IIR RLS filter is superior for echo suppression. This is also evident from Table 1.

| | AEC System | Echo Suppression [dB] | Computational Complexity MUL's per T |
|---|---|---|---|
| 1 | FIR RLS ( $L_{AEC} = 115$) | -15.536 | 13225 |
| 2 | FIR RLS ( $L_{AEC} = 256$) | -23.486 | 65536 |
| 3 | FIR RLS ( $L_{AEC} = 512$) | -24.899 | 262144 |
| 4 | IIRRLS ( $N$ =100, $M$ =15) | -23.55 | 13225 |

Table 1: Echo Suppression and Complexities Of Different Algorithms.

Table 2 illustrates the effect of the number of feedback coefficients M on the echo suppression performance for the proposed IIR RLS algorithm. It can be seen, the adding feedback to the IIR RLS filter will allow an improvement in echo suppression while keeping the number of overall filter coefficients constant.

| | Filter Length | Echo Suppression [dB] |
|---|---|---|
| Proposed IIR RLS Filter | $N$ =114 taps, $M$ =1 taps | -18.57 |
| | $N$ =110 taps, $M$ =5 taps | -20.70 |
| | $N$ =105 taps, $M$ =10 taps | -22.19 |
| | $N$ =100 taps, $M$ =15 taps | -23.53 |
| | $N$ =58 taps, $M$ =57 taps | -21.31 |

Table 2: Echo Suppression for Different Number of Feedback Coefficients.

## IV. CONCLUSIONS

In this letter, an adaptive IIR RLS filter based on output error approach for AEC has been investigated. The RLS adaptation algorithm was used to derive the feed forward and feedback coefficients on a sample by sample basis. From simulation results, it seems that the proposed IIR RLS algorithm has good tracking performance for time-varying unknown system and remain stable at each iteration of the coefficient update equation. For real data test signals in car environment, it is observed that the IIR models of acoustic echo paths, is outperform over their FIR counterparts. Furthermore, numerical comparisons show that the proposed IIR RLS requires fewer number of filter coefficients and by consequence lesser computational complexity to obtain the specified echo suppression performance. In addition the other main advantage of the proposed algorithm is that it processes data on sample by sample basis, which lends itself to a more efficient real time implementation.

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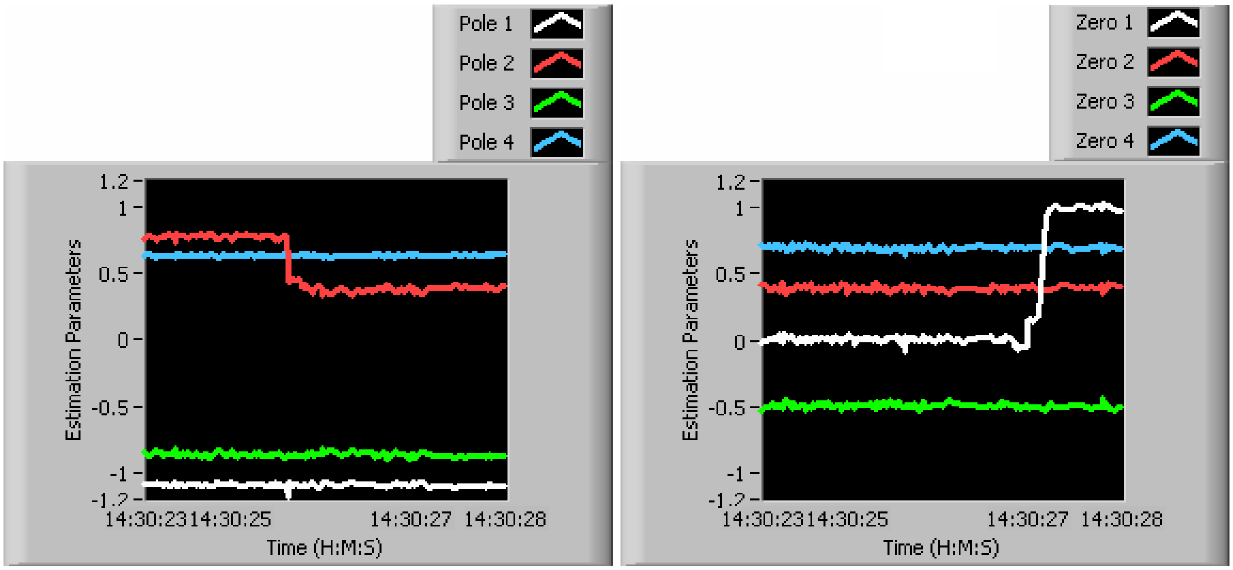
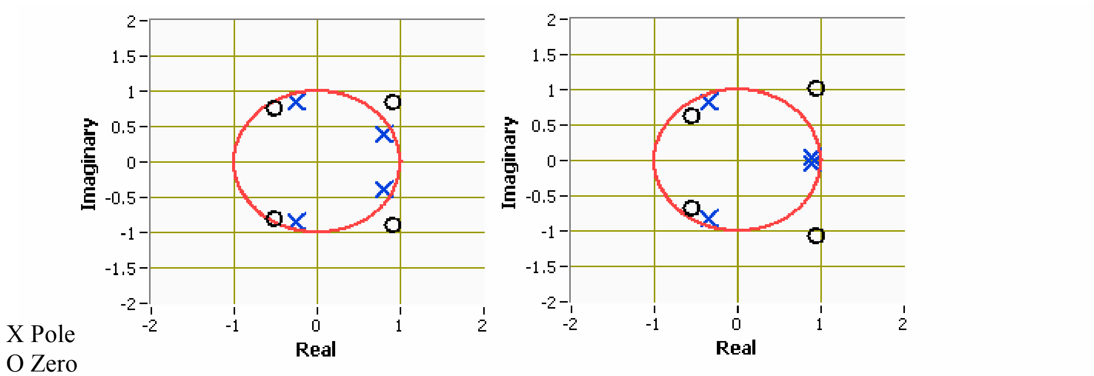



**(a)** **(b)**

Fig. 2 Evolution Curves Of The Estimated Parameters To Identify The Linear Time-Varying System, (a) Poles. (b) Zeros.




(a) (b)

Fig. 3 Pole-Zero Plot, (a) For Linear Time-Invariant System. (b) For Linear Time-Variant System.

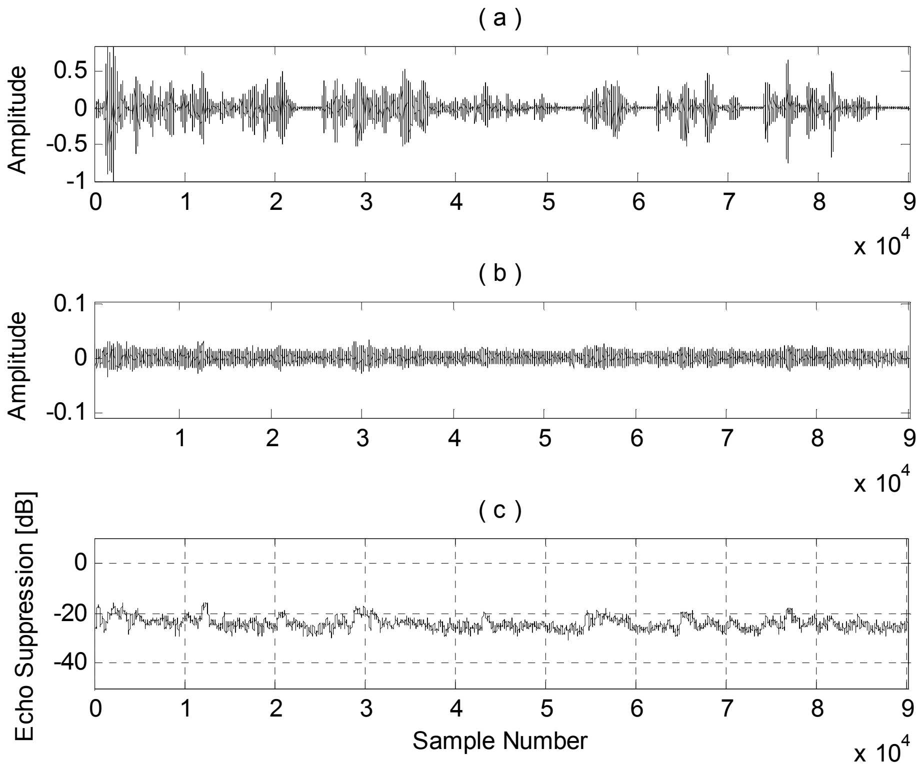



Fig. 4 FIR RLS Filter, (a) Echo From far-End Speech. (b) Residual Echo. (c) Echo Suppression. ( $L_{AEC} = 256$ taps, forgetting factor $\lambda = 0.98$ )

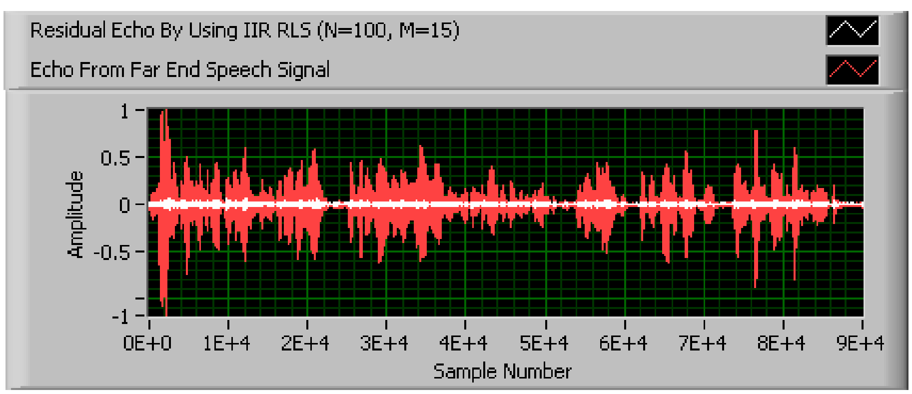



Fig. 5 Acoustic Echo Cancellation By Using Proposed IIR RLS Filter With Compare To Real Far-End Echo In Car Environment.

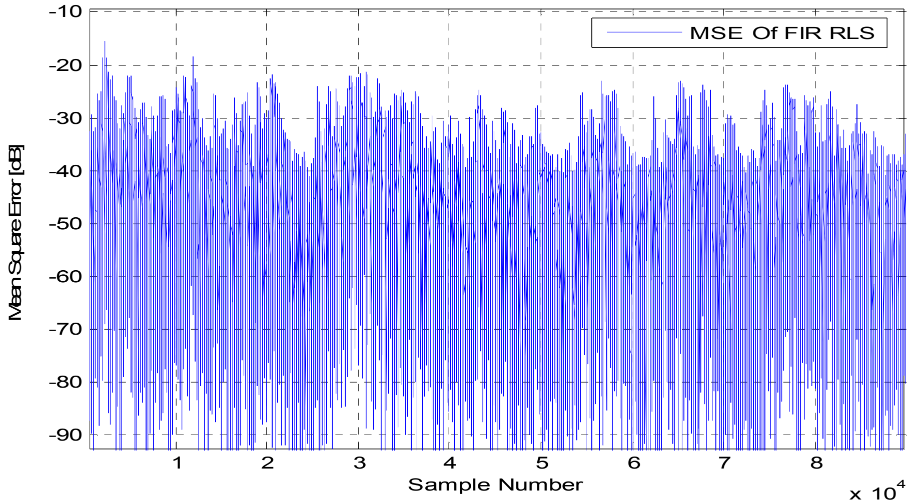



Fig. 6 Mean Square Error (MSE) Of FIR RLS Algorithm, Filter Length $L_{AEC}$ =115 taps.

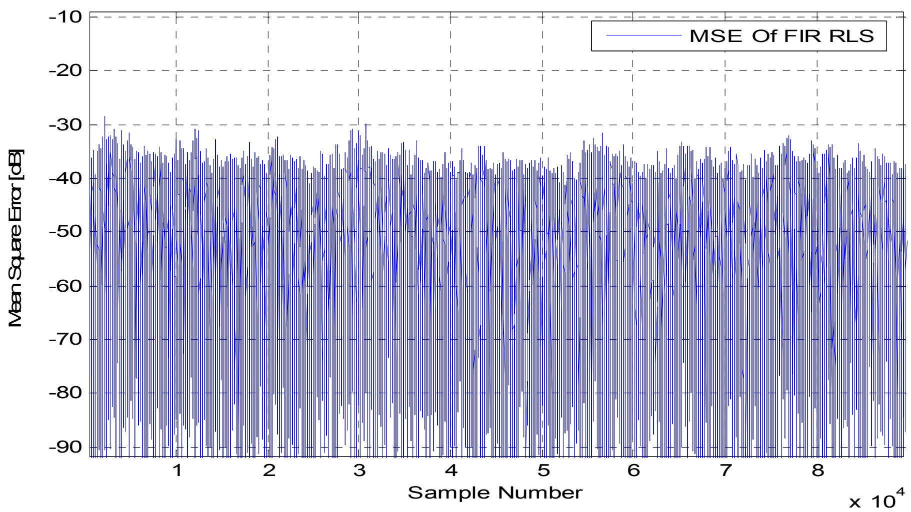



Fig. 7 Mean Square Error (MSE) Of FIR RLS Algorithm, Filter Length $L_{AEC}$ =256 taps.

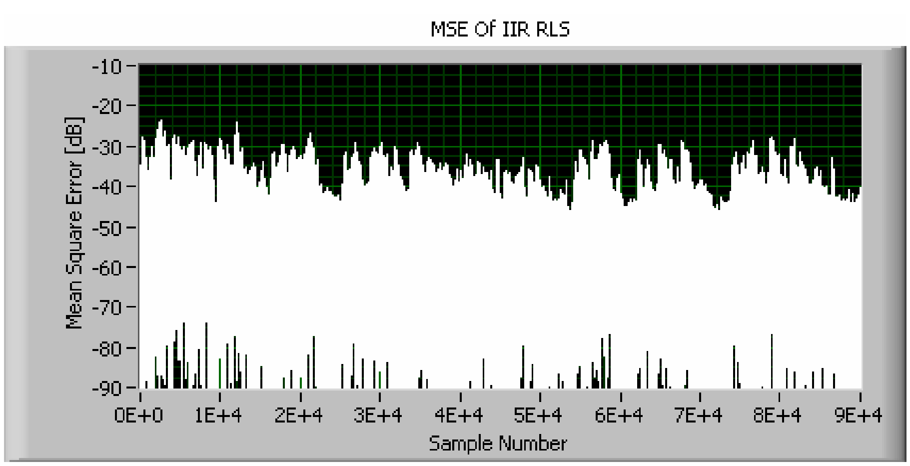



Fig. 8 Mean Square Error (MSE) Of Proposed IIR RLS Algorithm,
(Feed Forward Coefficients $N$ =100, Feed Back Coefficients $M$ =15).