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Conflict Analysis Based on Discernibility and Indiscernibility

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***Abstract*— The dual notions of discernibility and indiscernibility play an important role in intelligent data analysis. While discernibility focuses on the differences, the indiscernibility reveals the similarities. By considering them together in a same framework, one is able to obtain new insight of data.**

The main objective of the paper is to apply discernibility and indiscernibility to conflict analysis, a theory dealing with opinions of a set of agents on a set of issues. In particular, we are interested in the problem of issue reduction, so that a reduced set of issues can be obtained without loss of crucial information of the original set of issues. Extending the results from rough set theory, three types of issue reducts are introduced. They correspond to discernibility, indiscernibility, and discernibility-and-indiscernibility reducts, respectively. The results of this paper may offer a new research direction in rough set analysis in general, and conflict analysis in particular.

I. Introduction

The dual notions of similarities and differences play a crucial role in many fields such as concept formation, machine learning, data mining, data analysis, cluster analysis, and many more [1], [13], [17], [18], [19]. The similarities of objects lead naturally to their grouping and integration, and the differences lead to group division and decomposition. It is important to extract similarity of objects by ignoring certain differences in order to form a useful cluster or a high level concept. It is also important to identify differences among a set of similar objects in order to form sub-concepts. The study of similarity and difference can find many real-life applications. For example, in social science or politics, one can emphasize the differences between entities and thus virtually enlarge and aggravate the conflicts and discordance. On the other hand, we can emphasize the commonness between entities, and therefore create a concordant atmosphere for negotiation and communication [2], [6], [8], [10], [11], [14], [15].

The theory of rough sets, as a theory of data analysis, models similarities and differences of objects based on the notions of indiscernibility and discernibility. There are two fundamental issues: representations of indiscernibility and discernibility, and attribute reduction (information table simplification) based on indiscernibility and discernibility. We suggest that based on indiscernibility and discernibility, rough-set based data analysis can be unified into one model. Furthermore, three different kinds of reducts based on indiscernibility and discernibility can be explored. They are the family of indiscernibility reducts, the family of discernibility reducts and the family of indiscernibility-and-discernibility reducts. Each is a minimum attribute set that preserves the indiscernibility relations, the discernibility relations, and both the indiscernibility and the discernibility relations, respectively. The paper applies these results to conflict analysis.

II. Conflicts and Information Tables

A conflict is involved by at least two parties, called *agents*, who are dispute over some *issues* [10]. In general, the agent may be interpreted as individuals, groups, companies, states, and political parties. The relationship of each agent to all the issues can be clearly depicted in the form of an information table. Information tables, also known as information systems, data tables, attribute-value systems, are investigated by many researchers of the rough set theory [4], [7], [9]. It is assumed that data are represented in a table form, where a set of objects (rows) are described by a finite set of attributes (columns).

Definition 1: An information table S is the tuple

$$S = (U, At, \{V_a \mid a \in At\}, \{I_a \mid a \in At\}),$$

where U is a finite nonempty set of objects called universe, At is a finite nonempty set of attributes, and V_a is a nonempty set of values for an attribute $a \in At$. $I_a : U \longrightarrow V_a$ is an information function, such that for an object $x \in U$, an attribute $a \in At$, and a value $v \in V_a$, $I_a(x) = v$ means that the object x has the value v on the attribute a.

The above definition is general. For conflict analysis we will need its simplified version, where the domain of each attribute is restricted to three values only. That is, for all $a \in At$, $V_a = \{1, 0, -1\}$. For an agent (object) $x \in U$, $I_a(x) = 1$ means that the agent x is in favour to the issue (attribute) a, $I_a(x) = -1$ means x is against to a, and $I_a(x) = 0$ means x is neutral toward a. In the rest of the paper, we suppose that these three assessments are conclusive, and we use object and agent, attribute and issue, interchangeably.

Example 1: The information Table I, cited from Pawlak [10], contains six nations (rows) of the Middle East region to five issues (columns).

The following auxiliary function expresses relations between any two agents $x, y \in U$ [10]:

$$R_a(x,y) = \begin{cases} 1, & \text{if } I_a(x)I_a(y) = 1, \\ 0, & \text{if } I_a(x)I_a(y) = 0, \\ -1, & \text{if } I_a(x)I_a(y) = -1. \end{cases}$$

TABLE I
AN INFORMATION TABLE FOR THE MIDDLE EAST CONFLICT

	At				
U	a	b	c	d	e
o_1	0	1	1	1	1
o_2	1	0	-1	-1	-1
o_3	1	-1	-1	-1	0
o_4	0	-1	-1	0	-1
o_5	1	-1	-1	-1	-1
o_6	0	1	-1	0	1

The three different values of the function $R_a(x, y)$ reveal the *alliance*, *neutrality* and *conflict* relations between two agents x and y regarding the issue a, respectively. The auxiliary function can be extended to an attribute set. It is important to note that the neutrality relation stands for neither an alliance nor a conflict relation. If one agent is, or two agents are both, neutral towards one issue, then two agents are neither allied nor conflicting.

An approach for conflict analysis is based on indiscernibility and discernibility. If two agents agree on one issue then they are indiscernible regarding this issue, otherwise, they are discernible. In this case, two agents are indiscernible if and only if they both agree, disagree or neutral towards a set of issues.

The proposed approach can be extended for two different interpretations. One interpretation is alliance-oriented. If two agents are allied on a set of issues, then they are considered indiscernible regarding this issue set; otherwise, they are discernible. In other words, two agents are indiscernible if and only if two agents both agree or both disagree on the set of issues. For the other situations, such as one agrees and one disagrees, one agrees and one is neutral, one disagrees and one is neutral, and both are neutral, they are discernible and non-allied. The other interpretation is conflict-oriented. That is, if two agents are conflicting on a set of issues, then they are considered discernible; otherwise, they are indiscernible. By this view, two agents conflict only if and only if they explicitly state confliction on issues, otherwise, they are indiscernible and non-conflicting. By putting the neutrality relation on either the discernible side (the alliance-oriented interpretation) or the indiscernible side (the conflict-oriented interpretation), we reduce conflict analysis to a binary case. The following three sections explore the proposed approach and two oriented interpretations in detail.

III. AN APPROACH BASED ON INDISCERNIBILITY AND DISCERNIBILITY

A. *Indiscernibility and discernibility relations*

Given a subset of attributes $A \subseteq At$, four binary relations between objects can be differentiated in an information table.

Definition 2: Given a subset of attributes $A \subseteq At$, four relations on U are defined by:

$$\begin{aligned}
\mathrm{IND}(A) &= \{(x,y) \in U \times U \mid \forall a \in A, I_a(x) = I_a(y)\},\\
\mathrm{WIND}(A) &= \{(x,y) \in U \times U \mid \exists a \in A, I_a(x) = I_a(y)\},\\
\mathrm{DIS}(A) &= \{(x,y) \in U \times U \mid \forall a \in A, I_a(x) \neq I_a(y)\},\\
\mathrm{WDIS}(A) &= \{(x,y) \in U \times U \mid \exists a \in A, I_a(x) \neq I_a(y)\}.
\end{aligned}$$

A strong indiscernibility relation with respect to A is denoted as $\mathrm{IND}(A)$. Two objects in U satisfy $\mathrm{IND}(A)$ if and only if they have the same values on all attributes in A. An indiscernibility relation is reflexive, symmetric and transitive, namely, it is an equivalence relation. On the other extreme, a weak indiscernibility relation $\mathrm{WIND}(A)$ with respect to A only requires that two objects have the same value on at least one attribute in A. A weak indiscernibility relation is reflexive, symmetric, but not necessarily transitive. Such a relation is known as a compatibility or a tolerance relation [3], [12], [18], [19]. The two types of relations are studied in the rough set theory for different types of approximation spaces.

As the complement of a strong indiscernibility relation, a weak discernibility relation $\mathrm{WDIS}(A)$ states that two objects are discernible if and only if they have different values on at least one attribute in A. A weak discernibility relation is irreflexive and symmetric, but not transitive. The complement of a weak indiscernibility relation is a strong discernibility relation $\mathrm{DIS}(A)$. $\mathrm{DIS}(A)$ states that two objects are strongly discernible with respect to A if they have different values on all attributes in A. A strong discernibility relation is irreflexive and symmetric, but not transitive. The strong and weak discernibility relations are also called as the strong and weak diversity relations [5].

The weak and strong versions are related by a subset relationship. This is, if two objects are strongly indiscernible/discernible, then they are weakly indiscernible/discernible. An indiscernibility relation is a subset of the weak indiscernibility relation defined by the same attribute set. Similarly, a discernibility relation is a subset of the weak discernibility relation defined by the same attribute set.

(SW1). $\mathrm{IND}(A) \subseteq \mathrm{WIND}(A)$;
(SW2). $\mathrm{DIS}(A) \subseteq \mathrm{WDIS}(A)$.

The indiscernibility and discernibility relations are related by a complementary relationship. That is, we have two pairs of complementary relations, the pair $(\mathrm{IND}(A), \mathrm{WDIS}(A))$ of the strong indiscernibility relation and the weak discernibility relation, and the pair $(\mathrm{DIS}(A), \mathrm{WIND}(A))$ of the strong discernibility relation and the weak indiscernibility relation.

(C1). $\mathrm{WDIS}(A) = \mathrm{IND}^c(A)$;
(C2). $\mathrm{WIND}(A) = \mathrm{DIS}^c(A)$.

Example 2: Examples of the strong and weak indiscernibility/discernibility relations defined by the three attribute sets are given in Figure 1, where $\bullet$ means that the two corresponding objects are related according to the relation.

Given an information table, we can obtain both indiscernibility relations and discernibility relations with respect to a subset of attributes. On the other hand, given all the indiscernibility relations and discernibility relations, we cannot recover

IND($\{a\}$)= WIND($\{a\}$)	o_1	o_2	o_3	o_4	o_5	o_6
o_1	•			•		•
o_2		•	•		•	
o_3		•	•		•	
o_4	•			•		•
o_5		•	•		•	
o_6	•			•		•

DIS($\{a\}$)= WDIS($\{a\}$)	o_1	o_2	o_3	o_4	o_5	o_6
o_1		•	•		•	
o_2	•			•		•
o_3	•			•		•
o_4		•	•		•	
o_5	•			•		•
o_6		•	•		•	

IND ($\{b\}$)= WIND($\{b\}$)	o_1	o_2	o_3	o_4	o_5	o_6
o_1	•					•
o_2		•				
o_3			•	•	•	
o_4			•	•	•	
o_5			•	•	•	
o_6	•					•

DIS($\{b\}$)= WDIS($\{b\}$)	o_1	o_2	o_3	o_4	o_5	o_6
o_1		•	•	•	•	
o_2	•		•	•	•	•
o_3	•	•				•
o_4	•	•				•
o_5	•	•				•
o_6		•	•	•	•	

IND($\{a,b\}$)	o_1	o_2	o_3	o_4	o_5	o_6
o_1	•					•
o_2		•				
o_3			•		•	
o_4				•		
o_5			•		•	
o_6	•					•

DIS($\{a,b\}$)	o_1	o_2	o_3	o_4	o_5	o_6
o_1		•	•		•	
o_2	•			•		•
o_3	•					•
o_4		•				
o_5	•					•
o_6		•	•		•	

WIND($\{a,b\}$)	o_1	o_2	o_3	o_4	o_5	o_6
o_1	•			•		•
o_2		•	•		•	
o_3		•	•	•	•	
o_4	•		•	•	•	•
o_5		•	•	•	•	
o_6	•			•		•

WDIS($\{a,b\}$)	o_1	o_2	o_3	o_4	o_5	o_6
o_1		•	•	•	•	
o_2	•		•	•	•	•
o_3	•	•		•		•
o_4	•	•	•		•	•
o_5	•	•		•		•
o_6		•	•	•	•	

Fig. 1. Examples of the indiscernibility and discernibility relations

the original information table. A relation only tells whether two objects are discernible or indiscernible with respect to the attribute set, but does not keep the attribute values.

B. Indiscernibility and discernibility matrices

The relationships between objects, i.e., the family of relations, can be alternatively expressed as matrices. Each cell of an indiscernibility matrix stores those attributes that are shared by any two objects of the universe.

Definition 3: Given an information table S, its indiscernibility matrix $\mathbf{im}$ is a $|U| \times |U|$ matrix with each element $\mathbf{im}(x, y)$ defined as:

$$\mathbf{im}(x, y) = \{a \in At \mid I_a(x) = I_a(y), x, y \in U\}.$$

The indiscernibility matrix $\mathbf{im}(x, y)$ is symmetric, i.e., $\mathbf{im}(x, y) = \mathbf{im}(y, x)$, and $\mathbf{im}(x, x) = At$.

In contrast to an indiscernibility matrix, each element of a discernibility matrix stores those attributes on which the corresponding two objects have distinct values [16].

Definition 4: Given an information table S, its discernibility matrix $\mathbf{dm}$ is a $|U| \times |U|$ matrix with each element $\mathbf{dm}(x, y)$ defined as:

$$\mathbf{dm}(x, y) = \{a \in At \mid I_a(x) \neq I_a(y), x, y \in U\}.$$

TABLE II

THE INDISCERNIBILITY MATRIX OF THE INFORMATION TABLE I.

	o_1	o_2	o_3	o_4	o_5	o_6
o_1	At	$\emptyset$	$\emptyset$	a	$\emptyset$	abe
o_2		At	acd	ce	$acde$	c
o_3			At	bc	$abcd$	c
o_4				At	bce	acd
o_5					At	c
o_6						At

TABLE III

THE DISCERNIBILITY MATRIX OF THE INFORMATION TABLE I.

	o_1	o_2	o_3	o_4	o_5	o_6
o_1	$\emptyset$	At	At	$bcde$	At	cd
o_2		$\emptyset$	be	abd	b	$abde$
o_3			$\emptyset$	ade	e	$abde$
o_4				$\emptyset$	ad	be
o_5					$\emptyset$	$abde$
o_6						$\emptyset$

The discernibility matrix $\mathbf{dm}$ is symmetric, i.e., $\mathbf{dm}(x, y) = \mathbf{dm}(y, x)$, and $\mathbf{dm}(x, x) = \emptyset$. By definition, the two matrices are complementary, i.e., for any $x, y \in U$, $\mathbf{im}(x, y) = (\mathbf{dm}(x, y))^c = At - \mathbf{dm}(x, y)$.

Similar to the indiscernibility and discernibility relations, given an information table, we can obtain its indiscernibility matrix and discernibility matrix. Conversely, given an indiscernibility matrix or a discernibility matrix, we cannot recover its information table. Each element of the matrix keeps only the names of attributes whose values are the same, or different regarding the involved two objects, but not the values of those attributes.

There is a close connection between a matrix and its corresponding strong and weak relations. From an indiscernibility or a discernibility matrix, we can easily obtain the indiscernibility and discernibility relations defined by any subset of attributes. On the other hand, from the family of all the strong indiscernibility or the strong discernibility relations defined by singleton subsets, we obtain the indiscernibility or the discernibility matrix.

Example 3: The indiscernibility and discernibility matrix of the information Table I are given in Tables II and III, where each matrix element is a set of attributes that are shared by and differentiate a pair of objects, respectively. In the tables, for simplicity, we write a set of attributes, for example, $\{a, b, c\}$ as abc. Since both matrices are symmetric, we only list the elements in the upper right half.

C. Attribute reduction based on indiscernibility and discernibility

Definition 5: For an information table $S = (U, At, \{V_a\}, \{I_a\})$, an attribute set $P \subseteq At$ is a reduct if it meets following two conditions:

(1.) $\mathcal{R}(P) = \mathcal{R}(At)$;

(2.) For any proper subset $P' \subset P$, $\mathcal{R}(P') \neq \mathcal{R}(At)$,

The first condition ensures that a certain property is preserved by a set of attributes. Therefore, the attribute set as a whole is sufficient for preserving the property. The second condition ensures the constructed attribute set is the minimum, i.e., each attribute of it is necessary for reserving the property.

When $\mathcal{R}$ represents IND and DIS, the corresponding reducts are called indiscernibility reducts and discernibility reducts, respectively. For an information table $S = (U, At, \{V_a\}, \{I_a\})$, an attribute set $P \subseteq At$ is an indiscernibility-and-discernibility reduct if it meets following three conditions:

(1.) $\text{IND}(P) = \text{IND}(At)$;
(2.) $\text{DIS}(P) = \text{DIS}(At)$;
(3.) For any proper subset $P' \subset P$, either $\text{IND}(P') \neq \text{IND}(At)$ or $\text{DIS}(P') \neq \text{DIS}(At)$.

An indiscernibility-and-discernibility reduct is a minimum attribute set that keeps both the indiscernibility and the discernibility relations of the original table. The family of all indiscernibility reducts of the information table S is denoted as $RED_{\text{IND}}(S)$, and the family of all discernibility reducts as $RED_{\text{DIS}}(S)$. The intersection of all indiscernibility reducts is called the IND core, and the intersection of all discernibility reducts is called the DIS core.

Obviously, the three reducts are defined by the strong relations. Alternatively, reducts can be defined by the weak relation counterparts and corresponding matrices. The following functions define for two matrices reveal and correlate the weak relations of discernibility and indiscernibility, respectively.

Skowron and Rauszer [16] define a *discernibility function* for a discernibility matrix $\mathbf{dm}$:

$$f_{\text{DIS}}(\mathbf{dm}) = \bigwedge\{\bigvee \mathbf{dm}(x,y) : x, y \in U,\ \mathbf{dm}(x,y) \neq \emptyset\},$$

where $\bigvee \mathbf{dm}(x,y)$ represents the logical disjunction of those attributes in an element $\mathbf{dm}(x,y)$, which means that x and y are discernible regarding any attribute in $\mathbf{dm}(x,y)$. A discernibility function is the conjunction of all logical disjunction of matrix elements. That means, it keeps all the weak discernibility relations in the given universe.

Similar to the discernibility function, we can define an *indiscernibility function* for an indiscernibility matrix $\mathbf{im}$:

$$f_{\text{IND}}(\mathbf{im}) = \bigwedge\{\bigvee \mathbf{im}(x,y) : x, y \in U,\ \mathbf{im}(x,y) \neq At\},$$

where $\bigvee \mathbf{im}(x,y)$ is the logical disjunction of those attributes in an element $\mathbf{im}(x,y)$, which means that x and y are indiscernible regarding any attribute in $\mathbf{im}(x,y)$. An indiscernibility function is the conjunction of all logical disjunction of matrix elements. Thus, it keeps all the weak indiscernibility relations in the given universe.

According to the definition of indiscernibility-and-discernibility reducts, we can define an *indiscernibility-and-discernibility function* as follows:

$$f_{\text{IND}-\text{DIS}}(\mathbf{im}-\mathbf{dm}) = f_{\text{IND}}(\mathbf{im}) \bigwedge f_{\text{DIS}}(\mathbf{dm}).$$

The significance of the matrix functions are that an indiscernibility reduct is a disjunct of the matrix function $f_{\text{DIS}}(\mathbf{dm})$ in a reduced disjunctive form. And a discernibility reduct is a disjunct of the matrix function $f_{\text{IND}}(\mathbf{im})$ in a reduced disjunctive form. Namely, we have the following equivalences:

$$\begin{aligned} &P \in RED_{\text{IND}}(S) \\ &\qquad \text{iff} \quad \bigwedge P \text{ is a prime implicant of } f_{\text{DIS}}(\mathbf{dm}); \\ &P \in RED_{\text{DIS}}(S) \\ &\qquad \text{iff} \quad \bigwedge P \text{ is a prime implicant of } f_{\text{IND}}(\mathbf{im}). \end{aligned}$$

Example 4: Based on the indiscernibility matrix in Table II, we can construct the following indiscernibility function:

$$\begin{aligned} f_{\text{IND}}\ (\mathbf{im}) &= a \wedge c \wedge (b \vee c) \wedge (c \vee e) \wedge (a \vee b \vee e) \\ &\wedge (a \vee c \vee d) \wedge (b \vee c \vee e) \wedge (a \vee b \vee c \vee d) \\ &\wedge (a \vee c \vee d \vee e) \\ = &\quad (a \wedge c). \end{aligned}$$

Based on the discernibility matrix in Table III, we can construct the following discernibility function:

$$\begin{aligned} f_{\text{DIS}}\ (\mathbf{dm}) &= b \wedge e \wedge (a \vee d) \wedge (b \vee e) \wedge (c \vee d) \wedge (a \vee b \vee d) \\ &\wedge (a \vee d \vee e) \wedge (a \vee b \vee d \vee e) \wedge (b \vee c \vee d \vee e) \\ &\wedge (a \vee b \vee c \vee d \vee e) \\ = &\quad b \wedge e \wedge (a \vee d) \wedge (c \vee d) \\ = &\quad (b \wedge d \wedge e) \vee (a \wedge b \wedge c \wedge e). \end{aligned}$$

It means that we obtain one indiscernibility reduct ($\{a, c\}$) and two discernibility reducts ($\{b, d, e\}$ and $\{a, b, c, e\}$). Based on these results,

$$\begin{aligned} &f_{\text{IND}-\text{DIS}}(\mathbf{im}-\mathbf{dm}) = f_{\text{IND}}(\mathbf{im}) \bigwedge f_{\text{DIS}}(\mathbf{dm}) \\ = &\quad (a \wedge c) \bigwedge ((b \wedge d \wedge e) \vee (a \wedge b \wedge c \wedge e)) \\ = &\quad (a \wedge b \wedge c \wedge e). \end{aligned}$$

The result shows that we can obtain one indiscernibility-and-discernibility reduct, i.e., $\{a, b, c, e\}$.

IV. Alliance-oriented Interpretation

A. Alliance and non-alliance relations and matrices

Given a subset of attributes $A \subseteq At$, four alliance-oriented relations between objects can be defined in an information table.

Definition 6: Given a subset of attributes $A \subseteq At$, four alliance-oriented relations on U are defined by:

$$\begin{aligned} \text{RA}(A) &= \{(x,y) \in U \times U \mid \forall a \in A, R_a(x,y) = 1\}, \\ \text{WRA}(A) &= \{(x,y) \in U \times U \mid \exists a \in A, R_a(x,y) = 1\}, \\ \text{NA}(A) &= \{(x,y) \in U \times U \mid \forall a \in A, R_a(x,y) \neq 1\}, \\ \text{WNA}(A) &= \{(x,y) \in U \times U \mid \exists a \in A, R_a(x,y) \neq 1\}. \end{aligned}$$

The weak and strong versions are related by a subset relationship, and the alliance and non-alliance relations are related by a complementary relationship.

Definition 7: Given an information table S, its alliance matrix $\mathbf{ma}$ is a $|U| \times |U|$ matrix with each element $\mathbf{ma}(x,y)$ defined as:

$$\mathbf{ma}(x,y) = \{a \in At \mid R_a(x,y) = 1, x, y \in U\}.$$

And its non-alliance matrix $\mathbf{mna}$ is a $|U| \times |U|$ matrix with each element $\mathbf{mna}(x,y)$ defined as:

$$\mathbf{mna}(x,y) = \{a \in At \mid R_a(x,y) \neq 1, x, y \in U\}.$$

TABLE IV

THE ALLIANCE MATRIX OF THE INFORMATION TABLE I.

	o_1	o_2	o_3	o_4	o_5	o_6
o_1	At	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	be
o_2		At	acd	ce	$acde$	c
o_3			At	bc	$abcd$	c
o_4				At	bce	c
o_5					At	c
o_6						At

TABLE V

THE NON-ALLIANCE MATRIX OF THE INFORMATION TABLE I.

	o_1	o_2	o_3	o_4	o_5	o_6
o_1	$\emptyset$	At	At	At	At	acd
o_2		$\emptyset$	be	abd	b	$abde$
o_3			$\emptyset$	ade	e	$abde$
o_4				$\emptyset$	ad	$abde$
o_5					$\emptyset$	$abde$
o_6						$\emptyset$

Both the alliance and the non-alliance matrices are symmetric.

Example 5: The alliance and the non-alliance matrices of the information Table I are given in Tables IV and V.

B. Alliance-oriented attribute reduction

According to the general Definition 5 of reduct, when $\mathcal{R}$ represents RA and NA, the corresponding reducts are called alliance reducts and non-alliance reducts, respectively. We can define the alliance-and-non-alliance reducts which are the minimum attribute sets that keep both the alliance relation and non-alliance relation at the same time. The family of all alliance reducts of the information table S is denoted as $RED_{\mathrm{RA}}(S)$, and the family of all non-alliance reducts as $RED_{\mathrm{NA}}(S)$. The intersection of all alliance reducts is called the RA core, and the intersection of all non-alliance reducts is called the NA core.

An *alliance function*, a *non-alliance function* and an *alliance-and-non-alliance function* can be defined as follows:

$$f_{\mathrm{RA}}(\mathbf{ma}) = \bigwedge\{\bigvee \mathbf{ma}(x,y) : x,y \in U,\ \mathbf{ma}(x,y) \neq At\},$$
$$f_{\mathrm{NA}}(\mathbf{mna}) = \bigwedge\{\bigvee \mathbf{mna}(x,y) : x,y \in U,\ \mathbf{mna}(x,y) \neq \emptyset\},$$
$$f_{\mathrm{RA-NA}}(\mathbf{ma}-\mathbf{mna}) = f_{\mathrm{RA}}(\mathbf{ma}) \bigwedge f_{\mathrm{NA}}(\mathbf{mna}).$$

We have $P \in RED_{\mathrm{RA}}(S)$ if and only if $\bigwedge P$ is a prime implicant of $f_{\mathrm{NA}}(\mathbf{mna})$, and $P \in RED_{\mathrm{NA}}(S)$ if and only if $\bigwedge P$ is a prime implicant of $f_{\mathrm{RA}}(\mathbf{ma})$.

Example 6: Based on the alliance matrix in Table IV, we can construct the following alliance function:

$$\begin{aligned} f_{\mathrm{RA}}\ (\mathbf{ma}) &= c \wedge (b\vee c) \wedge (b \vee e) \wedge (c \vee e) \wedge (a \vee c \vee d) \\ &\quad \wedge (b \vee c \vee e) \wedge (a \vee b \vee c \vee d) \wedge (a \vee c \vee d \vee e) \\ &= c \wedge (b \vee e) \\ &= (b \wedge c) \vee (c \wedge e). \end{aligned}$$

Based on the non-alliance matrix in Table V, we can construct the following non-alliance function:

$$\begin{aligned} f_{\mathrm{NA}}\ (\mathbf{mna}) &= b \wedge e \wedge (a \vee d) \wedge (b \vee e) \wedge (a \vee b \vee d) \\ &\quad \wedge (a \vee c \vee d) \wedge (a \vee d \vee e) \wedge (a \vee b \vee d \vee e) \\ &\quad \wedge (a \vee b \vee c \vee d \vee e) \\ &= b \wedge e \wedge (a \vee d) \\ &= (a \wedge b \wedge e) \vee (b \wedge d \wedge e). \end{aligned}$$

That means that we obtain two alliance reducts ($\{b,c\}$ and $\{c,e\}$) and two non-alliance reducts ($\{a,b,e\}$ and $\{b,d,e\}$). Based on these results,

$$\begin{aligned} & f_{\mathrm{RA-NA}}(\mathbf{ma}-\mathbf{mna}) = f_{\mathrm{RA}}(\mathbf{ma}) \bigwedge f_{\mathrm{NA}}(\mathbf{mna}) \\ = \ & ((b \wedge c) \vee (c \wedge e)) \bigwedge ((a \wedge b \wedge e) \vee (b \wedge d \wedge e)) \\ = \ & (a \wedge b \wedge c \wedge e) \vee (b \wedge c \wedge d \wedge e). \end{aligned}$$

The result shows that we can obtain two alliance-and-non-alliance reducts, i.e., $\{a,b,c,e\}$ and $\{b,c,d,e\}$.

V. CONFLICT-ORIENTED INTERPRETATION

A. Conflict and non-conflict relations and matrices

Given a subset of attributes $A \subseteq At$, four conflict-oriented relations between objects can be defined in an information table.

Definition 8: Given a subset of attributes $A \subseteq At$, four conflict-oriented relations on U are defined by:

$$\begin{aligned} \mathrm{RC}(A) &= \{(x,y) \in U \times U \mid \forall a \in A, R_a(x,y) = -1\}, \\ \mathrm{WRC}(A) &= \{(x,y) \in U \times U \mid \exists a \in A, R_a(x,y) = -1\}, \\ \mathrm{NC}(A) &= \{(x,y) \in U \times U \mid \forall a \in A, R_a(x,y) \neq -1\}, \\ \mathrm{WNC}(A) &= \{(x,y) \in U \times U \mid \exists a \in A, R_a(x,y) \neq -1\}. \end{aligned}$$

The weak and strong versions are related by a subset relationship, and the conflict and non-conflict relations are related by a complementary relationship.

Definition 9: Given an information table S, its conflict matrix $\mathbf{mc}$ is a $|U| \times |U|$ matrix with each element $\mathbf{mc}(x,y)$ defined as:

$$\mathbf{mc}(x,y) = \{a \in At \mid R_a(x,y) = -1, x,y \in U\}.$$

And its non-conflict matrix $\mathbf{mnc}$ is a $|U| \times |U|$ matrix with each element $\mathbf{mnc}(x,y)$ defined as:

$$\mathbf{mnc}(x,y) = \{a \in At \mid R_a(x,y) \neq -1, x,y \in U\}.$$

Both the conflict and the non-conflict matrices are symmetric.

Example 7: The conflict and the non-conflict matrices of the information Table I are given in Tables VI and VII.

B. Conflict-oriented attribute reduction

According to the general Definition 5 of reduct, when $\mathcal{R}$ represents RC and NC, the corresponding reducts are called conflict reducts and non-conflict reducts, respectively. We can define the conflict-and-non-conflict reducts which are the minimum attribute sets that keep both the conflict relation and non-conflict relation at the same time. The family of all conflict reducts of the information table S is denoted as $RED_{\mathrm{RC}}(S)$, and the family of all non-conflict reducts as $RED_{\mathrm{NC}}(S)$.

A *conflict function*, a *non-conflict function* and a *conflict-and-non-conflict function* are defined as follows:

$$f_{\mathrm{NC}}(\mathbf{mnc}) = \bigwedge\{\bigvee \mathbf{mnc}(x,y) : x,y \in U,\ \mathbf{mnc}(x,y) \neq \emptyset\};$$
$$f_{\mathrm{RC}}(\mathbf{mc}) = \bigwedge\{\bigvee \mathbf{mc}(x,y) : x,y \in U,\ \mathbf{mc}(x,y) \neq At\};$$
$$f_{\mathrm{RC-NC}}(\mathbf{mc}-\mathbf{mnc}) = f_{\mathrm{RC}}(\mathbf{mc}) \bigwedge f_{\mathrm{NC}}(\mathbf{mnc}).$$

TABLE VI

THE CONFLICT MATRIX OF THE INFORMATION TABLE I.

	o_1	o_2	o_3	o_4	o_5	o_6
o_1	$\emptyset$	cde	bcd	bce	$bcde$	c
o_2		$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	e
o_3			$\emptyset$	$\emptyset$	$\emptyset$	b
o_4				$\emptyset$	$\emptyset$	be
o_5					$\emptyset$	be
o_6						$\emptyset$

TABLE VII

THE NON-CONFLICT MATRIX OF THE INFORMATION TABLE I.

	o_1	o_2	o_3	o_4	o_5	o_6
o_1	At	ab	ae	ad	a	abd
o_2		At	At	At	At	$abcd$
o_3			At	At	At	$acde$
o_4				At	At	acd
o_5					At	acd
o_6						At

We have $P \in RED_{\rm RC}(S)$ if and only if $\bigwedge P$ is a prime implicant of $f_{\rm NC}(\mathbf{mnc})$, and $P \in RED_{\rm NC}(S)$ if and only if $\bigwedge P$ is a prime implicant of $f_{\rm RC}(\mathbf{mc})$.

Example 8: Based on the matrices in Table VI and Table VII, we obtain:

$$f_{\rm RC}\ (\mathbf{mc}) = b \wedge c \wedge e \wedge (b \vee e) \wedge (b \vee c \vee d) \wedge (b \vee c \vee e) \wedge (c \vee d \vee e) \wedge (b \vee c \vee d \vee e)$$
$$= (b \wedge c \wedge e);$$
$$f_{\rm NC}\ (\mathbf{mnc}) = a \wedge a \vee b \wedge (a \vee d) \wedge (a \vee e) \wedge (a \vee b \vee d) \wedge (a \vee c \vee d) \wedge (a \vee b \vee c \vee d) \wedge (a \vee c \vee d \vee e)$$
$$= a;$$
$$f_{\rm RC-NC}(\mathbf{mc} - \mathbf{mnc}) = f_{\rm RC}(\mathbf{mc}) \bigwedge f_{\rm NC}(\mathbf{mnc})$$
$$= ((b \wedge c \wedge e)) \bigwedge ((a))$$
$$= (a \wedge b \wedge c \wedge e).$$

VI. CONCLUSION

Based on the similarities or the differences between agents, we can have an approach that distinguishes the indiscernibility and discernibility relations among agents. It can be used to analyze the dual alliance and non-alliance relations, and the dual conflict and non-conflict relations, respectively. For each pair of dual relations, four relations can be observed in the given universe for capturing strong version and weak version of the dual views. Two complementary matrices can be applied for conflict analysis. Based on any pair of views, three different types of reducts can be constructed. The study provides a systematic methodology for conflict analysis using rough set theory.

We use the strong versions of indiscernibility, discernibility, alliance, non-alliance, conflict, and non-conflict relations for conflict analysis, respectively. According to those relations, the constructed reducts are not necessarily the same. By using the same argument, the complement weak relations also can be used.

REFERENCES

[1] A. Gomolinska (2006) Approximation spaces based on similarity and dissimilarity, *Proceedings of Workshop on Concurrency, Specification, and Programming CS&P'2006*, pp.446-457.

[2] H. Hart (1974) Structures of influence and cooperation-conflict, *International Interactions*, vol.1, pp.141-162.

[3] T.Y. Lin (1989) Neighborhood systems and approximation in database and knowledge base system, *Proceedings of the International Symposium of Methodologies for Intelligent Systems*.

[4] M. Novotný and Z. Pawlak (1983) On a representation of rough sets by means of information systems, *Fundamenta Informaticae*, vol.6, pp.286-296.

[5] E. Orlowska (1998) Introduction: what you always wanted to know about rough sets, in: E. Orlowska (Ed.) *Incomplete Information: Rough Set Analysis*, Physica Verlag, Heidelberg, New York, pp.1-20.

[6] Z. Pawlak (1984) On conflicts, *International Journal of Man-Machine Studies*, vol. 21, pp.127-134.

[7] Z. Pawlak (1991) *Rough Sets: Theoretical Aspects of Reasoning About Data*, Kluwer Academic Publishers, Dordrecht, MA.

[8] Z. Pawlak (1998) An inquiry into anatomy of conflicts, *Information Sciences*, vol. 109, pp.65-78.

[9] Z. Pawlak (1998) Granularity of knowledge, indiscernibility and rough sets, *Proceedings of the IEEE International Conference on Fuzzy Systems*, pp.106-110.

[10] Z. Pawlak (2006) Conflicts and negotations, *Proceedings of Rough Sets and Knowledge Technology*, pp.12-27.

[11] Z. Pawla and A. Skowron (2007) Rough sets and Boolean reasoning, *Information Science*, vol. 177, pp.41-73.

[12] L. Polkowski, A. Skowron and J. Zytkow (1995) Tolerance based rough sets, in: T.Y. Lin and A.M. Wildberger (Eds.) *Soft Computing: Rough Sets, Fuzzy Logic, Neural Networks, Uncertainty Management*, pp.55-58.

[13] G.F. Qiu, W.X. Zhang, and W.Z. Wu (2005) Characterizations of attributes in generalized approximation representation spaces, *Proceedings of RSFDGrC'05*, pp.84-93.

[14] S. Ramanna, J.F. Peters and A. Skowron (2006) Generalized conflict and resolution model with approximation spaces, *Proceedings of RSCTC 2006*, pp.274-283.

[15] A. Skowron, S. Ramanna and J.F. Peters (2006) Conflict analysis and information systems: A rough set approach, *Proceedings of RSKT 2006*, pp.233-240.

[16] A. Skowron and C. Rauszer (1992) The discernibility matrices and functions in information systems, in: R. Slowiński (Ed.) *Intelligent Decision Support, Handbook of Applications and Advances of the Rough Sets Theory*, Kluwer Academic Publishers, Dordrecht.

[17] R. Slowinski and D. Vanderpooten (2000) A generalized definition of rough approximation based on similarity, *IEEE Transaction on Data and Knowledge Engineering*, vol.12, pp.331-336.

[18] J. Stefanowski and A. Tsoukias (2000) Valued tolerance and decision rules, *Proceedings of Rough Sets and Current Trends in Computing*, pp.170-181.

[19] R.W. Swiniarski (2001) Rough sets methods in feature reduction and classification, *International Journal of Applied Mathematics and Computer Science*, vol.11, pp.565-582.