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Investigation of Fault-Tolerant Adaptive Filtering for Noisy ECG Signals

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Abstract- **Studies shows that Electrocardiogram (ECG) computer programs perform at least equally well as human observers in ECG measurement and coding, and can replace the cardiologist in epidemiological studies and clinical trials [1]. However, in order to also replace the cardiologist in clinical settings, such as for out–patients, better systems are required in order to reduce ambient noise while maintaining signal sensitivity. Therefore the objective of this work was to develop an adaptive filter to remove the contaminating signal in order to better obtain and interpret the electrocardiogram (ECG) data. To achieve reliability, the real-time computing systems must be fault-tolerant. This paper proposed a fault-tolerant adaptive filter for noise cancellation of ECG signals. Comparison of the performance and reliability of non-fault-tolerant and fault-tolerant adaptive filters are performed. Experimental results showed that the fault-tolerant adaptive filter not only successfully extract the ECG signals, but also is very reliable.**

Keywords: ECG, adaptive filter, noise cancellation, fault tolerant.

I. INTRODUCTION

Electrocardiogram is the body-surface manifestation of the electrical potentials produced by the heart. The ECG is acquired by placing electrodes on the patient's skin. In a resting setting, the principal technical issue in interpreting ECG waveforms arise from the existence of ambient or background "noise" emanating from other electromagnetic sources, including (1) signals generated by the other organs, muscles and systems of the body, whether from movement or the performance by those organs of their bodily functions, and (2) signals generated by sources external to the body, such as electronic equipment, lights or engines. Cardiologists can identify irregularities in the heart's rate and rhythm, known as arrhythmia, by examining changes in the 0.67 to 40 Hz frequency range. Because of the relatively large amplitudes of these waveforms in this range, cardiologists can easily identify arrhythmia notwithstanding the existence of electromagnetic ambient noise from other sources. However, it is very difficult for cardiologists to distinguish physiological signals from ambient noise in the broader frequency ranges used to identify different types of heart disease, including cardiac ischemia, hypertrophy and the existence of past or presently occurring heart attacks. The reason for this difficulty is that the physiological signals associated with these other heart diseases are of a much lower amplitude or strength in the lower 0.05 to 0.67 Hz and upper 40 to 150 Hz portions of the frequency range, meaning that they do not stand-out from the ambient noise in these portions and therefore cannot be easily discriminated from that ambient noise. In order to minimize ambient noise in the clinical setting, ECGs are normally taken in the hospital or physician offices. Cardiologists instruct the patient to lie in the supine position, being as still as possible while a reading is taken to reduce ambient noise caused by physical movement. Another method to reduce ambient noise is to reduce the sensitivity of the monitoring equipment, although this alternative results in a loss of signal quality and the ability to read certain signal intricacies. Although diagnostic criteria have been improved by computerization, many of these techniques have not been widely applied, due to the described limitations [2]. Therefore, adaptive filtering [3] to remove artifact noise without distorting the actual signal is crucial to enable computer based clinical ECG.

Subsequently advances in filtering techniques will also improve ambulatory ECG recording routinely used to detect infrequent, and asymptomatic arrhythmias [4], [5] and to trace heart activities in fetals [6], [7]. Furthermore, it will enhance ECG editing [8] used to supplement advances in other technologies such as computer tomography used for cardiology [9]. Microprocessor-based even recorders have been commonly developed and used that carry out online signal processing, data reduction, and arrhythmia detection [10]. Computational power of the microprocessor makes them feasible to implement digital filters for noise cancellation and arrhythmia detection [11].

Adaptive filtering technique using neural networks has been shown to be useful in many biomedical applications [3]. The basic idea behind adaptive filtering has been summarized by Widrow *et al*. [12]. It reduces the mean-squared error between a primary input, which is the noisy ECG, and a reference input, which is either noise that is correlated in some way with the noise in the primary input or a signal that is correlated only with ECG in the primary input [13]. Adaptive filters permit to detect time-varying potentials and to track the dynamic variations of the signal. These types of filters learn the deterministic signal and remove the noise. Besides, they modify their behavior according to the input signal. Therefore, they can detect shape variations in the ensemble and thus can obtain a better signal estimation.

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Different filter structures are presented to eliminate the diverse form of noise: baseline wander, 60 Hz power line interference, muscle noise, and motion artifact [14], [15]. 60 Hz powerline interference cancellation is a simple but important application. In the paper, this kind of noise source is used to demonstrate the effectiveness of the adaptive filters we introduced.

The first aim of this paper is to construct an adaptive filter and demonstrate its application in noise cancellation. We combine a tapped delay lines to a tapped delay line with an ADALINE network to create an adaptive filter. The adaptive filter weights are updated by using the Least Mean Square algorithm. The constructed filter is proved and demonstrated with a single frequency noise source. The second aim is to introduce a fault-tolerant adaptive filter and demonstrate its improved reliability. A parallel construction is adopted for the fault-tolerant adaptive filter whose reliability is compared with that of the non-fault-tolerant adaptive filter.

II. ADAPTIVE NOISE CANCELLATION

When doctors are examining a patient on-line and want to review the electrocardiogram (ECG) of the patient in real-time, there is a good chance that the ECG signal has been contaminated by a 60-Hz noise source. To allow doctors to view the best signal that can be obtained, we need to develop an adaptive filter to remove the contaminating signal in order to better obtain and interpret the ECG data.

A. Adaptive Filter without Fault Tolerance

The adaptive filter without fault tolerance is designed to remove the contaminating signal, as shown in Fig. 1. The ECG signal, s is the original uncontaminated input signal to the network. The desired output is the contaminated ECG signal *t*. The adaptive filter will do its best to reproduce this contaminated signal, but it only knows about the original 60 Hz noise source, *v*. Thus, it can only reproduce the part of *t* that is linearly correlated with *v*, which is *m*. In effect, the adaptive filter will attempt to mimic the noise path filter, so that the output of the filter *a* will be close to the contaminating noise *m*. In this way the error e will be close to the original uncontaminated ECG signal *s*. We call *(s+m)* the primary input, and *a* the reference signal.

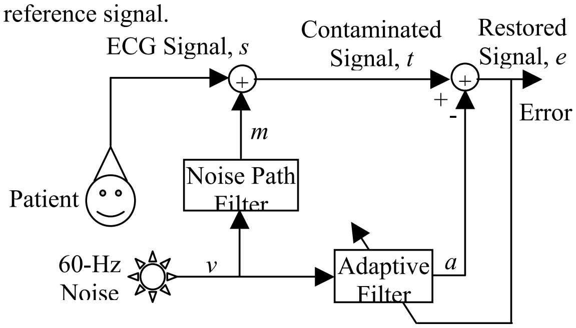


Fig. 1 Noise Cancellation System

Since the adaptive filter output is a and the error is $e = (s+m) - a$, then the mean square error (MSE) is

$$e^2 = ((s+m)-a)^2 = (s+m)^2 - 2\cdot(s+m)\cdot a + a^2$$

$$= (m-a)^2 + s^2 + 2\cdot s\cdot m - 2\cdot s\cdot a \qquad (1)$$

Since signal and noise are uncorrelated, the MSE is

$$E[e^2] = E[(m-a)^2] + E[s^2] \qquad (2)$$

Minimizing the MSE results in a filter error that is the best least squares estimate of the signal s. The adaptive filter extracts the signal, or eliminates noise, by iteratively minimizing the MSE between the primary and the reference inputs.

B. The Least Mean Square (LMS) Algorithm

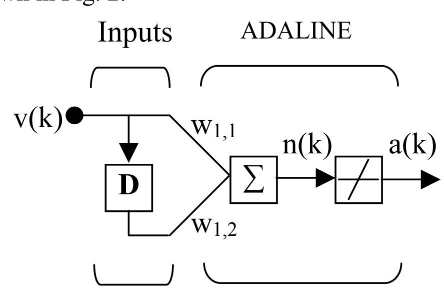
The LMS algorithm is an iterative technique for minimizing the mean square error (MSE) between the primary input and the reference signal [8]. The adaptive filter weights are updated by using the LMS algorithm. The LMS algorithm can be written in matrix notation:

$$W(k+1) = W(k) + 2\alpha\cdot e(k)\cdot p^T(k) \qquad (3)$$

and $b(k+1) = b(k) + 2\alpha\cdot e(k)$ (4)

where $W(k) = [{}_1w(k)\ \ {}_2w(k)\cdots{}_iw(k)]^T$ is a set of filter weights at time k, and ${}_iw(k)$ is the ith row of the weight matrix. $p^T(k) = [{}_1p(k)\ \ {}_2p(k)\cdots{}_ip(k)]^T$ is the input vector at time k of the samples from the reference signal. The error is $e(k) = t(k) - a(k) = (s(k)+m(k)) - a(k)$, where t is the desired primary input from the ECG to be filtered, and $a(k)$ is the filter output that is the best least-squared estimate of $t(k)$.

For simplicity, we use a single sine wave noise source. In this case a neuron with two weights and no bias is sufficient to implement the adaptive filer. The inputs to the filter are the current and previous values of the noise source. Such a two-input filter can attenuated and phase-shift the noise *v* in the desired way. The adaptive filter is shown in Fig. 2.



$$a(k) = w_{1,1}v(k) + w_{1,2}v(k-1)$$

Fig. 2. Adaptive Filter for Sine Wave Noise Source

C. Proof of Concept

We will first need to find the input correlation matrix R and the input/target cross-correlation vector h:

$$R = E[zz^T] \text{ and } h = E[tz]. \quad (5)$$

In our case the input vector is given by the current and previous values of the noise source:

$$z(k) = \begin{bmatrix} v(k) \\ v(k-1) \end{bmatrix}, \quad (6)$$

while the target is the sum of the current signal and filtered noise:

$$t(k) = s(k) + m(k) . \quad (7)$$

Now expand the expressions for R and h to give

$$R = \begin{bmatrix} E[v^2(k) & E[v(k)v(k-1) \\ E[v(k)v(k-1) & E[v^2(k-1) \end{bmatrix}, \quad (4)$$

and

$$h = \begin{bmatrix} E[s(k)+m(k))v(k)] \\ E[s(k)+m(k))v(k-1)] \end{bmatrix}. \quad (8)$$

To obtain specific values for these two quantities we must define the noise signal v, the ECG signal s and the filtered noise m. We will assume: the ECG signal is a white (uncorrelated from one time step to the next) random signal uniformly distributed between the values -0.2 and +0.2, the noise source (60-Hz sine wave sampled at 180 Hz) is given by

$$v(k) = 1.2\sin(\frac{2\pi k}{3}), \quad (9)$$

and the filtered noise that contaminates the ECG is the noise source attenuated by a factor of 10 and shifted in phase by $\pi / 20$:

$$m(k) = 0.5\sin(\frac{2\pi k}{3} + \frac{\pi}{20}). \quad (10)$$

Now calculate the elements of the input correlation matrix R:

$$E[v^2(k)] = \frac{1}{3}\sum_{k=1}^{3}(\sin(\frac{2\pi k}{3}))^2$$

$$= \frac{1}{3}(\sin\frac{2\pi}{3}\cdot\sin\frac{2\pi}{3} + \sin\frac{4\pi}{3}\cdot\sin\frac{4\pi}{3} + \sin 2\pi\cdot\sin 2\pi)$$

$$= \frac{1}{3}(0.75 + 0.75 + 0) = 0.5 \quad (11)$$

$$E[v^2(k-1)] = E[v^2(k)] = 0.5 \quad (12)$$

$$E[v(k)v(k-1)] = \frac{1}{3}\sum_{k=1}^{3}(\sin\frac{2\pi k}{3})(\sin\frac{2\pi(k-1)}{3})$$

$$= \frac{1}{3}(\sin\frac{2\pi}{3}\cdot\sin 0 + \sin\frac{4\pi}{3}\cdot\sin\frac{2\pi}{3} + \sin 2\pi\cdot\sin\frac{4\pi}{3})$$

$$= \frac{1}{3}(0 + (-0.75) + 0) = -0.25 \quad (13)$$

Thus R is

$$R = \begin{bmatrix} 0.72 & -0.36 \\ -0.36 & 0.72 \end{bmatrix}. \quad (14)$$

The terms of h can be found in a similar manner. We will consider the top term in Eq. (8) first:

$$E[s(k)+m(k))v(k)] = E[s(k)v(k)] + [E[m(k)v(k)]$$

Here the first term on the right is zero because s(k) and v(k) are independent and zero mean. The second term is also zero:

$$E[m(k)v(k)] = \frac{1}{3}\sum_{k=1}^{3}(0.5\sin(\frac{2\pi k}{3}+\frac{\pi}{20}))\cdot(\sin(\frac{2\pi k}{3}))$$

$$= \frac{0.5}{3}(\sin(\frac{2\pi}{3}+\frac{\pi}{20})\cdot\sin\frac{2\pi}{3} + \sin(\frac{4\pi}{3}+\frac{\pi}{20})\cdot\sin(\frac{4\pi}{3})$$

$$+\sin(2\pi+\frac{\pi}{20})\cdot\sin 2\pi) \quad (15)$$

$$= \frac{0.5}{3}(0.6730 + 0.8085 + 0) = \frac{0.5}{3}\times 1.4815 = 0.2469$$

Next consider the second element of h:

$$E[s(k)+m(k))v(k-1)] = E[s(k)v(k-1)] + E[m(k)v(k-1)]$$

As with the first element of h, the first term on the right is zero because s(k) and v(k-1) are independent and zero mean. The second term is evaluated as follows:

$$E[m(k)v(k-1) = \frac{1}{3}\sum_{k=1}^{3}0.5\sin(\frac{2\pi k}{3}+\frac{\pi}{20})\cdot\sin\frac{2\pi(k-1)}{3}$$

$$+\sin(2\pi+\frac{\pi}{20})\cdot\sin\frac{4\pi}{3}) \quad (16)$$

$$=\frac{0.5}{3}(\sin(\frac{2\pi}{3}+\frac{\pi}{20})\cdot\sin 0+\sin(\frac{4\pi}{3}+\frac{\pi}{20})\cdot\sin\frac{2\pi}{3}$$

$$=\frac{0.5}{3}(0+(-0.8085)+(-0.1355))=\frac{0.5}{3}\times(-0.9440)=-0.1573$$

Thus, h is

$$h=\begin{bmatrix} 0.2469 \\ -0.1573 \end{bmatrix} \quad (17)$$

The minimum mean square error solution for the weights is given by:

$$x^* = R^{-1}h = \begin{bmatrix} 0.72 & -0.36 \\ -0.36 & 0.72 \end{bmatrix}^{-1}\begin{bmatrix} 0.2469 \\ -0.1573 \end{bmatrix}$$

$$= \begin{bmatrix} 0.4487 \\ -0.0903 \end{bmatrix} \quad (18)$$

To find the minimum mean square error, consider the performance index:

$$F(x) = c - 2x^t h + x^T Rx \quad (19)$$

We have just found x^*, h and R, so we only need to find c:

$$c = E[t^2(k)] = E[(s(k)+m(k))^2] \quad (20)$$
$$= E[s^2(k)] + 2E[s(k)m(k)] + E[m^2(k)]$$

The middle term is zero because s(k) and m(k) are independent and zero mean. The first term, the expected value of the random signal, can be calculated as follows:

$$E[s^2(k)] = \frac{1}{0.4}\int_{-0.2}^{0.2} s^2\,ds = \frac{1}{3(0.4)} s^3 \Big|_{-0.2}^{0.2} = 0.0133 \quad (21)$$

The mean square value of the filtered noise is

$$E[m^2(k) = \frac{1}{3}\sum_{k=1}^{3}(0.5\sin(\frac{2\pi k}{3}+\frac{\pi}{20}))(0.5\sin(\frac{2\pi k}{3}+\frac{\pi}{20})]$$
$$= \frac{0.5\times 0.5}{3}(\sin(\frac{2\pi}{3}+\frac{\pi}{20})\cdot\sin(\frac{2\pi}{3}+\frac{\pi}{20}) \quad (22)$$
$$+\sin(\frac{4\pi}{3}+\frac{\pi}{20})\cdot\sin(\frac{4\pi}{3}+\frac{\pi}{20})+\sin(2\pi+\frac{\pi}{20})\cdot\sin(2\pi+\frac{\pi}{20}))$$
$$= \frac{0.25}{3}(0.6040+0.8716+0.0245) = \frac{0.25}{3}\times 1.5001 = 0.125$$

so that c=0.0133+0.125=0.1383

$$F(x) = c - 2x^t h + x^T Rx$$
$$= 0.1383 - 2\begin{bmatrix} w_{1,1} & w_{1,2}\end{bmatrix}\begin{bmatrix}0.2469\\ -0.1573\end{bmatrix}$$
$$+\begin{bmatrix} w_{1,1} & w_{1,2}\end{bmatrix}\begin{bmatrix}0.5 & -0.25\\ -0.25 & 0.5\end{bmatrix}\begin{bmatrix}w_{1,1}\\ w_{1,2}\end{bmatrix}$$
$$= 0.1383 - 0.4938w_{1,1} + 0.3146w_{1,2}$$
$$+[0.5w_{1,1} - 0.25w_{1,2} \quad -0.25w_{1,1}+0.5w_{1,2}]\begin{bmatrix}w_{1,1}\\ w_{1,2}\end{bmatrix}$$
$$= 0.1383 - 0.4938w_{1,1} + 0.3146w_{1,2}$$
$$+(0.5w_{1,1} - 0.25w_{1,2})w_{1,1} + (-0.25w_{1,1}+0.5w_{1,2})w_{1,2}$$
$$= 0.1383 - 0.4938w_{1,1} + 0.3146w_{1,2} + 0.5w_{1,1}^2$$
$$-0.25w_{1,1}w_{1,2} - 0.25w_{1,1}w_{1,2} + 0.5w_{1,2}^2$$
$$=0.1383-0.4938w_{1,1}+0.3146w_{1,2}+0.5w_{1,1}^2+0.5w_{1,2}^2$$
$$-0.5w_{1,1}w_{1,2}$$

Substituting x^*, h and R, we find that the minimum mean square error is

$$F(x^*) = 0.1383 - 0.4938\times 0.4487 + 0.3146\times(-0.0903)$$
$$+0.5\times 0.4487^2+0.5(-0.0903)^2-0.5(0.4487)(-0.0903)$$
$$=0.0133$$

The minimum mean square error is the same as the mean square value of the ECG signal. This is what we expected, since the 'error' of this adaptive noise canceller is in fact the reconstructed ECG signal. Fig. 3 illustrates the mean square error performance index surface contour.

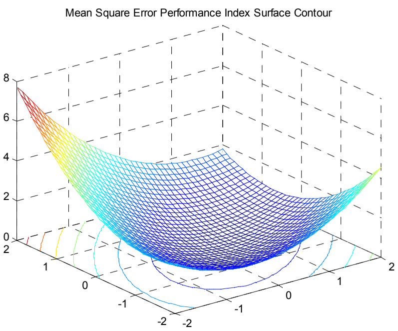


Fig. 3. Mean Square Error Performance Index Surface Contour

D. Adaptive Filter with Fault Tolerance

Real-time computing systems must be fault-tolerant: they must be able to continue operating despite the failure of a limited subset of their hardware or software. A fault is a physical defect, imperfection or flaw that occurs within some hardware or software component. A fault can be caused by specification mistakes, implementation mistakes, component defects or external disturbance. Fault tolerance is the ability of a system to continue to perform its tasks after the occurrence of faults. The fault tolerant adaptive filter is shown in Fig. 4.

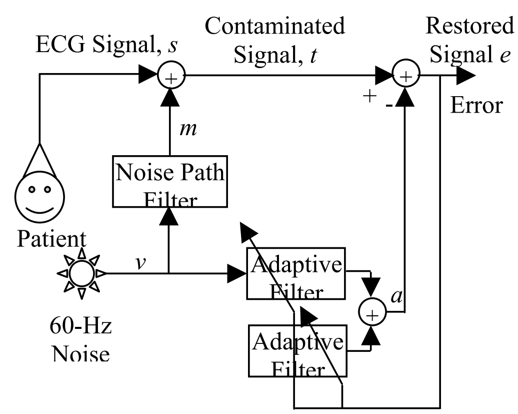


Fig. 4 Fault Tolerant Noise Cancellation System

E. Reliability Analysis of Fault Tolerant Adaptive Filter

The reliability at time t, R(t), is the conditional probability that the system performs correctly during the period [0,t], given that the system was performing correctly at time 0. The unreliability, F(t), is equal to 1-R(t). Often referred to as the probability of failure. Now we compare the reliability of a non-fault-tolerant adaptive filter and that of a fault-tolerant adaptive filter.

$$R = 1 - F \quad (23)$$

For a parallel construction, as shown in Fig. 4, two parts are considered to be operating in parallel if the combination is considered failed when both parts fail. The combined system is operational if either is available. From this it follows that the combined availability is 1 - (both parts are unavailable). The combined availability is shown by the equation below:

$$R = 1 - F_1 \cdot F_2 = 1 - (1 - R_1)(1 - R_2) \quad (24)$$

When the redundancy of a parallel construction is N, the reliability is

$$R = 1 - F_1 \cdot F_2 \cdot \ldots \cdot F_N$$
$$= 1 - \prod_{i=1}^{N} F_i = 1 - \prod_{i=1}^{N} (1 - R_i) \quad (25)$$

The reliability of fault-tolerant adaptive filter is

$$R = 1 - (1 - R_1)(1 - R_2) \quad (26)$$

where R_1 is the reliability of adaptive filter 1, and R_2 is the reliability of adaptive filter 2.

Assume the reliability of the two filters are equal, the reliability of the fault-tolerant adaptive filter is simplified as

$$R = 1 - (1 - R_1)^2 = (1 + (1 - R_1)) \cdot (1 - (1 - R_1))$$
$$= R_1 \cdot (2 - R_1) \quad (27)$$

$$\frac{R}{R_1} = 2 - R_1 \quad (28)$$

Since $0 \le R_1 \le 1$, so $1 \le 2 - R_1 \le 2$. In other word, the reliability of fault-tolerant adaptive filter is greater than or equal to that of the non-fault-tolerant adaptive filter.

III. EXPERIMENTAL RESULTS

In the first experiment, the input signal is a white (uncorrelated from one time step to the next) random signal uniformly distributed between the values -0.2 and +0.2, the noise source (60-Hz sine wave sampled at 180 Hz). A non-fault-tolerant adaptive filter is used. In order to judge the performance of the noise canceller, the original random signals, noise, contaminated signals (i.e. random signals + noise), and the restored signals (i.e. filtered signals) were plotted in Fig. 5. From the fourth subplot, we can see that: at first the restored signal is a poor approximation of the original random signals. It takes about 0.2 second for the filter to adjust to give a reasonable restored signal. The fifth subplot compares the original random signals and the restored signal. It shows that the restored signal favorably matches the original signal.

In the second experiment, MIT-BIH Arrhythmia Database data was used as the input: reference annotation (100.atr), data file (100.dat), and header file (100.hea). The result is shown in Fig. 6. A non-fault-tolerant adaptive filter is used. At the 500th time step, the weights of adaptive filter 1 were all set to 0s. From the fourth subplot we can see that: the filtered ECG decayed to zero at about the 550th time step. The fifth subplot compares the original ECG signal and the restored signal. It shows that the adaptive filter cannot give the right response after the 550th time step.

In the third experiment, the same MIT-BIH Arrhythmia Database data was used as the input: reference annotation (100.atr), data file (100.dat), and header file (100.hea). The result is shown in Fig. 7. This time a fault-tolerant adaptive filter is used. At the 500th time step, the weights of adaptive filter 1 were all set to 0s. However, from the fourth subplot we can see that: the system was not affected by the failure of the first adaptive filter and operated normally. The fifth subplot compares the original ECG signal and the restored signal. It shows that the restored ECG signal exactly matches the original ECG signal.

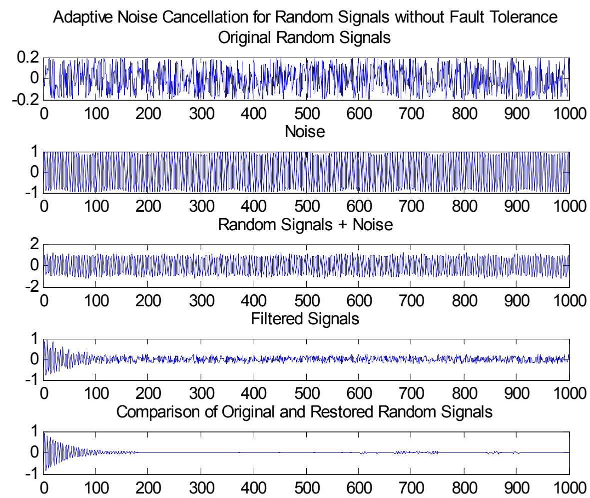


Fig. 5. Adaptive Noise Cancellation for Random Signal without Fault Tolerance

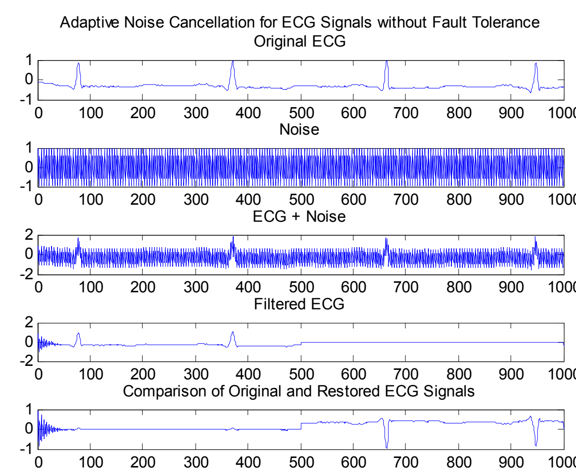


Fig. 6. Adaptive Filter Cancellation for ECG Signal without Fault Tolerance

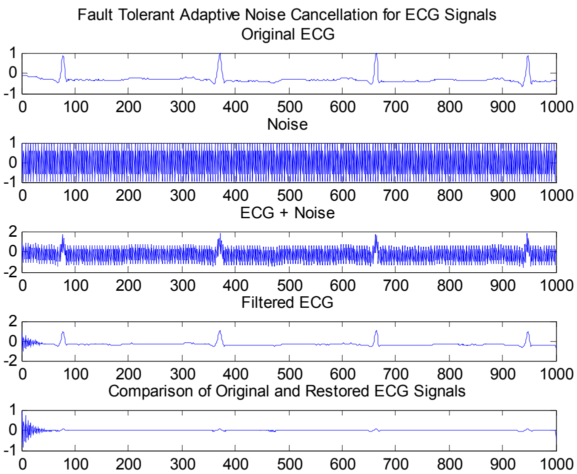


Fig. 7. Adaptive Filter Cancellation for ECG Signal with Fault Tolerance

IV. CONCLUSIONS

A reliable neural network based fault-tolerant adaptive filter was designed. The filter does not need computation for voting and error detection. As a result, it requires very little computational power or memory while still maintaining the ability to handle complex signal processing. We analyzed the reliability of the non-fault-tolerant and fault-tolerant adaptive filters. The experimental results showed that the fault-tolerant adaptive filter is highly reliable after a permanent fault occurs. Thus the adaptive filter approach as described herein can be applied to readily remove 60Hz artifact noise while minimally distorting the true ECG signals.

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