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A Study on MMSE Hartley Algorithm

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***Abstract*—In this paper, we present a real signal parameter estimation algorithm based on the MMSE algorithm which called the MMSE Hartley algorithm. Firstly, we introduce the processing steps of MMSE Hartley algorithm. Secondly, compare the main difference in the processing steps between MMSE and MMSE Hartley algorithm. Lastly, compare the performance between Hartley transform and MMSE Hartley algorithm by computer simulation. The computer simulation result proves the validity of our algorithm.**

Keywords-parameter estimation; MMSE; MMSE Hartley algorithm; Hartley transform

I. INTRODUCTION

In paper [1], Erik G. Larsson *et al.* presented a parameter estimation algorithm that called MMSE. The MMSE algorithm is derived according to the complex signal model.

In this paper, we extend the MMSE algorithm to the processing of real signals and present a real counterpart of MMSE algorithm that we call the MMSE Hartley algorithm. The MMSE Hartley algorithm is an algorithm devised to process the real signal and is totally based on the MMSE algorithm.

Because the derivation procedure of MMSE Hartley algorithm is similar to the MMSE algorithm [1], we compare the main difference in the processing step between MMSE and the MMSE Hartley algorithm. The processing steps of MMSE Hartley algorithm are similar to these of MMSE except that some processing steps need to be modified to match the processing of real signal.

In order to verify our method, we compare the performance ability of Hartley and MMSE Hartley algorithm through computer simulation.

This paper is organized as follows. In the later section, the MMSE Hartley algorithm is introduced. The computer simulation is introduced in the third section. The last section is the conclusions.

II. MMSE HARTLEY ALGORITHM

Assume the signal model can be expressed by (1) ~ (3). For different signal models, the MMSE Hartley algorithm has different performance formula.

$$s_1(i) = A(\xi)\operatorname{cas}(i\xi/I) + \theta_i(\xi), i = 0,1,\cdots,I-1 \quad (1)$$

$$s_2(i) = A(\xi)\cos(i\xi/I) + \theta_i(\xi), i = 0,1,\cdots,I-1 \quad (2)$$

$$s_3(i) = A(\xi)\sin(i\xi/I) + \theta_i(\xi), i = 0,1,\cdots,I-1 \quad (3)$$

In (1) ~ (3) and the other part of this paper, $\operatorname{cas}(x) = \cos(x) + \sin(x)$, $\theta_i(\xi)$ is the error, $A(\xi)$ and I are the amplitude and length of the signal respectively.

The MMSE Hartley filter is completely similar to the MMSE algorithm. For the derivation of MMSE algorithm in detail, please refer to [1]. Because the processing of MMSE Hartley algorithm is very similar to the MMSE algorithm, we mainly discuss the difference of the processing step between MMSE Hartley algorithm and MMSE algorithm.

Now we discuss the MMSE Hartley algorithm with the method similar to [1]. We use the forward-only [1] method to introduce the MMSE Hartley algorithm, the extension to forward-backward case [1] is easy.

Let $\mathbf{s}_1(i)$ and $\mathbf{s}_2(i)$ denote the forward data vector that is the same as the MMSE algorithm [1]

$$\mathbf{s}_2(i) = \left[s_2(i), s_2(i+1), \cdots, s_2(i+J-1)\right]^T, \quad (4)$$

$$\mathbf{s}_3(i) = \left[s_3(i), s_3(i+1), \cdots, s_3(i+J-1)\right]^T. \quad (5)$$

In (4) ~ (5), $s_2(i)$ and $s_3(i)$ are the signal of (2) and (3) respectively; J is the length of the filter, for (4) ~ (5) and the other section of this paper, "T" denotes transposition.

The aim of the MMSE Hartley algorithm is to estimate the frequency and amplitude of signal in (1). Because the phase shift of the signal in (1) can be formulated as

$$\cos\left(\left(i+1\right)\omega\right)+\sin\left(\left(i+1\right)\omega\right)$$
$$=\cos\left(i\omega\right)\left(\cos\left(\omega\right)+\sin\left(\omega\right)\right)+\sin\left(i\omega\right)\left(\cos\left(\omega\right)-\sin\left(\omega\right)\right). \tag{6}$$

We can find that (6) can be decomposed into three steps. The first and second step is to compute (7) and (8) respectively.

$$\cos\left(i\omega\right)\left(\cos\left(\omega\right)+\sin\left(\omega\right)\right) \tag{7}$$

$$\sin\left(i\omega\right)\left(\cos\left(\omega\right)-\sin\left(\omega\right)\right) \tag{8}$$

The third step is the summation of the results of step 1 and step 2.

Use the method similar to [1] [2] and the former three steps method, we can compute the MMSE Hartley algorithm in three steps.

Step 1: Filter the signal in (4) uses the MMSE quasi-cosine algorithm.

The MMSE quasi-cosine algorithm can be explained by a filter that is similar to [1], and it's defined in (9).

$$\tilde{A}_1\left(\xi\right)=\mathbf{w}^T_{\text{MMSE-quasi-cosine}}\left(\xi\right)\cdot \frac{1}{I-J+1}\sum_{i=0}^{I-J}\mathbf{s}_2\left(i\right)\operatorname{cas}\left(\xi i/I\right), \tag{9}$$

where $\tilde{A}_1\left(\xi\right)$ is the MMSE quasi-cosine estimator.

According to the method similar to [1], the MMSE quasi-cosine filter is similar to [1], which is [1]

$$\mathbf{w}_{\text{MMSE-quasi-cosine}}\left(\xi\right)=\frac{\tilde{\mathbf{B}}_1^{-1}\left(\xi\right)\mathbf{k}_1\left(\xi\right)}{\mathbf{k}_1^T\left(\xi\right)\tilde{\mathbf{B}}_1^{-1}\left(\xi\right)\mathbf{k}_1\left(\xi\right)}, \tag{10}$$

where

$$\mathbf{k}_1\left(\xi\right)=\left[1,\operatorname{cas}\left(\xi/I\right),\operatorname{cas}\left(2\xi/I\right),\cdots,\operatorname{cas}\left(\left(J-1\right)\xi/I\right)\right]^T. \tag{11}$$

From (9) we can find that in order to compute the MMSE quasi-cosine transform, need to compute the discrete quasi-cosine transform firstly, the discrete quasi-cosine transform is defined in (12).

$$S_{\text{quasi-cosine}}\left(\xi\right)=\frac{1}{I-J+1}\sum_{i=0}^{I-J}\mathbf{s}_2\left(i\right)\operatorname{cas}\left(\xi i/I\right), \tag{12}$$

where $\mathbf{s}_2\left(i\right)$ is the signal of (4).

Step 2: Filter the signal in (5) uses the MMSE quasi-sine algorithm that is defined in (13).

$$\tilde{A}_2\left(\xi\right)=\mathbf{w}^T_{\text{MMSE-quasi-sine}}\left(\xi\right)\cdot \frac{1}{I-J+1}\sum_{i=0}^{I-J}\mathbf{s}_3\left(i\right)\left(\cos\left(\xi i/I\right)-\sin\left(\xi i/I\right)\right), \tag{13}$$

where $\tilde{A}_2\left(\xi\right)$ is the MMSE quasi-sine estimator.

Similarly to step 1, the MMSE quasi-sine filter is similar to [1], which is [1]

$$\mathbf{w}_{\text{MMSE-quasi-sine}}\left(\xi\right)=\frac{\tilde{\mathbf{B}}_2^{-1}\left(\xi\right)\mathbf{k}_2\left(\xi\right)}{\mathbf{k}_2^T\left(\xi\right)\tilde{\mathbf{B}}_2^{-1}\left(\xi\right)\mathbf{k}_2\left(\xi\right)}, \tag{14}$$

where

$$\mathbf{k}_2\left(\xi\right)=\left[1,\cos\left(\xi/I\right)-\sin\left(\xi/I\right),\cos\left(2\xi/I\right)-\sin\left(2\xi/I\right),\cdots,\cos\left(\left(J-1\right)\xi/I\right)-\sin\left(\left(J-1\right)\xi/I\right)\right]^T. \tag{15}$$

Similarly, we need to compute the discrete quasi-sine transform before compute the MMSE quasi-sine transform. The discrete quasi-sine transform is defined in (16).

$$S_{\text{quasi-sine}}\left(\xi\right)=\frac{1}{I-J+1}\sum_{i=0}^{I-J}\mathbf{s}_3\left(i\right)\left(\cos\left(\xi i/I\right)-\sin\left(\xi i/I\right)\right). \tag{16}$$

From (9) and (13) we can find that in order to compute $\tilde{A}_1\left(\xi\right)$ and $\tilde{A}_2\left(\xi\right)$, need to use the method similar to the discrete Hartley transform [3] firstly, which is denoted in (12) and (16) respectively, that is the reason we name our algorithm the MMSE Hartley algorithm.

Step 3: add the result of Step 1 and Step 2 and get the final result.

The format of (10) and (14) is the same as the MMSE algorithm except that the element of it is different from MMSE for the need of processing real signal, now we discuss the differences between them.

Firstly, in (10) and (14), $\mathbf{k}_1\left(\xi\right)$ and $\mathbf{k}_2\left(\xi\right)$ are the phase difference vector due to the data shift and are illuminated in (11) and (15) respectively.

Secondly, in (10) and (14),

$$\tilde{\mathbf{B}}_1(\xi)=\frac{1}{I-J+1}\sum_{i=-(I-J+1)+1}^{I-J} r_i\tilde{\mathbf{D}}_1(i)\operatorname{cas}(i\xi/I), \quad (17)$$

$$\tilde{\mathbf{B}}_2(\xi)=\frac{1}{I-J+1}\cdot \sum_{i=-(I-J+1)+1}^{I-J} r_i\tilde{\mathbf{D}}_2(i)\left[\cos(i\xi/I)-\sin(i\xi/I)\right]. \quad (18)$$

In (17) ~ (18), $\tilde{\mathbf{D}}_1(i)$ and $\tilde{\mathbf{D}}_2(i)$ are the covariance matrixes defined similar to the MMSE algorithm [1]. The computation of $\tilde{\mathbf{B}}_1$ and $\tilde{\mathbf{B}}_2$ is the same as [1] except for the computation of the Toeplitz matrix [1]. For MMSE Hartley algorithm, in the computation of step 1 and step 2, the element of Toeplitz matrix is $T_1(i,j)$, $T_2(i,j)$ respectively, which is

$$T_1(i,j)=r_{j-i}\operatorname{cas}\left(\xi(j-i)/I\right), \quad (19)$$

$$T_2(i,j)=r_{j-i}\left(\cos\left(\xi(j-i)\right)-\sin\left(\xi(j-i)\right)\right). \quad (20)$$

In (17) ~ (20), r_i is defined the same as the Eq.28 of [1].

From the former discussion, we can find that the MMSE quasi-cosine, quasi-sine algorithm are similar to the MMSE algorithm completely except the computation of (9) and (13), $\mathbf{k}_1(\xi)$, $\mathbf{k}_2(\xi)$ and $T_1(i,j)$, $T_2(i,j)$.

The concrete processing procedure of MMSE Hartley algorithm is:

1) Compute the data matrix [1] (4) ~ (5) of the signal in (2) ~ (3) respectively. This step is completely the same as the MMSE algorithm.

2) Compute the data's discrete quasi-cosine, quasi-sine transform according to (12) and (16) respectively. This step is the main difference from the MMSE algorithm.

3) Compute the data phase move matrix $\mathbf{K}_1$ and $\mathbf{K}_2$, which are the matrixes composed by the vector of $\mathbf{k}_1(\xi)$ and $\mathbf{k}_2(\xi)$ respectively when the variable is ξ. It is another different step from the MMSE algorithm because the vector of (11) and (15) is different from MMSE algorithm.

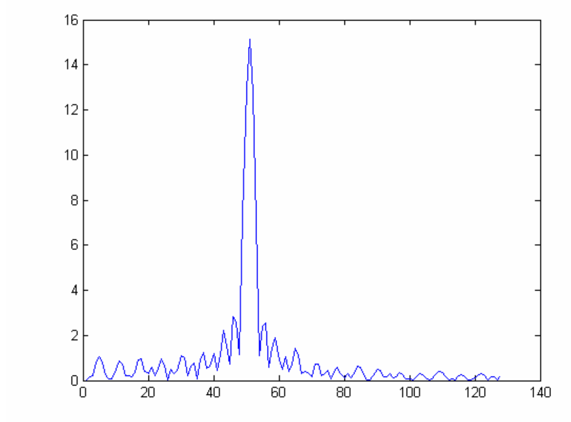
4) Compute the Toeplitz matrix according to (19) ~ (20). It is another different step from the MMSE algorithm.

5) Compute the adaptive filter $\mathbf{w}_{\text{MMSE-quasi-cosine}}$, $\mathbf{w}_{\text{MMSE-quasi-sine}}$ and the estimated result $\tilde{A}_1(\xi)$, $\tilde{A}_2(\xi)$. This step is also the same as the MMSE algorithm.

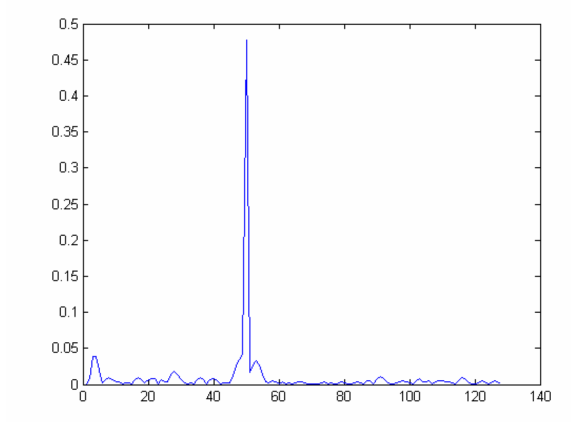
6) Add $\tilde{A}_1(\xi)$ and $\tilde{A}_2(\xi)$ to get the final result.

III. Computer Simulation

In the simulation, we use the forward-backward method (Refer to [1] for detail of the forward-backward method). In the simulation, the noise is the white gauss noise with zero mean and the variance is 2; the length of the signal is $I=256$.



(a)The spectrum estimated by Hartley transform.



(b) The spectrum estimated by MMSE Hartley algorithm.

Fig.1. The estimation of Hartley transform and MMSE Hartley algorithm.

In this simulation, the signal to be estimated is denoted by (21) ~ (23),

$$s_1(i) = 15\,\mathrm{cas}(50i/I), \quad (21)$$

$$s_2(i) = 15\cos(50i/I), \quad (22)$$

$$s_3(i) = 15\sin(50i/I). \quad (23)$$

In (21) ~ (23), I is the length of the signal.

We use the Hartley transform and the MMSE Hartley algorithm to estimate the parameter of the signal. The estimate results of the Hartley transform and the MMSE Hartley algorithm is illustrated in Fig.1 (a), (b) respectively. In Fig.1, the horizontal axis is the frequency ξ, the vertical axis is the absolute value of the estimated result.

Compare Fig.1 (a) and (b) we can find that the MMSE Hartley algorithm has a thinner spectrum line and lower sidelobe than the Hartley transform.

IV. CONCLUSIONS

In this paper, we introduced the MMSE Hartley algorithm that is the real counterpart of the MMSE algorithm. We introduced the processing steps of MMSE Hartley algorithm and compare the differences between the processing steps of MMSE and MMSE Hartley algorithm. On the other hand, we compare the performance ability between Hartley transform and MMSE Hartley algorithm by computer simulation. From the analysis we can find that similar to the complex MMSE algorithm and the Fourier transform [1], the MMSE Hartley algorithm has a better resolution in the estimation of frequency but poor amplitude estimate ability than the Hartley transform.

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