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Fourier Approach to Moving Target Indication and Detection in Multichannel SAR Data

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Abstract— **This paper outlines a solution to the multi-channel synthetic aperture radar (SAR) moving target indication and detection (MTI/MTD) by means of inverse systems approach. A novel model of the problem is presented and an approximate analytic solution to it will be given. It will be demonstrated how a moving target indicator can benefit from a multichannel SAR system as opposed to a traditional approach that separates MTI and SAR systems. It will be shown that the problem of separation of moving targets from stationary ones can be solved completely by using multi-channel approach and in such a way that a spatial distribution of the stationary targets does not play a role.**

I. Introduction

Extraction of moving targets from SAR imagery is of great interest in civil and military applications. Recent methods are based on multi-channel processing to achieve optimal and complete separation of moving targets from the stationary ones. The technology is currently utilized in experimental systems such as PAMIR (Germany) and MCARM (USA). Since multiple sensors distributed in space are used to record information in time, a term *space-time processing* is commonly being used for techniques processing acquired data. Space-time processing methods mostly involve inversion of large matrices. In order to make this inversion possible, adaptive (that is, iterative) methods are employed. Then, one speaks of *space-time adaptive processing* or STAP for short.

STAP methods for radar MTI are chiefly based on spectral estimators [1]–[3]. Since SAR signal is *non-stationary* (it is a chirp-like signal), it is necessary to use very short time durations, where the signal can be considered *stationary* (harmonic). Only then one gets sharp peaks marking the targets. This naturally decreases resolution of such a system. To counter that, elaborate spectral estimators – also called super-resolution techniques – are necessary. However, these estimators still require a formation of large matrices at one point or another.

The goal of this paper is to describe the problem by means of a generalized model, to formulate a solution to this model as an inverse problem and to demonstrate its approximate solution by means of the Fourier transform. That will be evaluated with the help of asymptotic expansions, namely the method of stationary phase. This approximate solution will be used to prove that one can resolve stationary targets and moving ones *completely* regardless their distribution in space or reflectivity. The only criterion used is the velocity of a target.

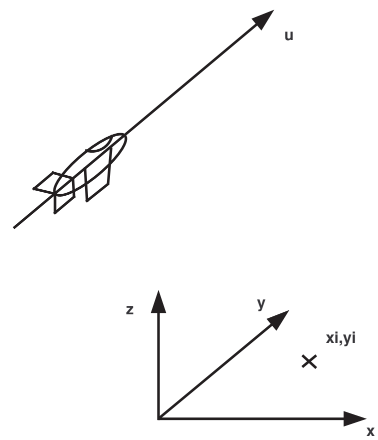

Fig. 1. Geometry of the problem.

II. Problem Formulation

The problem at hand is depicted in figures 1 and 2. A side-looking phased array is placed on aircraft. It is moving along the coordinate u, on a straight line, on an interval $[-L, L]$. The radar footprint is binding a region to be imaged. The targets are located in the x, y plane, y coordinate is parallel to the u coordinate. Targets can be moving or stationary. Stationary and moving targets will be denoted with i and j subscripts, respectively. The phased array consists of N sensors that are spaced with a distance $\Delta d = D/N$. Each sensor takes M measurements along the u coordinate. The standard SAR processing uses the following process for a single-sensor image formation. It is assumed that because radar impulses travel at the speed of light, a radar platform will move only slightly during acquisition of all the echos coming back as a response to each pulse transmitted. Thus, one can imagine that radar stops at positions u_m spaced $\Delta u = 2L/M$ apart, and it records all the responses for a time interval $\Delta\tau = \Delta u/v_r$ sampling it at a rate of P samples per $\Delta\tau$. Variable τ is essentially a time variable t, since $\tau = u/v_r$, where v_r is the radar velocity. τ is sampled at much slower rate than t ($\Delta\tau$ is also called *pulse repetition interval*), τ is referred to as *slow*

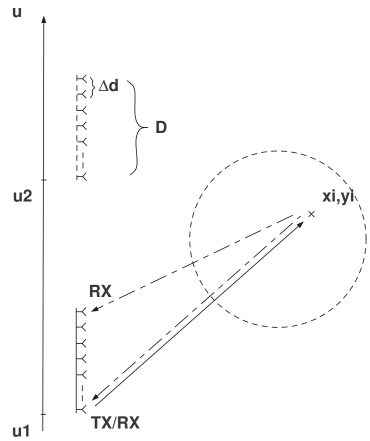

Fig. 2. Geometry of the problem.

time and the sampled time variable t as *fast time*. So, with M sensors, N space, and P time acquisitions, one obtains a three-dimensional array called *datacube* of a size MxNxP.

Solving a three-dimensional inverse problem is a tedious task. In order to avoid this, one can reduce the problem with the following assumptions: Let us take a slice of the datacube at a particular range, that means we have taken a two-dimensional array MxN for a certain p. Now, since the range is known – it is supposed that a SAR image can be formed beforehand and hence all ranges will be known – one can use a model that supposes targets to be located on *one range line only*, that is they will be positioned on a line $x = const.$ in the (x, y) plane. A case when this assumption is violated – and when so-called *range migration* occurs – will be studied numerically in section IV. Finally, since one does not need a range resolution if the range is known, the radar signal needs not be modulated and its bandwidth will be of zero width located at ω_c, the angular carrier frequency. This reasoning is consistent with the theory outlined in [4, chapter 2].

The model used is an extended version of the bi-static SAR model derived in [4, chapter 8]: Please note that a *continuous* model with infinite aperture lengths L and D will be used. Also all amplitude functions with the exception of target reflectivity σ are suppressed, as they do not play a role in the image formation. This approach greatly simplifies the mathematics of the problem with the results still applicable to a discrete case. Reference [5] provides a discussion of sampling and finite aperture effects in great detail. Given the assumptions made above, a signal recorded by the radar can be written as

$$s(u, d) = \sum_i \sigma_i e^{-\jmath k r(u,d)} \tag{1}$$

with r, the range given as

$$\begin{aligned} r(u, d) &= \sqrt{x_i^2 + (y_i - u)^2} \\ &+ \sqrt{x_i^2 + (y_i - u + d)^2}, \end{aligned} \tag{2}$$

where $k = \omega_c/c$ in our case. c is the speed of light. σ_i is target reflectivity. One can observe that function $s(u, d)$ is *spatially variant*. In the following text we are going to show that the two-dimensional Fourier transform of this function is spatially invariant. That will allow us to devise a focusing scheme that is based on the fast convolution.

Let us proceed with writing the two-dimensional Fourier transform of a response of a unit scatterer located at coordinates (x, y):

$$S(k_u, k_d) = \iint_{-\infty}^{\infty} e^{-\jmath k r(u,d)} e^{-\jmath(k_u u + k_d d)}\, \mathrm{d}u\, \mathrm{d}d \tag{3}$$

It will be shown in section III that this double integral has an approximate closed-form solution:

$$S(k_u, k_d) = e^{-\jmath x\left(\sqrt{k^2-(k_u+k_d)^2}+\sqrt{k^2-k_d^2}\right)-\jmath k_u y} \tag{4}$$

It is one of the principal results derived in this paper. Equation 4 contains a linear phase function with respect to (x, y) as opposed to the phase function in expression 1. This fact can and will be used to construct focusing algorithms based on fast convolution. It should be also observed that, unlike in expression 1, the linear phase $\psi(x, y)$ in equation 4 now allows for *separation of variables*, in this case: $\psi(x, y) = \psi_1(x)\psi_2(y)$. This fact will be used to demonstrate separability of moving targets from the stationary ones.

III. Solution

This section provides derivations of closed-form formulas for Fourier spectra of the radar target responses. These formulas will be used to show how moving targets can be separated from the stationary ones in Fourier domain. First, a frequency response of a multi-channel SAR system to a stationary target given by equation 3 will be derived. Then, the model for stationary target responses expressed by equation 1 will be extended to moving targets and its frequency response will be evaluated. Finally, it will be demonstrated that moving and stationary targets can be efficiently separated in the spectral domain.

A. Stationary targets

The solution of equation 3 can be found by means of the *method of stationary phase* [6]–[8]. The idea is that the argument of the integral in equation 3 is a rapidly oscillating function for large k. Integrals of such functions tend to vanish outside of so-called *stationary points*. Thus, it makes sense to evaluate the argument of the integral at the stationary points only. These points are found as maxima, minima or saddle points of a phase function ψ which is composed of arguments of the exponential functions in equation 3. First, a more suitable expression of the phase function will be written. By substitution $v = u - d/2$ one gets:

$$S(k_u, k_d) = \iint_{-\infty}^{\infty} e^{\jmath\psi(v,d)}\, \mathrm{d}v\, \mathrm{d}d, \tag{5}$$

where

$$\begin{aligned}\psi(v,d) &= -k\sqrt{x^2+(y-v-d/2)^2} \\ &\quad -k\sqrt{x^2+(y-v+d/2)^2} \\ &\quad -k_u(v+d/2)-k_d d \end{aligned} \tag{6}$$

If we denote values of the phase function at the stationary points as $\psi_l(v_l,d_l)$, then the value of the integral in equation 3 will be constant and can be calculated as follows:

$$S(k_u,k_d)=\sum_l e^{-\jmath\psi_l(v_l,d_l)} \tag{7}$$

In our case, the phase function is hyperbolic and will have one extremum. This means there will be only one stationary point. Partial derivatives of the phase function will be solved to find this point:

$$\frac{\partial\psi}{\partial v}=0, \qquad \frac{\partial\psi}{\partial d}=0 \tag{8}$$

This gives

$$k_u = k\sin\phi_1 + k\sin\phi_2 \tag{9}$$

$$2k_d+k_u = k\sin\phi_1 - k\sin\phi_2 \tag{10}$$

With

$$\sin\phi_1 = \frac{y-v-d/2}{\sqrt{x^2+(y-v-d/2)^2}} \tag{11}$$

$$\sin\phi_2 = \frac{y-v+d/2}{\sqrt{x^2+(y-v+d/2)^2}} \tag{12}$$

Expressions that depend on v and d were obtained. However, the phase function needs to be constant with respect to v,d. To achieve this, ψ will be expressed as $\psi(k_u,k_d)$. Using the fact that $\sin^2\phi=1-\cos^2\phi$ and with

$$\cos\phi_1 = \frac{x}{\sqrt{x^2+(y-v-d/2)^2}} \tag{13}$$

$$\cos\phi_2 = \frac{x}{\sqrt{x^2+(y-v+d/2)^2}} \tag{14}$$

equations 9 to 14 will yield:

$$\sqrt{x^2+(y-v-d/2)^2} = \frac{kx}{\sqrt{k^2-(k_u+k_d)^2}} \tag{15}$$

$$\sqrt{x^2+(y-v+d/2)^2} = \frac{kx}{\sqrt{k^2-k_d^2}} \tag{16}$$

and also

$$d = -x\left(\frac{k_u+k_d}{\sqrt{k^2-(k_u+k_d)^2}}+\frac{k_d}{\sqrt{k^2-k_d^2}}\right) \tag{17}$$

$$v = y-\frac{x}{2}\left(\frac{k_u-k_d}{\sqrt{k^2-(k_u+k_d)^2}}-\frac{k_d}{\sqrt{k^2-k_d^2}}\right) \tag{18}$$

Finally, substituting previous expressions back into equation 6, one obtains:

$$\psi(k_u,k_d)=-x\left(\sqrt{k^2-(k_u+k_d)^2}+\sqrt{k^2-k_d^2}\right)-k_u y \tag{19}$$

Thus, the approximate solution of the double integral in equation 3 is

$$S(k_u,k_d)=e^{-\jmath x\left(\sqrt{k^2-(k_u+k_d)^2}+\sqrt{k^2-k_d^2}\right)-\jmath k_u y} \tag{20}$$

There are two things to be observed. Firstly, unlike the phase function in expression 6, expression 19 is a linear function of x and y. This shows that equation 19 describes a spatially invariant (shift invariant) system. Secondly, the x coordinate is known because it is calculated for a p-th slice of the datacube: $x=const$. Thus, the first term can be removed. What remains is a linear phase function of y which for $y=y_i$ will give a peak after the inverse Fourier transform at a position of a stationary target. Therefore, the following focusing scheme can be proposed:

$$\begin{aligned} f(x,y) &= \mathrm{DFT}_{2d}^{-1}\left\{\mathrm{DFT}_{2d}\left\{s(u,d)\right\}\cdot S_0^*(k_u,k_d)\right\} \\ S_0^*(k_u,k_d) &= e^{\jmath x_p\left(\sqrt{k^2-(k_u+k_d)^2}+\sqrt{k^2-k_d^2}\right)}, \end{aligned} \tag{21}$$

which is the same as

$$f(x,y)=\mathrm{DFT}_{2d}^{-1}\left\{\mathrm{DFT}_{2d}\left\{s(u,d)\right\}\cdot \mathrm{DFT}_{2d}\left\{s_0(u,d)\right\}^*\right\}, \tag{22}$$

where

$$s_0(u,d) = e^{-\jmath k r_p(u,d)} \tag{23}$$

$$\begin{aligned} r_p(u,d) &= \sqrt{x_p^2+u^2} \\ &\quad +\sqrt{x_p^2+(d-u)^2} \end{aligned} \tag{24}$$

DFT_{2d} denotes the two-dimensional discrete Fourier transform, and * conjugation. All the stationary targets located at a line $x=x_p$ will again lie in a straight line $x=0$ after focusing.

Now, let us show that when applied to moving targets, the same focusing scheme will displace them outside the line where stationary targets are located. This will show that their complete separation is possible.

B. Moving targets

The model of the signal from a moving target located at coordinates (x_j,y_j) is given as [4, Chapter 8]:

$$s(u,d)=\sum_j \sigma_j e^{\jmath\psi(u,d)} \tag{25}$$

with the phase function

$$\begin{aligned} &\psi(u,d)= \\ &-k\sqrt{(x_j-v_x\tau)^2+(y_j-v_y\tau-v_r\tau)^2} \\ &-k\sqrt{(x_j-v_x\tau)^2+(y_j-v_y\tau-v_r\tau+d)^2} \\ &-k_u u-k_d d, \end{aligned} \tag{26}$$

where v_x,v_y are velocities of a moving target in x and y directions, respectively. v_r is the radar velocity, and $u=v_r\tau$. This is an extended version of the stationary target model defined by equation 1. The aim of this subsection is to find the spectrum of the moving target signal. Using the same approach as in the previous subsection, this result can be obtained:

$$\begin{aligned}\psi(k_u,k_d) \quad \approx \\ & -X\sqrt{4k^2-\left(\frac{k_u}{\alpha}\right)^2}-\frac{k_u}{\alpha}Y \\ & -\frac{x}{4k}\left(\frac{2kv_x}{v_r}-\frac{k_u}{\alpha}-2k_d\right)^2, \end{aligned} \tag{27}$$

where:

$$X = \frac{(v_y+v_r)x - v_x y}{\sqrt{v_x^2+(v_y+v_r)^2}} \tag{28}$$

$$Y = \frac{v_x x + (v_y+v_r)y}{\sqrt{v_x^2+(v_y+v_r)^2}} \tag{29}$$

$$\alpha = \frac{\sqrt{v_x^2+(v_y+v_r)^2}}{v_r}. \tag{30}$$

So, the spectrum of a moving target will be given by

$$S(k_u,k_d) = e^{j\psi(k_u,k_d)} \tag{31}$$

C. Stationary and moving targets separation

In this subsection, it will be shown that a moving target will be displaced after focusing. The focusing scheme given by formula 21 will be applied. In order to simplify the solution, however, an approximate version of formula 21 will be used. Namely, we will use equation 27. The complex conjugate of this phase function for the case of a broadside stationary target located at coordinates $(x_p, 0)$ can be written as

$$\psi_{\text{focus}} = x_p\sqrt{4k^2-k_u^2} - \frac{x_p}{4k}\left(-k_u-2k_d\right)^2 \tag{32}$$

As shown before, stationary targets will be focused and located on a line $x = 0$, moving targets should be off that line. Suppose a moving target at a coordinate (x_p, y_j) with reflectivity $\sigma_j = 1$. Let its phase function be

$$\begin{aligned}\psi_{\text{mov}} = & -X\sqrt{4k^2-k_u^2}-k_uY \\ & +\frac{x_p}{4k}\left(\frac{2kv_x}{v_r}-k_u-2k_d\right)^2 \end{aligned} \tag{33}$$

It is also assumed that for slow speeds, one can have $\alpha \approx 1$. This assumption will further simplify the derivation. It makes sense to investigate slowly moving targets, as they are expected to be closest to the line $x = 0$. The resulting phase function of the moving target's spectrum after the focusing will take the form:

$$\begin{aligned}\psi_{\text{mov}}+\psi_{\text{focus}} = \\ & (x_p-X)\sqrt{4k^2-k_u^2}-k_uY \\ & +\frac{x_pv_x}{v_r}\left(\frac{kv_x}{v_r}-k_u-2k_d\right) \end{aligned} \tag{34}$$

Further, we can set $v_y = 0$. For slow targets $v_r \gg v_x$ and so

$$X \approx x_p - \frac{v_x}{v_r}y_j, \qquad Y \approx \frac{v_x}{v_r}x_p + y_j,$$

because v_y only causes de-focusing [5]. Actually, it also changes the slope on which a moving target signature appears for various v_x, but it does not change the fact that moving targets will be displaced. If we also assume a broadside moving target, i.e. $y_j = 0$, then a reduced expression appears:

$$\psi_{\text{mov}}+\psi_{\text{focus}} = kx_p\frac{v_x^2}{v_r^2} - \frac{2x_pv_x}{v_r}(k_u+k_d) \tag{35}$$

The first term in this equation is a constant phase shift. The second term is a linear phase function of k_u and k_d.This term will cause the shift of a moving target signature in x and y direction, therefore away from the line $x = 0$.

The proposed scheme for moving targets extraction is obvious: form the datacube, take one range slice after another, perform focusing, separate stationary and moving targets. Alternatively, stationary or moving targets can be removed, data inverted back to the time domain, and additional processing performed. The advantage of using Fourier imaging is is also clear: it is the possibility to use the fast Fourier transform (FFT).

There is also another attractive possibility to perform the moving-stationary targets separation offered by this approach. Since it is supposed there are enough data to form N SAR images, one may ask if it is possible to reuse a SAR processor somewhere in the algorithm. Indeed, this is the case. We call this method an *interferometric approximation*, since it resembles radar interferometry. Recall formula 26. It can be shown, that for or stationary targets, the phase function will take this form:

$$\begin{aligned}&\psi(u,d) \approx \\ &-2k\sqrt{x^2+\left(y-u+\frac{d}{2}\right)^2}-\frac{kd^2}{4x} \\ &-k_uu-k_dd \end{aligned} \tag{36}$$

In a single-channel SAR system, only one-dimensional Fourier transform in the u domain is calculated to perform focusing in the spectral domain. The same approach will be taken here. An approximate solution for the Fourier integral of a complex function $\exp(j\psi)$ in u domain, where ψ is given by equation 36, will be shown. The aim is to see whether the result contains the focusing phase function used in conventional SAR processing. Before the start, yet again some manipulations to ease the calculations. Substitute $v = u - d/2$:

$$\begin{aligned}&\psi(v,d) \approx \\ &-2k\sqrt{x^2+(y-v)^2}-\frac{kd^2}{4x} \\ &-k_u\left(v+\frac{d}{2}\right)-k_dd \end{aligned} \tag{37}$$

We wish to evaluate the following integral

$$S(k_u,d) = \int_{-\infty}^{\infty} e^{j\psi(v,d)}\,\mathrm{d}v \tag{38}$$

As before, the method of stationary phase will be used. Partial derivative $\partial\psi/\partial v = 0$ produces

$$k_u = \frac{2k(y-v)}{\sqrt{x^2+(y-v)^2}} \tag{39}$$

By the same way as before, the following phase function is found

$$\begin{aligned}\varphi(k_u, d) &= -x\sqrt{4k^2 - k_u^2} - k_u(y + d/2) \\ &\quad -k_d d - \frac{kd^2}{4x} \end{aligned} \quad (40)$$

Not only the first term is known and can be removed, it is also used for SAR image formation via spherical wave decomposition and spatial Doppler phenomenon [5]. The first two terms represent the model used for bi-static SAR in [4, chapter 8].

After deduction of phase terms related to SAR processing:

$$\varphi(k_u, d) = -k_u y - k_d d - \frac{kd^2}{4x} \quad (41)$$

The integral

$$S(k_u, k_d) = \int_{-\infty}^{\infty} e^{\jmath\varphi(k_u,d)}\, \mathrm{d}d \quad (42)$$

will again be solved by calculating

$$\frac{\partial\varphi}{\partial d} = 0$$

That will give

$$d = \frac{-2xk_d}{k} \quad (43)$$

Substituting d back into 41 yields:

$$\varphi(k_u, k_d) = -k_u y + \frac{k_d^2 x}{k} \quad (44)$$

Removing the second term in this expression, one again obtains a linear phase function dependent on k_u only.

This analysis suggests the following approach: Focus a SAR image recorded by each sensor, shift it $d/2$ relatively to the transmitter, take the Fourier transform with respect to d for each range slice, multiply with the reference function $\exp(-\jmath k_d^2 x_p/k)$, and take the inverse Fourier transform with respect to d. This will again produce focused stationary targets located at one line $x = 0$ and moving target signatures located off that line. Accuracy of this approximation is treated by means of numerical experiments in section IV.

IV. SIMULATIONS

Simulations were performed to test some of the findings from section III. The parameters of all simulations, unless stated otherwise, were as follows: $\omega_c = 2\pi 10^9\,\mathrm{rad/s}$, $L = 30\,\mathrm{m}$, $x_p = 1000\,\mathrm{m}$, $D = 120\,\mathrm{m}$, $M = 256$, $N = 256$, $v_r = 100\,\mathrm{m/s}$, $v_x = -1\,\mathrm{m/s}$, $v_y = 5\,\mathrm{m/s}$ The size of an imaged area was 50x50 meters. High numbers of samples and large apertures were chosen to comfortably demonstrate all properties of the solutions. Stationary and moving target models from equations 1 and 25 were used in all simulations. Figures labeled 'interferometric approximation' were created using the interferometric approximation presented in subsection III-C. To create the rest of the figures, focusing scheme expressed by equation 21 was applied.

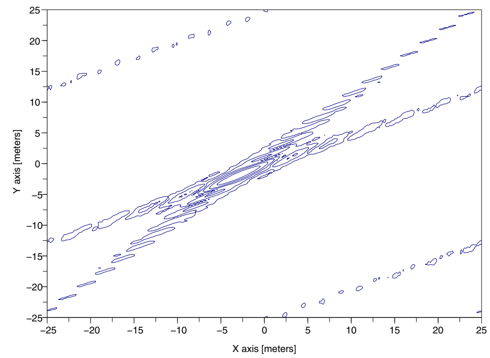

Fig. 3. Point spread function.

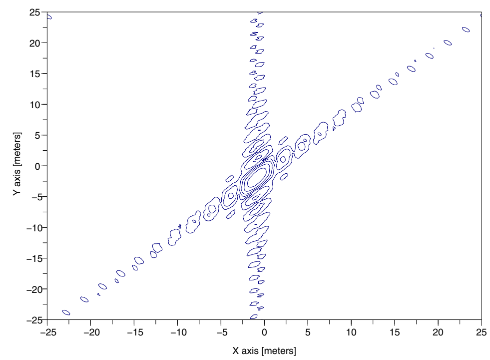

Fig. 4. Point spread function, interferometric approximation.

A. Point spread functions

Figures 3 and 4 show point spread functions of a single stationary target located in the center of the imaged area (broadside). The range migration effect as described in section II, occurs when signal of a target located in a range cell p leaks into the neighboring range cell $p-1$. To test the influence of range migration, it was supposed that a target was located 100 m further than its actual position. Please note that given other parameters of the simulation, this could be considered an extremely large and unrealistic value. It was chosen in order to demonstrate the effect visibly. One can see that range migration has a negative impact on the point spread function shape which is now wider with higher level of sidelobes. The result is in figure 5. Figure 6 shows what happens when aperture D is halved. As expected, one observes a signature that is more spread in x direction. Since this direction indicates targets with nonzero velocities, the velocity resolution is now coarser.

B. Focused images

In figure 7, a case of ten stationary and one moving target is depicted. As predicted, stationary targets appear on a vertical

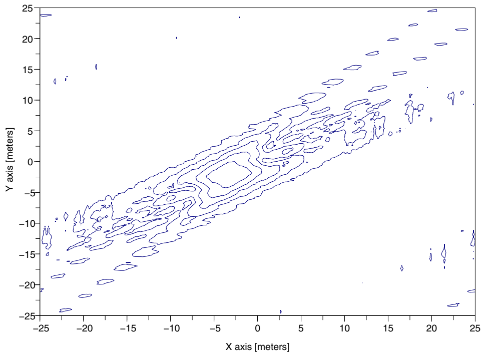

Fig. 5. Point spread function with range migration 10%.

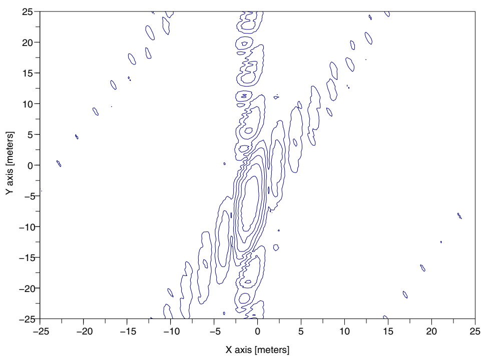

Fig. 6. Point spread function with reduced aperture $D = 60$ m, interferometric approximation.

line in the center of images $x = 0$. The moving target is shifted due to its cross-track velocity v_x and slightly de-focused due to its along-track velocity v_y.

V. Conclusions

An approximate analytic solution to the multi-channel SAR MTI/MTD problem was presented. It was demonstrated that a combination of a SAR and MTI processing techniques is possible. Thus, one can reuse existing software and hardware used in SAR imagery. Hence, it is advantageous to design one multichannel SAR system, rather than two separate MTI and SAR systems or a system operating in multiple modes.

It follows from the approach taken that the separation scheme does not depend on a spatial or power distribution of the targets, but merely on their velocity. This means that under conditions defined here, moving targets can always be extracted and their detection depends only on the resolution of a system, given by its point spread function, and additional noise in the system, such as thermal noise. This is one of the major advantages of the Fourier approach in this case, for it

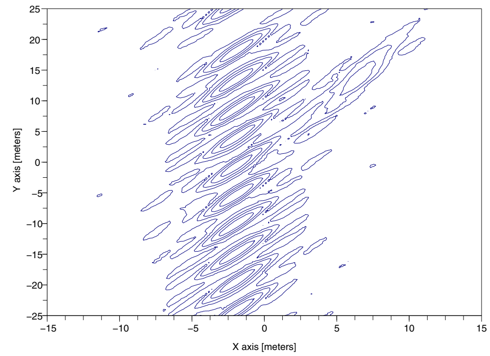

Fig. 7. Focused image, stationary and moving targets.

clearly demonstrates capabilities of a multichannel SAR system to perform optimal separation without using advanced stochastic approaches.

Thanks to Fourier approach to the solution of this problem, it is also evident that the larger the aperture, the better the resolution, as clearly demonstrated by the simulations. High resolution Fourier focusing is possible in the u domain because synthetic aperture L can be made sufficiently long. Actually, it was one of the goals to take the advantage of this fact. The problem is in the d domain. The aperture of a phased array D is physically limited and one indeed needs to use some high resolution spectral estimators, rather than power spectrum obtained by FFT.

However, the use of a larger aperture L reduces the problem into having to perform a high-resolution Fourier transform in *one dimension only*. Such approach is called *post-Doppler processing* in STAP literature [1], [3]. The processing power reduction is tremendous when using this approach; a size of a covariance matrix featured in most high-resolution spectral estimators reduces from MxNxMxN to MxN.

VI. Acknowledgments

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