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Studies on Extremal Optimization and Its Applications in Solving Real World Optimization Problems

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***Abstract*—Recently, a local-search heuristic algorithm called Extremal Optimization (EO) has been proposed and successfully applied in some NP-hard combinatorial optimization problems. This paper presents an investigation on the fundamentals of EO with its applications in discrete and numerical optimization problems. The EO was originally developed from the fundamental of statistic physics. However, in this study we also explore the mechanism of EO from all three aspects: statistical physics, biological evolution or co-evolution and ecosystem. Furthermore, we introduce our contributions to the applications of EO in solving traveling salesman problem (TSP) and production scheduling, and multi-objective optimization problems with novel perspective in discrete and continuous search spaces, respectively. The simulation results demonstrate the competitive performance with EO optimization solutions due to its extremal dynamics mechanism.**

***Index Terms*—Extremal optimization, Self-organized criticality, Multiobjective optimization, Traveling salesman problem, Production scheduling.**

I. Introduction

THE studies on NP-hard optimization problems have been a challenge subject in optimization community. In addition to traditional operations research, the modern heuristics [1] have been attractive in fundamental research and real applications. The approaches to evolutionary algorithm (EA) [2], artificial life [3], simulated annealing (SA) [4] and Tabu search [5] *et al*. are developed from the natures of biological evolution, statistical physics and artificial intelligence *et al.* In recent years, a novel general-purpose local search optimization approach, so-called "Extremal Optimization (EO)" has been proposed by Boettcher and Percus [6, 32, 34, 35] based on the fundamentals of statistical physics and self-organized criticality (SOC) [8]. In contrast to SA which is inspired by equilibrium statistical physics, EO is based on Bak-Sneppen (BS) model [7] of biological evolution which simulates far-from equilibrium dynamics in statistical physics. The BS model is one of the models that show the nature of SOC [8, 33, 37, 40]. The SOC means that regardless of the initial state, the system always tunes itself to a critical point having a power-law behavior without any tuning control parameter. In BS model, species has an associated fitness value between 0 and 1 representing a time scale at which the species will mutate to a different species or become extinct. The species with higher fitness has more chance of surviving. Species in this model is located on the sites of a lattice. Each species is assigned a fitness value randomly with uniform distribution. At each update step, the worst adapted species is always forced to mutate. The change in the fitness of the worst adapted species will cause the alteration of the fitness landscape of its neighbors. This means that the fitness values of the species around the worst one will also be changed randomly, even if they are well adapted. After a number of iterations, the system evolves to a highly correlated state known as SOC. In that state, almost all species have fitness values above a certain threshold. In the SOC state, a little change of one species will result in co-evolutionary chain reactions called "avalanches" [8]. The probability distribution of the sizes "*K*" of these avalanches is depicted by a power law $P(K) \propto K^{-\tau}$, where τ is a positive parameter. That is, the smaller avalanches are more likely to occur than those big ones, but even the avalanches as big as the whole system may occur with a small but non-negligible probability. Therefore, the large avalanches make any possible configuration (i.e., so-called "solution" in the evolutionary algorithms) accessible [9].

In contrast to genetic algorithms (GAs) which operate on an entire "gene-pool" of huge number of possible solutions, EO successively eliminates those worst components in the sub-optimal solutions. Its large fluctuations provide significant hill-climbing ability, which enables EO to perform well particularly at the phase transitions [9, 38, 39]. EO has been successfully applied to some NP-hard combinatorial optimization problems such as graph bi-partitioning [9], TSP [9], graph coloring [10], spin glasses [11], MAX-SAT [12] and dynamic combinatorial problems [42].

In this paper, we make a deep investigation on the fundamental of EO from three points of view: statistical physics, biological evolution and ecosystem. In this work, we also introduce our contributions to the applications of EO in discrete and numerical optimization problems. Finally, the advantages and disadvantages of EO are discussed.

II. Extremal optimization

Unlike GAs, which work with a population of candidate solutions, EO evolves a single individual (i.e. chromosome) *S*. In EO, each decision variable in the current individual *S* is considered "species". There is only mutation operator in EO. Through always performing mutation on the worst species and its neighbors successively, the individual can improve its components and evolve itself toward the optimal solution

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generation by generation. This requires that a suitable representation should be selected which permits each species to be assigned a quality measure (i.e. fitness). This differs from holistic approaches such as evolutionary algorithms that assign equal-fitness to all species of a solution based on their collective evaluation against an objective function.

For a minimization problem with *n* decision variables, EO proceeds as follows [9]:

1. Randomly generate an individual *S*. Set the optimal solution $S_{best} = S$.
2. For the "current" individual *S*
(a) evaluate the fitness λ_i for each decision variable $x_i, i \in (1, \cdots, n)$
(b) find *j* satisfying $\lambda_j \leq \lambda_i$ for all *i*, i.e., x_j creates the "worst fitness",
(c) choose $S' \in N(S)$ such that x_j *must* change its state, *N(S)* is the neighborhood of *S,*
(d) accept $S = S$ *unconditionally,*
(e) if the current cost function value is less than the minimum cost function value, i.e. $C(S) < C(S_{best})$, then set $S_{best} = S$
3. Repeat at Step 2 as long as desired.
4. Return S_{best} and $C(S_{best})$.

To avoid getting stuck into a local optimum [9], a single parameter τ is introduced into EO and the new algorithm is called τ -EO. In τ -EO, according to fitness λ_i , all x_i are ranked, i.e., a permutation Π of the variable labels *i* with

$$\lambda_{\Pi(1)} \leq \lambda_{\Pi(2)} \leq \cdots \leq \lambda_{\Pi(n)} . \quad (1)$$

is found. The worst variable x_j is of rank 1, $j = \Pi(1)$, and the best variable is of rank n. Consider a scale-free probability distribution over the ranks k,

$$P_k \propto k^{-\tau}, \quad 1 \leq k \leq n \ , \quad (2)$$

for a fixed value of the parameter τ . At each update, select a rank *k* according to P_k. Then, modify Step 2 (c) so that the variable x_j with $j = \Pi(k)$ changes its state.

III. Fundamental Aspects of Extremal Optimization

From Fig. 1, it can be seen that EO is a multi-disciplinary technique which is based on the fundamental and knowledge of statistical physics, biological evolution and ecosystem.

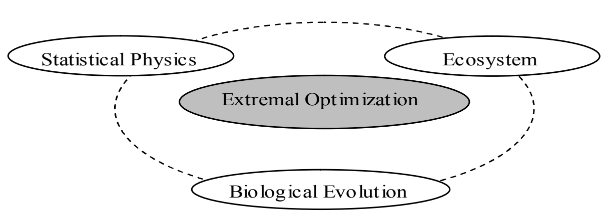


Fig. 1. Extremal Optimization: A Multi-disciplinary Technique.

First, we will make an investigation on EO from statistical physics point of view. EO is motivated by the SOC, which is a statistical physics concept to describe a class of dynamical systems that have a critical point as an attractor. Their macroscopic behavior exhibits the spatial and temporal scale-invariance characteristics of the critical point of a phase transition [8]. It is interesting to note that in SOC, there is no need to tune control parameters to precise values. SOC is typically observed in slowly-driven non-equilibrium systems with extended degrees of freedom and a high level of nonlinearity [8]. Inspired by SOC, EO drives the system far from equilibrium: aside from ranking, there exists no adjustable parameter, and new solutions are accepted indiscriminately.

Second, we can study the mechanism of EO from the perspective of biological evolution. The EO heuristic was motivated by the BS model which shows the emergence of SOC in ecosystems. The fundamental idea behind this model is that of co-evolutionary avalanches. It is well known that the competitive and co-evolutionary activities are regarded as two important factors that help the organisms evolve generation by generation in the nature. Although co-evolution does not have optimization as its exclusive goal, it serves as a powerful paradigm for EO [13]. EO follows the spirit of the BS model in that it merely updates those components with worst fitness in the current solution, replacing them by random values without ever explicitly improving them. At the same time, EO changes the fitness of all the species connected to the weakest one randomly. Thus, the fitness of the worst species and its neighbors will always change together, which can be considered a co-evolutionary activity. This co-evolutionary activity gives rise to chain reactions or "avalanches": large (non-equilibrium) fluctuations that rearrange major parts of the system, potentially making any configuration accessible. Large fluctuations allow the method to escape from local minima and explore the configuration space efficiently, while the extremal selection process enforces frequent returns to near-optimal solutions.

Third, the mechanism of EO can be studied from the perspective of ecosystem. An ecosystem is defined as a biological community of interacting organisms and their surrounding environment. That is to say, the fitness of any species living in an ecosystem will be affected by the fitness of any other species in the same ecosystem, whereas the change in the fitness of any species will affect the fitness landscape (i.e. environment) of the whole ecosystem. The interaction relationship between any two species in the ecosystem can be regarded as the inherent fundamental mechanism which drives all the species to co-evolving. The food chain may be one of the ways in which the interaction between any two species takes place. The food chain provides energy that all living things in the ecosystem must have in order to survive. In the food chain, there exit direct or intermediate connections between any species. According to natural selection or 'survival of the fittest' proposed by Darwin, those species with higher fitness will survive while those with lower fitness will die out. Thus, the species with the lower fitness will die out with larger probability than other species. When one species with lower fitness dies out, those species above the extinct species in the food chain will be also in threat of extinction, no matter how high the fitness value of them is. Similarly, EO considers those species with lower fitness more easily to die out than others. Hence EO always selects those "weakest" to update. The change in the fitness of the worst species will impact the fitness landscape of the whole system. At the same time, the fitness of those species connected to the weakest species will also be affected by the altered environment and be changed simultaneously.

IV. Applications of EO in Solving Discrete Optimization

EO and its variations have been successfully applied to solve some combinatorial optimization problems as follows.

A. Representative Applications

So far, most of the implementation of EO in solving discrete optimization is quite straightforward. Following will briefly review the EO implementations on those well-studied combinatorial optimization problems.

1) The graph bi-partitioning [9] is to partition a set of N points into two $N/2$ subsets, in which edges connects certain pairs of points. The optimization objective is to minimize the number of edges cutting across the partition (so called "cutsize"). From the local evaluation criteria of EO, the fitness of each point x_i is defined as $\lambda_i = b_i/2$, where b_i is the number of bad edges connecting x_i to the other subset. The next feasible solution is generated by exchanging the least fit point with a random point from the other subset. The results on a series of well-studied large graphs proved that EO can provide superior performance.

2) A spin glass [11] consists of a lattice or a graph with a spin variable $\sigma_i \in \{-1,1\}$ placed on each vertex $i, 1 \le i \le n$. Every spin is connected to each of its nearest neighbors j via a fixed bond variable J_{ij}, drawn at random from a distribution of zero mean and unit variance. Spins may be coupled to an arbitrary external field h_i. The optimization objective to find the minimum cost states $S_{\min}$. In the EO implementation, the energy function can be defined as $C(S) = \sum_{i=1}^{n} \lambda_i$, where λ_i represents the fitness for each spin variable σ_i.

3) The graph coloring problem K-COL [10] is to give K different colors to label the vertices of a graph, the objective is to find a coloring solution that minimizes the number of edges connecting vertices with identical color. The fitness for each vertex x_i is defined as $\lambda_i = b_i/2$, where b_i is the number of equally colored vertices connected to it.

In all the above implementations, EO drives the optimizing process through sequential changes on a single species. The cost $C(S)$ is assumed to consist of the individual cost contributions λ_i for each variable x_i, which correspond to the fitness values in the BS model. The fitness λ_i of variable x_i depends on its state in relation to other variables to which x_i is connected. Straightforwardly, EO can play its overwhelming advantages while the optimization objective $C(S) = \sum_{i=1}^{n} \lambda_i$.

So, the definition of fitness function and neighbor $N(S)$ are crucial to the implementation of EO for solving various combinatorial optimization problems. The latter have been widely studied and the former need further researches for the aspects of local evaluation.

B. EO Implementations on TSP

The TSP is probably the most famous combinatorial optimization problem [31]. It has been frequently used as a test bed for the study of optimization techniques.

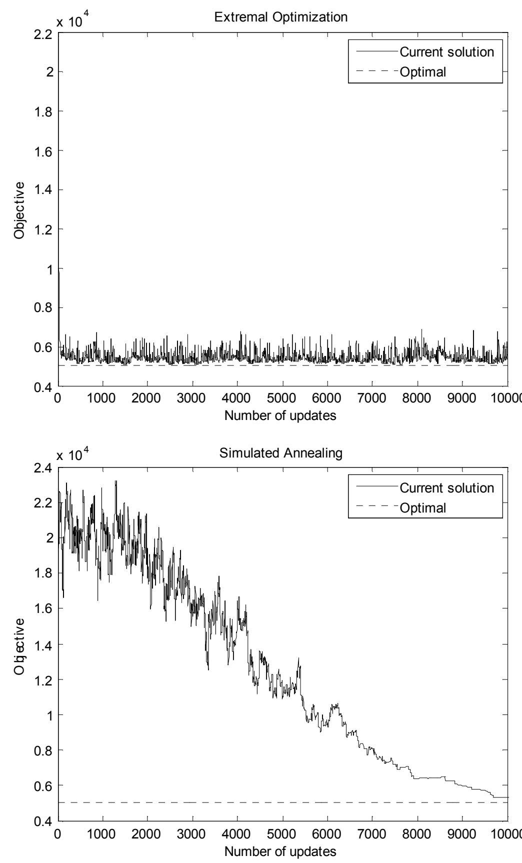


Fig. 2. Comparative computational results of EO and SA on TSP instance-gr48 from TSPLIB. The vertical axis stands for the objective values for the current generation.

In the TSP, N points ("cities") are given, and every pair of cities i and j is separated by a distance d_{ij}. The objective is to connect the cities using the shortest closed "tour", passing through each city exactly once.

Ideally, a city would want to be connected to its first and second nearest neighbor, but is often "frustrated" by the competition of other cities, causing it to be connected instead to (say) its pth and qth neighbors, Boettcher [9] defined the fitness of city i to be $\lambda_i = 3/p_i + q_i$. Compared the simulation results of EO to other methods, the author asserted that EO is not competitive for this problem. However, the key point is that the fitness function has no close relationship with the global optimization objective.

Alternatively, we can define the fitness of city i to be the potential optimized length, i.e. $\lambda_i = d_i - \min(d_{ij})$, where d_i is the length of edge starting from city i, So, the optimization objective $C(S) = \sum_{i=1}^{n} \lambda_i + \sum_{i=1}^{n} \min(d_{ij})$, and $\sum_{i=1}^{n} \min(d_{ij})$ is a constant for a given distance matrix. The comparative computation results on TSP instance-gr48 from TSPLIB [31] are illustrated in Fig.2.

In the simulation, the neighbor is the 2-*opt* solution space in both SA and EO. In contrast to SA, which has large fluctuations in early stages and then converges to a local optimum, EO quickly approaches an acceptable solution where broadly distributed fluctuations allow it to exploit the global optima.

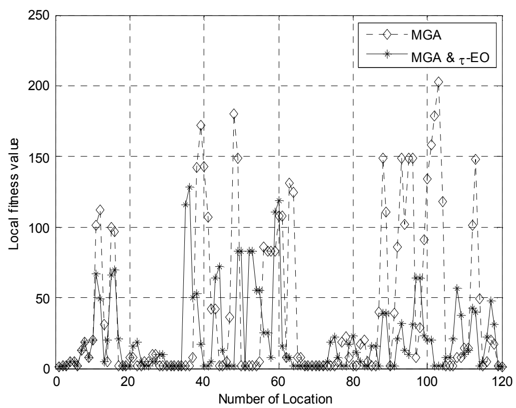


Fig. 3. Comparisons in the localized fitness between MGA and MGA - τ-EO

C. Industrial Application

Consider the EO implementation on a practical scheduling problem arisen from steel hot strip mill [15]. A set of manufacturing orders $N = \{1,2,\ldots,n\}$ is to be processed, and the sequence-dependent transition cost c_{ij} is incurred while processing order j immediately after order i ($i, j \in N$). The scheduling objective is to find an optimal production sequence $S^* = \{j_1, j_2, \ldots, j_m\}$ ($m \le n$) from the feasible solution space. The local fitness of each manufacturing orders are defined as $\lambda_i = c_{is(i)}$, where $c_{is(i)}$ is sequence-dependent transition cost between order i and its successor $s(i)$ in the production sequence. The move class is selecting an unscheduled order to replace the least fit order, which was usually employed by manual schedulers.

Through simulating the production scale data, Fig.3 shows the comparisons in the localized fitness between MGA and MGA - τ-EO, in which the scheduling solution of MGA is further optimized by τ-EO. τ-EO can significantly improve the scheduling solution inherited from modified GA (MGA) by successively eliminating a majority of undesirable variables [41].

V. Applications of EO in Solving Numerical Optimization Problems

Up to now, some variations of EO have been proposed to solve the numerical optimization problems. The Generalized Extremal Optimization (GEO) [16] was developed to operate on bit strings where the component quality is determined by the bits contribution to holistic solution quality. This work includes the applications to numerical optimization problems [16] as well as engineering problem domains [17]. Another extension to EO is the continuous extremal optimization (CEO) algorithm [18]. CEO consists of two components: one is the classical EO algorithm responsible for global searching and the other is a certain local searching algorithm. The effectiveness of CEO is demonstrated via solving the Lennard-Jones cluster optimization problem.

It is worth reminding that EO performs a search through sequential changes on a single individual, namely, the point-to-point search rather than the population based search applied in GAs. In order to accelerate the convergence speed, we developed a novel real-coded EO search algorithm, so-called Population-based Extremal Optimization (PEO) [19], through introducing the population search strategies being popularly used in evolutionary algorithms to EO. Similar to the evolutionary algorithms, the PEO operates on the evolution of chromosomes generation after generation. By uniformly placing the population of initial random solutions on the search space, PEO can explore the wide search space, avoiding getting trapped into local optima. On the other hand, similar to EO, the PEO performs only one operation, i.e. mutation, on each variable. In addition, we adopted the adaptive Lévy mutation operator, which makes our approach able to carry out not only coarse-grained but also fine-grained search. It is worth noting that there exists no adjustable parameter in our approach, which makes our approach more charming than other methods. Our approach was successfully applied in solving constrained numerical optimization and shows competitive performance in comparison with three state-of-the-art search methods, i.e., SR [20], ASCHEA [21] and SMES [22].

Recently, there have been some papers studying on the multiobjective optimization using extremal dynamics. Ahmed *et al.* [23] and Elettreby[36] introduced random version of BS model. They also generalized the single objective BS model to a multiobjective one by weighted sum of objectives method. The method is easy to implement but its most serious drawback is that it can not generate proper members of the Pareto-optimal set when the Pareto front is concave regardless of the weights used [24]. Galski *et al*. [25] applied the Generalized Extremal Optimization (GEO) algorithm to design a spacecraft thermal control system. The design procedure is tackled as a multiobjective optimization problem. They also resorted to the weighted sum of objectives method to solve the problem.

To extend EO to handle multiobjective problems in an effective and efficient way, we presented a novel algorithm, called Multiobjective Population-based Extremal Optimization (MOPEO) [26]. Being different from the aforementioned methods, the fitness assignment of MOPEO is based on the Pareto domination, which is popularly used by many existing multiobjective evolutionary algorithms.

For a multiobjective optimization problem, the flowchart of the MOPEO algorithm as shown in Fig.4 is developed in terms of the marriage of EO and evolutionary algorithms.

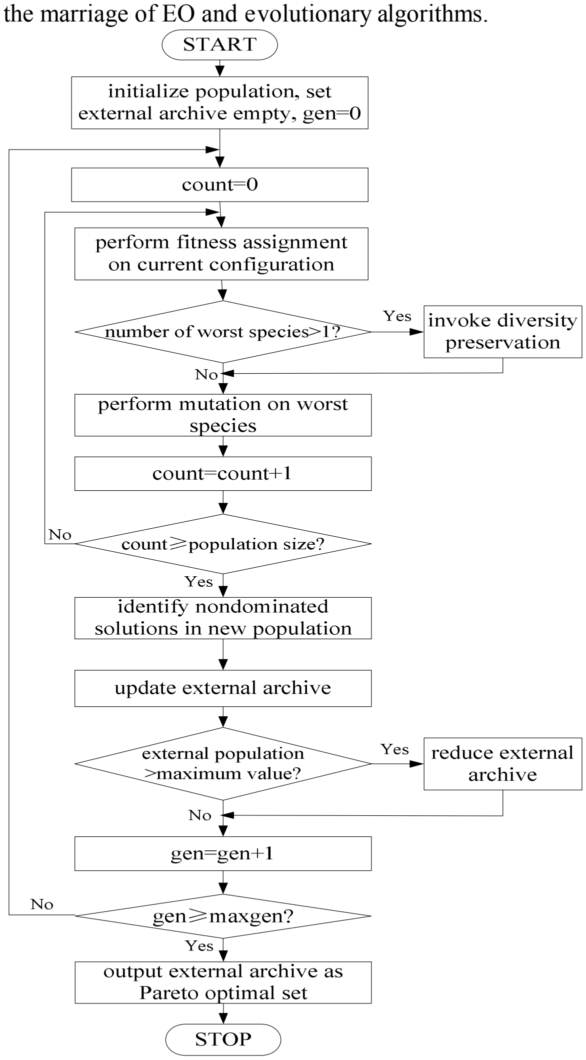
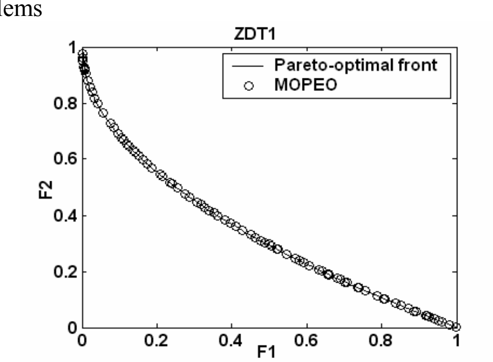
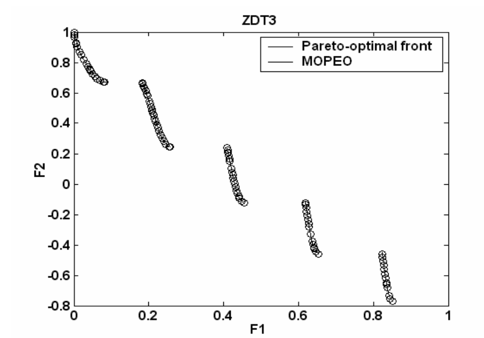


Fig. 4. Flowchart of MOPEO algorithm.

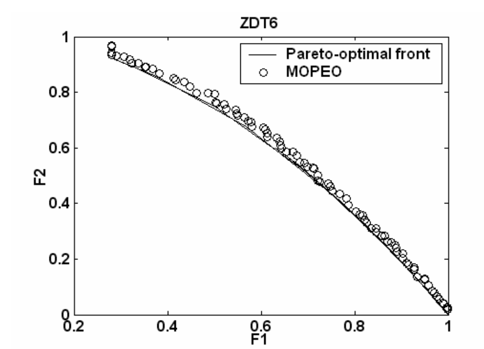
Our approach has been validated by three test functions ZDT1, ZDT3 and ZDT6 proposed by Zitzler *et al*. [27]. These problems are minimization problems with two objectives and described in Table I. Fig.5 shows one of ten runs with MOPEO on these problems. From Fig. 5, we can see that, with respect to convergence and diversity of the obtained set of nondominated solutions, MOPEO performs well in the three test problems. MOPEO is also compared against three highly competitive multiobjective evolutionary algorithms: the real-coded Nondominated Sorting Genetic Algorithm-II (NSGA-II) [28], the Strength Pareto Evolutionary Algorithm (SPEA) [29] and the Pareto Archived Evolution Strategy (PAES) [30]. The comparison results is shown in Table II, where we use two performance metrics proposed by Deb *et al*. [28] to assess the performance of our approach. The experimental results of NSGA-II, SPEA and PAES in Table II come from [28]. As can be seen from Table II, MOPEO outperforms any other algorithm in both aspect of convergence and diversity of nondominated solutions. In all the cases with MOPEO, the variance in ten runs is small. The simulation results illustrate that MOPEO is less susceptible to the shape or continuity of the Pareto front and has good performance in both aspects of convergence and distribution of solutions. Hence, MOPEO may be a good alternative to solve the multiobjective optimization problems



(a)



(b)



(c)

Fig. 5 (a)~(c) show the nondominated solutions with MOPEO on problems ZDT1, ZDT3 and ZDT6, respectively. The horizontal and vertical axes stand for the values of two objective functions, respectively.

TABLE I
TEST PROBLEMS

Problem	n	Variable bounds	Objective functions	Optimal solutions	Comments
ZDT1	30	[0,1]	$f_1(X)=x_1$ $f_2(X)=g(X)[1-\sqrt{x_1/g(X)}]$ $g(X)=1+9(\sum_{i=2}^{n}x_i)/(n-1)$	$x_1\in[0,1]$ $x_i=0,$ $i=2,\cdots,n$	convex
ZDT3	30	[0,1]	$f_1(X)=x_1$ $f_2(X)=g(X)[1-\sqrt{x_1/g(X)}-\frac{x_1}{g(X)}\sin(10\pi x_1)]$ $g(X)=1+9(\sum_{i=2}^{n}x_i)/(n-1)$	$x_1\in[0,1]$ $x_i=0,$ $i=2,\cdots,n$	convex, disconnected
ZDT6	10	[0,1]	$f_1(X)=1-\exp(-4x_1)\sin^6(6\pi x_1)$ $f_2(X)=g(X)[1-(f_1(X)/g(X))^2]$ $g(X)=1+9[(\sum_{i=2}^{n}x_i)/(n-1)]^{0.25}$	$x_1\in[0,1]$ $x_i=0,$ $i=2,\cdots,n$	nonconvex, nonuniformly spaced

TABLE II
MEAN (FIRST ROWS) AND VARIANCE (SECOND ROWS) OF THE TWO PERFORMANCE METRICS

Algorithm	convergence metric Υ			diversity metric Δ		
	ZDT1	ZDT3	ZDT6	ZDT1	ZDT3	ZDT6
MOPEO	**0.001062**	**0.004175**	**0.013400**	**1.24E-05**	**0.249489**	**0.047618**
	7.53E-05	0.012972	0.020698	1.18E-04	0.036808	0.123937
NSGA- II	0.033482	0.114500	0.296564	0.390307	0.738540	0.668025
	0.004750	0.007940	0.013135	0.001876	0.019706	0.009923
SPEA	0.001799	0.047517	0.221138	0.784525	0.672938	0.849389
	0.000001	0.000047	0.000449	0.004440	0.003587	0.002713
PAES	0.082085	0.023872	0.085469	1.229794	0.789920	1.153052
	0.008679	0.00001	0.006664	0.004839	0.001653	0.003916

VI. ADVANTAGES AND DISADVANTAGES OF EO

From the above descriptions, it can be seen that EO has the following advantages:

-- The co-evolutionary mechanism makes EO able to find the near-optimal solutions quickly.

-- The extremal driving mechanism generates long-term memory for EO. Most species preserve a good fitness for long times unless they are connected to poorly adapted species, providing the system with a long memory.

-- The system retains a potential for large, hill-climbing fluctuations at any stage. The large fluctuations make any configuration accessible.

-- There is no any control parameters in EO except in τ -EO.

-- There exists only mutation operation. This makes EO simple and convenient to be implemented.

Every coin has two sides. There also exist some shortcomings in EO. For example, it is hard to give an appropriate definition to the fitness in many specific cases.

VII. CONCLUDING REMARKS

In this paper, we make studies on the fundamental of EO and its applications in solving the real world optimization problems. First, we explore the mechanism of EO from three points of view: statistical physics, biological evolution and ecosystem.

In this work, we also introduce our contributions to the applications of EO in discrete and numerical optimization problems.

On the aspect of discrete optimization, the definition of local evaluation fitness function and neighbor $N(S)$ is especially important for EO to solve the discrete optimization problem. Through an effective implementation on TSP and a scheduling problem, we illuminate that the optimization method inspired by extremal dynamics has great potentials to solve combinatorial optimization problems.

On the aspect of numerical optimization, we proposed two novel real-coded algorithms to solve numerical constrained optimization and multiobjective optimization problems, respectively. One is the PEO algorithm, which is able to solve numerical constrained optimization problem, through introducing population search strategy associated with adaptive Lévy mutation to EO. The experimental results demonstrate that PEO is competitive in comparison with three state-of-the-art algorithms. The other is MOPEO, in which the fitness assignment is based on the Pareto domination and the population search strategy is also introduced to EO. The proposed approach has been validated by five benchmark functions and compared with three highly competitive algorithms. The simulation results show that MOPEO has good performance in both aspects of convergence and diversity of nondominated solutions. Thus MOPEO can be considered as a good alternative to handle multiobjective optimization problems.

There are many advantages of EO such as extremal dynamics mechanism, co-evolution, only mutation operator and long-term memory. Thus, EO can be considered a good heuristic method which is competitive in comparison with many state-of-the-art heuristics.

More fundamental research on EO dynamics, dealing with the optimization problem with phase transitions and real-world applications is the major future work for EO optimization.

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