

Automatic FIR Filter Design Method and Tool based on Genetic Algorithms

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Abstract—A FIR Filter design method and its respective tool using genetic algorithms were developed. The main feature of this method is to offer a transparent mode for the user who doesn't know evolutionary computation, as well as its parameters. The user inputs the filter specifications and gets a sub-optimal result in an average number of four attempts. The sub-optimal criterion was based on the Rabiner, Parks and McClellan algorithm and the implemented software was built using the GALOPPS tool.

Index Terms—FIR filter design, genetic algorithms, Galopps, Rabiner Parks and McClellan algorithm, Remez.

I INTRODUCTION

The design of FIR (*Finite Impulse Response*) digital filters using techniques of DSP is an automatic procedure. There are computer programs like MATLAB and DSPLAY that offer this facility. Some of these techniques use methods like window, remez and frequency sampling. Details about these methods can be found in [1]. The only thing the user needs to concern is the FIR filter specification which can involve some few additional parameters related to the method chosen.

FIR filter design using GAs (*genetic algorithms*) has been studied at least for 20 years [2]. But choosing this way to get a digital filter the user usually has to know a considerable number of additional parameters besides the FIR filter specifications. They are the GA parameters. Some of them need to be adjusted at each new filter specification. Another feature of a method based on GAs is the stochastic behavior of this kind of application: the user previously needs to know that it can be necessary to run more than one execution to get an interesting solution. And this solution can be different at each time the application is run. These two features of a FIR filter design tool based on GAs tend to restrict it for people that have some knowledge of evolutionary computation. One illustration of a tool like this is presented in Fig. 1. The main contributions of this work are two: a) the user doesn't need to specify any GA parameter. All of them are already fixed in optimal values or are self-adjustable; b) and the quality of the

results on an average number of four attempts are sub-optimal. Some of these results can be used in hard FIR filter specifications in a better condition than the specialist methods. A sub-optimal pattern was created and the reference for this was the Parks-McClellan method implemented in MATLAB through the `remez` command [3].

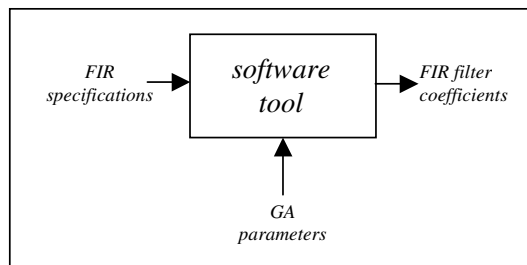


Figure 1: One model of a GA FIR filter design tool

II FIR FILTER DESIGN

Details about the topics covered by this section can be found in [4]. A FIR digital filter frequency response can be calculated from:

$$H(k) = \sum_{n=0}^{M-1} h(n) \times e^{(-j \frac{\pi}{N}) \times kn} \quad (1)$$

In eq. (1): $H(k)$ is the Discrete Fourier Transform complex vector. T is the FIR digital filter frequency response; N is the number of collected points during the sampling process; k is an index varying from zero to $N-1$; $h(n)$ is the FIR filter response vector to the unit impulse. This vector corresponds to the FIR filter coefficients; and M is the number of the FIR filter coefficients.

A digital filter gives a realizable version of a desired frequency response that was specified as part of the filter specifications. This happens because an ideal digital filter response is unrealizable.

To express $H(k)$ as a function of the normalized frequency it can be used [5]:

$$f = \frac{k}{N-1} \quad ()$$

In eq () f is the normalized frequency ranging from 0 to 0.5 cycles/sample

Depending on the number of coefficients and the symmetry of $h(n)$ the FIR filters can be classified in four categories (types I to IV) three of them requiring some restrictions to give a specific frequency response This work covers the four categories

The complex vector $H(k)$ is more useful when viewed as separated in magnitude and phase frequency responses There is a symmetry in the FIR filters coefficients that guarantees a linear phase frequency response So a FIR filter specification is often expressed only by its magnitude frequency response One set of FIR filter parameters often used in a specification is presented in Fig

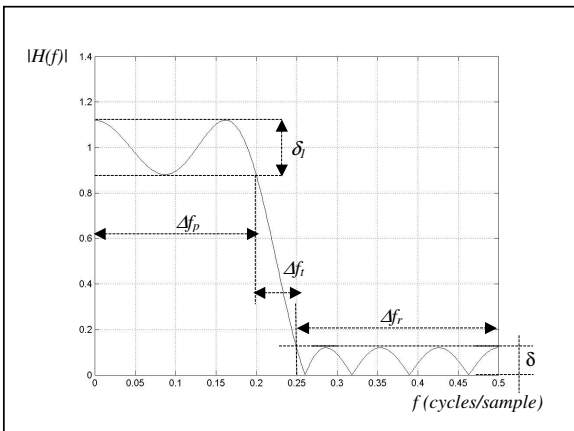


Figure 2: One set of FIR filter parameters

The description of these parameters are: a) δ_1, δ_2 : Maximum ripple allowed for bands pass and stop respectively; b) $\Delta f_p, \Delta f_s$: bandwidth of bands pass and stop respectively; c) Δf_t : transition bandwidth between bands

There are also other specification parameters needed which were not presented in Fig They are the amplitude levels of bands pass A_1 and A_2 respectively and the number of FIR filter coefficients M In this work a third band was used to provide passband and stopband filters Returning to Fig these parameters are the FIR specifications

FIR filters are Linear and Time Invariant systems So the filtering process can be made through the convolution of the FIR filter coefficients $h(n)$ and the signal $x(n)$ to be filtered:

$$y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k) \quad ()$$

In eq (): $y(n)$ is the filtered signal; $h(n)$ is the FIR filter unit response; $x(n)$ is the signal to be filtered; $*$ is the

convolution operator symbol; U is a number which is the sum of the number of samples found in $h(n)$ and in $x(n)$; and n varies from zero to $U-1$

It is possible to get from eq () the following conclusion: the larger number of $h(n)$ coefficients (M) the more precise will be the filtering process Larger values of M offer a better quality of filtering Better here can be understood as a FIR filter frequency response with a minimum transition width and a minimum value of ripple But also through eq () it is possible to conclude that for a same signal $x(n)$ to be filtered the number of products between $x(n)$ and $h(n)$ is regulated by the number of FIR filter coefficients M But larger values of M cause larger delays during the filtering process This can be unacceptable in some real time applications

One of the main advantages of the Parks McClellan method is the possibility of previously calculate the value of M that will satisfy a FIR filter specification which will here be called the recommended M or M_{rec} This is done through empirical formulas Smaller values for δ_1, δ_2 or Δf_t will require a larger value for M_{rec}

This work also tries to enforce the following area of study: the search for an acceptable FIR filter response in terms of quality with a minimum value of FIR filter coefficients

III GENETIC ALGORITHMS AND THE PROPOSED MODELING

Details about the topics covered in this section can be found in [4]

A GA is based on a sequence of actions that among others can be represented by Fig A brief description of these actions with the model adopted by this work is:

a) a possible numerical solution of the problem is codified as an individual Such representation usually adopts symbols to codify the numerical solution The vector of the individual corresponds to a chromosome In this work an individual is the $h(n)$ vector represented by the binary alphabet either in Gray or binary positional As a chromosome $h(n)$ is yet represented by a bit string of integers The integer to decimal decodification is made by portions of bits Each portion gives a precision number ranging from -1 to 1 which is one FIR filter coefficient;

b) a set of individuals is generated at random This set is called population and corresponds to the step in Fig It is called the search space S that corresponds to all possible solutions that can be formed with the chosen alphabet In this work the search space is variable with the number of FIR filter coefficients according to:

$$S = 2^l \quad (4)$$

In eq (4): S is the search space; and l is the size in bits of the chromosome l depends of M and only one half of the coefficients must be codified in the chromosome because of the FIR filter symmetry propriety For example for $M=14$ only the first seven coefficients must be codified In this way $l = 7 \times 2 = 14$ So $S = 2^{14} = 16384$;

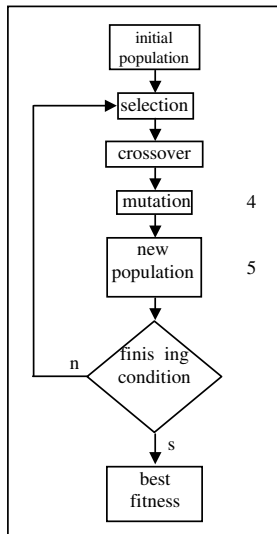


Figure 3: A GA basic flow

c) this population is submitted to an evaluation. Each individual is tested according to how good it is as an optimal or sub optimal numerical solution for the proposed problem. In this way each individual receives a score. This evaluation is called fitness. In this work the fitness function chosen is:

$$fitness = \sum_{i=1}^{N-1} \frac{1}{\left(\left| H_d(f_i) \right| - \left| H_i(f_i) \right| \right)} \quad (5)$$

In eq (5): $fitness$ is the fitness function; $|H_d(f)|$ is the desired (given by user specification) magnitude frequency response; and $|H_i(f)|$ is the magnitude frequency response from each individual. $|H_i(f)|$ is calculated through Eq () because an individual corresponds to one vector $h(n)$. This fitness function numerically indicates how close an answer from an individual is from the desired answer. This happens in step one of Fig in the first time;

d) it is applied in this evaluated population some mechanism of selection. This is an attempt to exclude from the next steps individuals with low values of fitness. In this work it was used the following selection processes: Roulette Wheel Stochastic Tournament and the Stochastic Universal Sampling. This happens in step two of Fig ;

e) the next step with the selected population is a recombination mechanism called crossover. A crossover consists basically of changes in portions of the chromosome between two individuals. The crossover is applied in this selected population with a probability $pcross$ between to

In this work the following crossover techniques were used: one point and two point crossover. This happens in step three of Fig ;

f) after this another operation in the chromosomes is applied. It is the mutation that consists basically of a change in one or more bits in an individual at random. This operation is applied with a probability $pmutation$ also ranging from to . In this work the following mutation operations were

used: single bit mutation and multiple field mutation. This is step four in Fig ;

g) the new population is evaluated in the same way as made in the step described in item c. This happens in step five of Fig ;

h) it is called *generation*. The steps covered by items d to g inclusive. Every time a generation is concluded a finishing condition is tested to end the GA execution. In this work the finishing condition is a maximum generation number. This is step six in Fig ;

i) if the finishing condition is not true there is a return to the step that corresponds to item d and a new generation cycle is executed. This happens in the decision structure in Fig ;

j) if the finishing condition is true the last population is obtained and the individual with the highest value of fitness in this population is the best solution the GA can give. In this case it will be the best $h(n)$ founded.

Returning to Fig these parameters are the ones automatically set by the tool called GA parameters.

IV FIR FILTER DESIGN WITH GENETIC ALGORITHMS

Counting from [] until these days more than one hundred FIR Filter Design methods using GAs are already published e.g. [5], [] and []. In these works the prior goal was not to fetch for a method or software based on GAs for give FIR filter coefficients without the need to adjust evolutionary parameters at each new specification and with a predetermined number of mean trials to get an acceptable answer.

Two related works were found. The first is a MATLAB toolbox []. Some differences between that tool and this work are: a) in that tool the GA parameters are configurable that is the user must know GAs. In this work this kind of knowledge is not necessary; b) The platform: to run that application it is necessary to have MATLAB. In this work the final version of the tool runs over Windows directly. The second work also a MATLAB toolbox is a tool called CSDFIR []. The final version of this tool is automatic but today it owned by a private company.

V METHODOLOGY

To achieve the proposed goals the following strategy was taken: a) two sub optimal conditions related to a well known specialist method were specified; b) and a bank of FIR filter frequency response specifications was specified. These filter specifications tried to cover all the possible kinds of hard and easy to solve FIR filters. All of them were specified with or amplitudes. Arbitrary levels were not proposed because the size of the filter bank would grow considerably.

With these conditions and the filter bank a test case group of tests was proposed: a) Phase : the filter bank was tested with a fixed number of coefficients from types I to IV. Several possibilities of combinations with GA parameters were tested in this phase considering the chromosome binary representation, selection process, crossover operator, mutation operator, probabilities of mutation and crossover and an auxiliary technique of selection called elitism [4]. A score

based on the quality of the results for each configuration was proposed. If the GA with the specific configuration passed (presented at least one acceptable result in an average number of four trials) to all the filters of the bank, it received a score based on the quality of the results. If the configuration didn't pass through all the proposed filters, its score was zero; b) *Pase*: with the best score approved configurations of *Pase* the same filter bank was tested, but with a variable number of coefficients, to check if the GA with the same configuration parameters is robust to support different search spaces. The GA parameters to be changed in this phase were the population size and the maximum number of generations; c) and *Pase*: with the most robust version of GA approved with a specific configuration of *Pase*, several FIR filter specifications different from the ones present in the filter bank were tested, e.g. with variable amplitudes and variable number of FIR filter coefficients. Also in this phase, the LTI superposition propriety was tested for hard to solve variable amplitude filters or filters that could not be solved when specified directly.

During phase two and three, it was looked for some pattern behavior in some GA parameters with the variation of the FIR filter coefficients. This was done as an attempt to find some mathematical relations between the number of coefficients and time.

The limitations of the proposed method and GA modeling were: a) do not cover more than three amplitude levels between 0 and 5 cycles/sample; b) do not cover any frequency range smaller than 0.5 cycles/sample; c) do not cover all the possibilities of arbitrary level response frequencies; d) and depending of the number of coefficients (which determine the search space), the execution cannot be processed in usual machines because of the processing time.

With this strategy, the expectation was to find a configuration that covered the requisites of any FIR filter specification through tests.

A. First and Second Sub-optimal Conditions

To make the quality response comparisons, it was chosen the MATLAB implemented version of the Parks McClellan method with equal weights for pass and reject bands. A FIR filter frequency response given by the GA that is considered accepted must satisfy two conditions.

The conditions are called the First Sub-optimal Condition (FSC) is:

$$\left| \sum_{i=1}^M \times \log(A_i + \delta_{iAG}) - \sum_{i=1}^M \times \log(A_i + \delta_{iR}) \right| \leq 5 \quad (1)$$

In eq (1): δ_{iAG} are the GA ripples of bands; δ_{iR} are the Parks McClellan ripples of bands; and A_i are the amplitude specifications of bands.

And the Second Sub-optimal Condition (SSC) is:

$$\left| f_{iAG} - f_{iR} \right| \leq 5 \quad (2)$$

In eq (1): f_{iAG} are the GA transition frequency edges between the bands; f_{iR} are the Parks McClellan transition frequencies edges between the bands.

B. The FIR Filter Bank

Some FIR filter specifications of the filter bank are presented in Table I.

	f_{12}	f_{21}	f_{22}	f_{31}	A_1	A_2	A_3
1	5	4	45	45			
5	5	9					
7	4	4	5				
8	5	5	4	4			
10	4	4	44	4			
12	5		9	45			
16	5		4	45			
18	5		44	45			

For all filter specifications: $f_{11} = 0$ and $f_{32} = 5$ cycles/sample.

The differences between this work and a previous one described in [8] are: a) in this work, the probabilities of crossover and mutation were considered fixed. In this one, these values are considered in the first phase; b) In this work, the number of attempts were fixed in four. If a configuration failed, it received a zero degree. In this work, it was used an average number of four attempts. The zero degree is set only if a configuration exceeds eight attempts for one FIR filter specification of the filter bank. With this change, the method became automatic.

Details about the software can be found in [9].

VI RESULTS

Table II presents some GA parameters configurations scores in *Pase*. It was used a population size of 100 individuals, a number of FIR filter coefficients M of 5, and the computer used was a Pentium notebook, 1.5 GHz with 512 Mb of RAM. It can be seen in configuration 1 of Table II that the canonic version of the model could not satisfy the proposed goals, requiring more advanced GA configurations.

The conversions for Table II are: type = filter type, selection = selection process, maxgen = max generation number, crossover = crossover operator, mutation = mutation operator, bin rep = binary representation, t_{AG} = time spent, suselect = Stochastic Universal Sampling process, rselect = Roulette Wheel process, tselect = tournament process, twoptx = point crossover, oneptx = one point crossover, bitmutat = single bit mutation operator, multmut = multiple field mutation operator, Gray = Binary Gray representation, Pos = Binary positional representation, pcross, pmut = probabilities of crossover, mutation.

It was observed that the elitism was always present with the configuration that passed through the filter bank. The binary Gray codification offered more resolution than the conventional binary positional codification. This was

expected because in Gray codification only one bit can change from one number and its next. The Stochastic Tournament selection process did not pass in Phase 1 but it presented very interesting results (more successful results than the Roulette Wheel selection process). The Stochastic Universal Sampling selection method presented the following useful behavior: at each attempt one different sub-optimal result was obtained. Comparing it to the Stochastic Tournament, this one presented always the same sub-optimal result for different attempts.

TABLE II
SOME GA CONFIGURATION SCORES FOR PHASE

<i>config #.</i>	1	2	3	4
<i>type</i>	I	I	I	I
<i>selection</i>	suselect	rselect	tselect 4	tselect 4
<i>maxgen</i>	5	5	5	5
<i>cross</i>	twoptx	oneptx	twoptx	oneptx
<i>pcross</i>	85	9	85	9
<i>mutation</i>	bitmutat	multimut	multimut	bitmutat
<i>pmut</i>				
<i>bin rep</i>	Gray	Pos	Pos	Gray
<i>elitism</i>	Yes	Yes	No	Yes
<i>t_{AG}</i>	s	s	s	s
<i>score</i>	5			
<i>config. #</i>	5	6	7	8
<i>type</i>	II	II	III	IV
<i>selection</i>	suselect	suselect	suselect	suselect
<i>maxgen</i>	5		5	5
<i>cross</i>	twoptx	twoptx	twoptx	twoptx
<i>pcross</i>	85	85	85	85
<i>mutation</i>	bitmutat	bitmutat	bitmutat	bitmutat
<i>pmut</i>				
<i>bin rep</i>	Gray	Gray	Gray	Gray
<i>elitism</i>	Yes	Yes	Yes	Yes
<i>t_{AG}</i>	s	s	s	s
<i>score</i>		5	8	5.4

A GA FIR filter magnitude frequency response in dB that does not satisfy Eq (1) is presented in Fig 4. An example of a FIR filter magnitude frequency response in dB that does not satisfy Eq (2) is presented in Fig 5. Figure 6 presents one result that satisfies both FSC and SSC.

Fig 6 is a result that demonstrates the applicability of the method for a hard FIR filter specification M in this figure is 5. The calculated value for M_{rec} that must be used in the Parks McClellan method to satisfy this FIR filter specification is

Observing the scores presented in Table II, one GA parameters configuration maintained its stability. It was the one presented in configurations 1 and 8, except for the maximum number of generations. This GA parameters configuration was the one used in Phase 1.

On Phase 2, only population size and maximum number of generations were changed. The GA parameters configuration maintained its stability of results, i.e. compliant with Eq (1) and Eq (2) for several values of M (several search spaces).

One example: population size = 50 and maximum number of generations = 50 for a number of coefficients (M) = 10.

In Phase 1 for hard to solve and arbitrary levels specifications, the LTI superposition principle was also valid. Among other tests, this was verified in Phase 1. The GA FIR filter frequency response presented in Fig 8 was obtained directly as well as through the sum of a low pass and a high pass FIR filter specification.

The final fixed GA parameters configuration obtained was: *binary representation* = Gray, *fitness function* = Eq (5), *selection* = Stochastic Universal Sampling, *crossover* = two point crossover, *mutation* = single bit mutation, *elitism* = yes, *probability of crossover* = 85 and *probability of mutation* = 9.

For all the filter specifications of the filter bank proposed and for more than fifty others, this GA did run on in an average number of four times to satisfy the conditions specified in Eq (1) and Eq (2). Returning to Fig 6, these were the GA parameters fixed in the tool.

It was possible to establish together with prior tests [9] the following relations:

$$popsize = 8 \times L^{-8} \quad (8)$$

$$maxgen = 5 + (L-8) \times 5 \quad L \geq 8 \quad (9)$$

In equations (8) and (9): *popsize* is the population size; $L = M/2$ for M even and $(M+1)/2$ for M odd; *maxgen* is the maximum number of generations. Returning to Fig 6, these equations were used in the GA parameters to adapt the GA tool to receive a variable FIR filter specification with a variable number of coefficients M .

VII CONCLUSIONS

A GA method and its respective tool with predefined GA parameters was obtained to solve variable coefficients, variable filter type, and fixed in zero or one amplitude FIR digital filter specifications. Future research will be the development of a tool with a more user-friendly interface and the automatic FIR filter design with coefficients expressed in powers of two samples.

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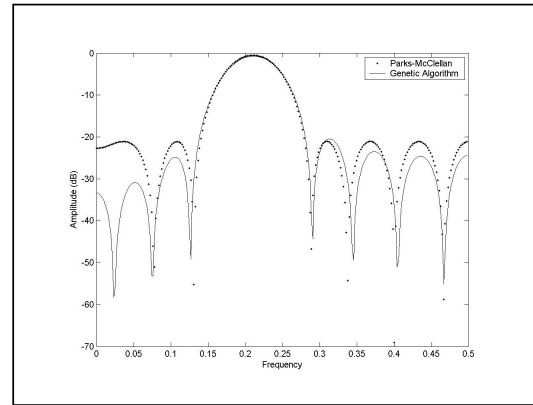


Figure 6: One result that satisfies the FSC and the SSC

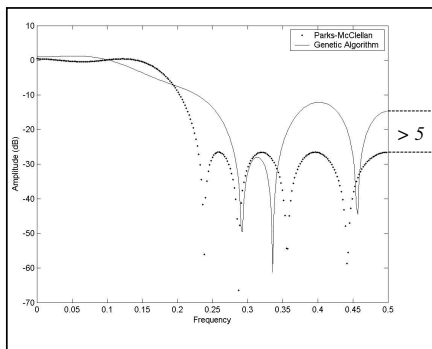


Figure 4: GA filter response that does not satisfy the FSC

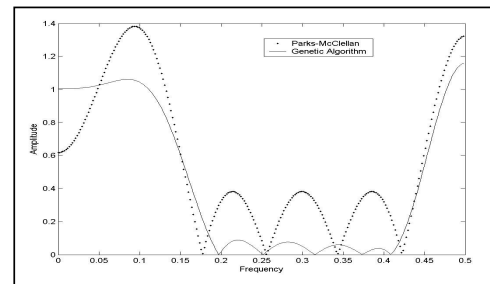


Figure 7: One hard FIR filter specification ($M < M_{rec}$)

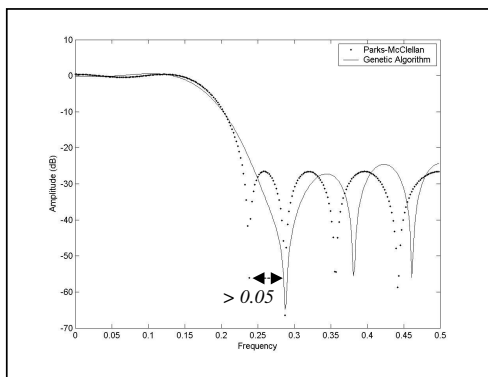


Figure 5: GA filter response that does not satisfy the SSC

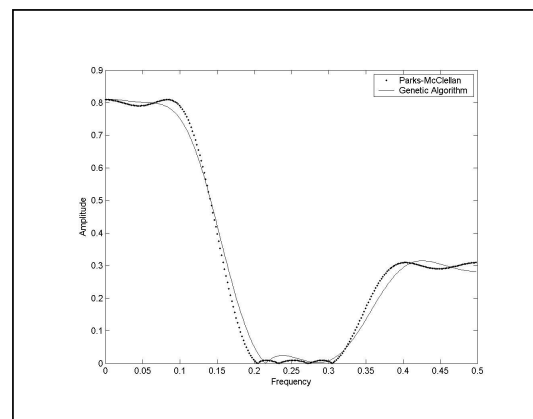


Figure 8: Arbitrary level filter response obtained in Pass