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Probabilistic Inference to the Problem of Inverse-halftoning based on Statistical Mechanics of the Q-Ising Model

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Abstract—**On the basis of statistical mechanics of the Q-Ising model, we formulate the problem of inverse-halftoning using the maximizer of the posterior marginal (MPM) estimate for halftone images obtained by the threshold mask method. Then, we estimate the performance of the method in terms of the mean square error and the histogram of the gray-level using the Markov-chain Monte Carlo simulation. The simulation for a set of snapshots of the Q-Ising model reveals the results that the MPM estimate works effectively for the problem of inverse-halftoning, if we appropriately set the parameters of the model prior expressed by the Boltzmann factor of the Q-Ising model. We then clarify that the model prior shifts the gray-level images from both sides to the middle range of the gray-level in the procedure of inverse-halftoning. Also, these properties are confirmed by the MCMC method even for real-world images.**

Index Terms—**inverse-halftoning, the Bayes inference, statistical mechanics, Monte Carlo simulation**

I. Introduction

In recent years, a lot of researchers have been working on problems of information sciences related to image analysis and the Markov-random fields [1]. In recent development of information sciences, an analogy between statistical mechanics and information processing based on the Bayes inference [2,3] has been clarified. Based on the analogy, various techniques in statistical mechanics have been applied to problems of information sciences, such as the mean-field theory and the replica theory. In the problem of image restoration, Geman and Geman [4] have formulated the problem of image restoration using the Markov-random field model which is closely related to statistical mechanics of spin systems. Then, Sourlas [5] has formulated the problem of error-correcting codes for the Sourlas' codes based on statistical mechanics of spin glasses. Next, based on statistical mechanics of the Ising model, Nishimori and Wong [6,7] constructed a unified framework of image restoration and error-correcting codes and they evaluated the performance of the MPM estimate using the replica theory. Following the strategy, statistical-mechanical techniques, such as the replica theory and the mean-field theory, have been used for various problems, such as the low-density parity-checking codes [7] and the mobile communication [8].

In the field of printing, it is well known that a lot of techniques in information processing play important roles to create printed matter with high quality. Especially, a technique of halftoning [9,10] has been developed to convert a multi-level image into a bi-level one. For this purpose, various techniques have been proposed, such as the threshold mask method and the clustered-dot ordered dither method. Also, a technique of inverse-halftoning [11] is important to reconstruct an original multi-level image from a bi-level one. For this purpose, various filters have been applied for inverse-halftoning, such as the conventional low-pass filter and its variants. In recent years, in order to improve the performance of inverse-halftoning, the MAP estimate based on Bayes inference has been applied to the problem.

In the present study, on the basis of the maximizer of the posterior marginal (MPM) estimate which corresponds to statistical mechanics of the Q-Ising model, we formulate the problem of inverse-halftoning for halftone images generated by the threshold mask method using a uniform threshold and the Bayer's and screw arrays. We try this approach in the hope that the MPM estimate is expected to achieve better performance than the conventional methods, because we can selectively choice the model. Then, in order to clarify the mechanism of the method for the problem of inverse-halftoning, we evaluate the mean square error and the histogram of the gray-level using the Markov-chain Monte Carlo simulation both for a standard image "girl" and a set of snapshots of the Q-Ising model. This approach derives the results that the Boltzmann factor of the Q-Ising model play a role to shift images from both sides to the middle range of the gray-level and therefore that the MPM estimate works effectively for images located in the middle range of the gray-level, such as the set of snapshots of the Q-Ising model. Then we also show that the MPM estimate works more effectively for a halftone image generated by the

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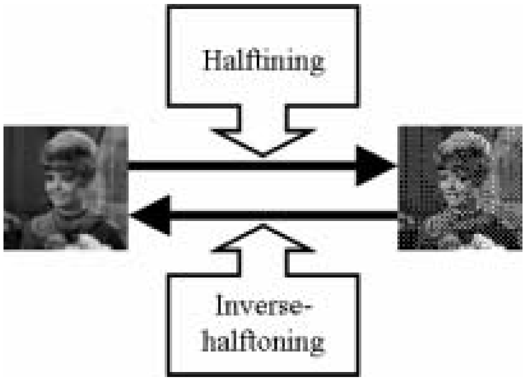

Fig. 1 Problems of halftoning and inverse-halftoning

threshold mask method using the Bayer's array than that using the screw array.

The content of the present paper is as follows. We first show our formulation for the problem of inverse-halftoning for a halftone image generated by the threshold mask method based on statistical mechanics of the Q-Ising model. Then we show the performance of the MPM estimate using the Markov-chain Monte Carlo simulation for a standard image and a set of the snapshots of the Q-Ising model. Finally, we summarize our study.

II. General formulation

Here we construct a statistical-mechanical formulation for the problem of inverse-halftoning for a halftone image which is generated by the threshold mask method using a uniform threshold and the Bayer's and screw arrays.

First, we consider a gray-scale image $\{\xi_{x,y}\}$, where $\xi_{x,y} = 0,\dots,Q-1$, x, $y = 1,\dots,L$. In this study, we treat two kinds of original images which are shown as follows. One is a set of original gray-level images (Fig.2(a)) generated by a true prior expressed by the Boltzmann factor of the Q-state Ising spin system:

$$P(\{\xi_{x,y}\}) = \frac{1}{Z_s}\exp\left[-\frac{h_s}{T_s}\sum_{n.n.}\left(\xi_{x,y}-\xi_{x',y'}\right)^2\right]. \tag{1}$$

The other is a standard image given in Fig. 2(b). Next, we construct halftone images $\{\tau_{x,y}\}$ ($\tau_{x,y}$=0, 255, x, y = 1, $\cdots$, L) by comparison of each pixel value of the original image with a corresponding threshold of the masks, such as the Bayer's and screw arrays shown in Fig. 2 (c) and (d). That is, the halftone image is obtained with the conditional probability as

$$P\left(255 \mid \xi_{x,y}\right) = \theta\left(\xi_{x,y} - M_{x,y}\right),\ P\left(0 \mid \xi_{x,y}\right) = 1 - P\left(255 \mid \xi_{x,y}\right) \tag{2}$$

where $\{M_{x,y}\}$ corresponds to the Bayer's and screw arrays. The obtained halftone images are shown in Fig. 2 (e), (f), (g) and (h).

Next, we reconstruct a gray-level image using the MPM estimate based on statistical mechanics of the Q-Ising model Here this model is constructed by a set of Q-state Ising spins $\{z_{x,y}\}$ ($z_{x,y}$= 0,…, Q-1, x, y = 1,…, L) located on the square lattice. We carry out inverse-halftoning so as to maximize the posterior marginal probability as

$$\hat{z}_{x,y} = \arg\max_{z_{x,y}} \sum_{\{z\}\neq z_{x,y}} P(\{z\} \mid \{J\}), \tag{3}$$

using the Q-Ising model $\{z_{x,y}\}$ ($z_{x,y}$=0.…,Q-1, x,y=1,…,L) The posterior probability is estimated based on the Bayes formula:

$$P(\{z\} \mid \{J\}) = P(\{z\})P(\{J\} \mid \{z\}) \tag{4}$$

using the models of the true prior and the noise probability. In this study, we assume the model prior expressed by the Boltzmann factor of the Q-Ising model:

$$P(\{z\}) = \frac{1}{Z_m}\exp\left[-\frac{J}{T_m}\sum_{n.n.}\left(z_{x,y}-z_{x',y'}\right)^2\right]. \tag{5}$$

This model prior is expected to enhance smooth structures, which is shown in natural images. Then, we assume the model of the noise probability:

$$P(\{z\} \mid \{\tau\}) \propto \exp\left[-\frac{h}{T_m}\sum_{x,y}\left(z_{x,y}-\hat{\tau}_{x,y}\right)^2\right] \tag{6}$$

so as to stabilize a image which is constructed by

$$\tau_{x,y} = \sum_{i,j=-1}^{1} a_{i,j}\tau_{x+i,y+j} \tag{7}$$

where coefficients $\{a_{i,j}\}$ are determined respective of the choice of the conventional filters. In this study, we use the halftone image obtained using the uniform threshold, the Bayer's array and the screw array.

Next, in order to estimate the performance of our method for a standard image, we evaluate the mean square error as

$$\sigma = \frac{1}{NQ^2}\sum_{x,y=1}^{L}\left(\hat{z}_{x,y}-\xi_{x,y}\right)^2 \tag{8}$$

where $\xi_{x,y}$ and $\hat{z}_{x,y}$ are the pixel values of the original gray-level and reconstructed images. On the other hand, if we estimate the performance of gray-level images generated by the true prior $P(\{\xi\})$, we evaluate the mean square error which is averaged over the true prior as

$$\sigma = \sum_{\{\xi\}} P(\{\xi\})\frac{1}{NQ^2}\sum_{x,y=1}^{L}\left(\hat{z}_{x,y}-\xi_{x,y}\right)^2. \tag{9}$$

III. Performance

In order to estimate the performance of the MPM estimate for the problem of inverse-halftoning, we carry out the Markov-chain Monte Carlo simulation both for a standard image "girl" and a set of images generated with the probability expressed by the Boltzmann factor of the Q-Ising model in the following way.

Here, we use a 256-level standard image "girl" with 100 ×100 pixels and a set of 16-level images with 100×100 pixels, which are generated with the prior probability distribution expressed by the Boltzmann factor of the 16-state Ising model on the square lattice. Here, parameters of the prior are set as h_s=1, T_s=1. Next, we convert the gray-level image into a bi-level one due to the threshold mask method using several kinds of masks, such as a uniform threshold, the 4×4 Bayer's and screw arrays. In this method, the halftone image is generated by comparing each pixel value of the original image with a corresponding threshold to the pixel.

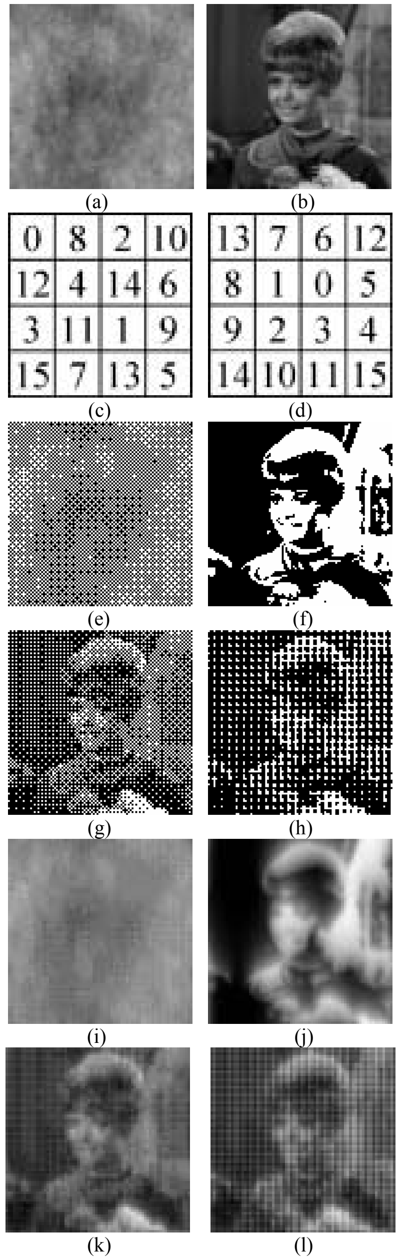

Fig.2 (a) a 16-level image generated by the Boltzmann factor of the Q-Ising model with Q=16, h=1, T_s=1, (b) a 100×100 standard image "girl" with 256 gray-levels, (c)the Bayer's array, (d) the screw mask, (e)a halftone image of (a) generated by the threshold mask method using the Bayer's array, (f)a halftone image of (b) generated by the threshold mask method using a uniform threshold with M=80, (g)a halftone image of (b) generated by the threshold mask method using the Bayer's array, (h) A halftone image of Fig. 2 (b) using the screw array, (i) a gray-level image reconstructed from (e) by the MPM estimate, (j)a gray-level image reconstructed from (f) by the MPM estimate, (k)a gray-level image reconstructed from (f), (l)a gray-level image reconstructed from (f) by the MPM estimate.

Next we reconstruct a gray-level image so as to maximize the marginal posterior probability using the 100×100 Q-state Ising spin system which is located on the square lattice. When we estimate the performance of our method, we evaluate the mean square error and the gray-level distribution averaged over the original gray-level images generated by the assumed prior.

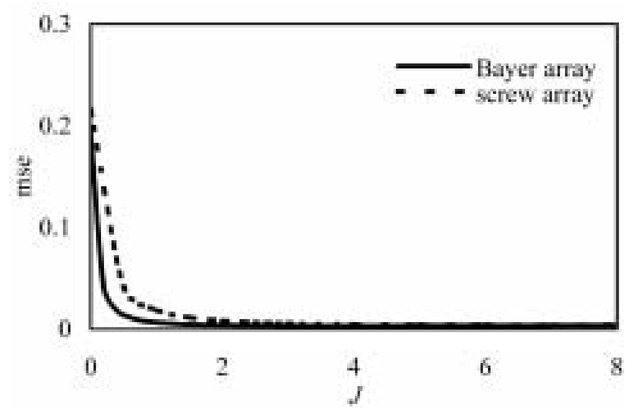

Fig. 3 The mean square error as a function of J obtained by the MPM estimate using halftone images converted from a set of gray-level images generated by the Boltzmann factor of the Q-Ising model when h_s=1, T_s=1, h=1, T=1.

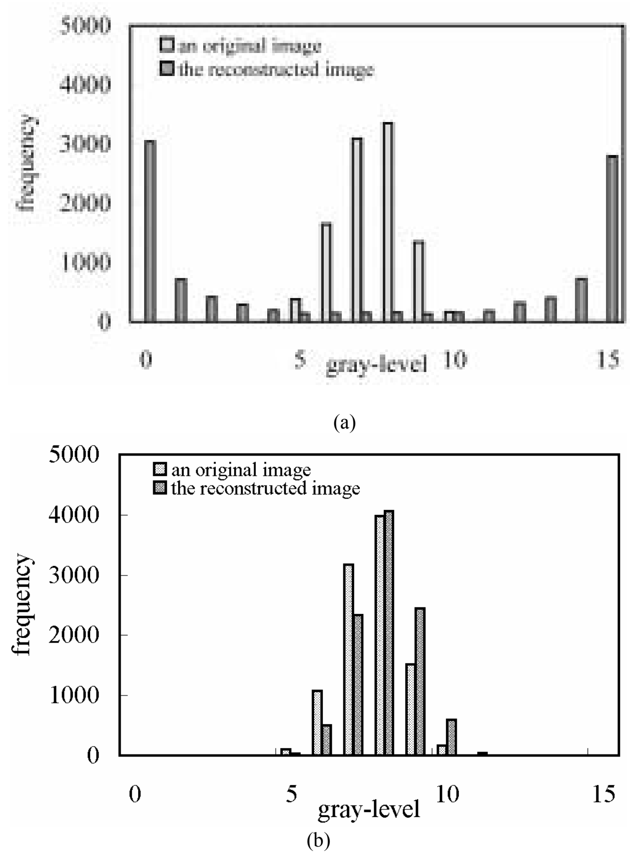

Fig.4 The histogram of the gray-level both of an original gray-level image generated by a ture prior expressed by the Boltzmann factor of the Q-Ising model and that of the reconstructed image by the MPM estimate (a) h_s=1, T_s=1, h=1, T=0.1, J=0.1, (b) h_s=1, T_s=1, h=1, T=0.1, J=6.0.

Under the above conditions, we estimate the performance of the MPM estimate for a set of gray-level images generated by the true prior expressed by the Boltzmann factor of the Q-Ising model. First, in order to clarify optimal performance of our method, we evaluate how the mean square error depends on the parameters. Figure3 shows the J-dependence of the mean square

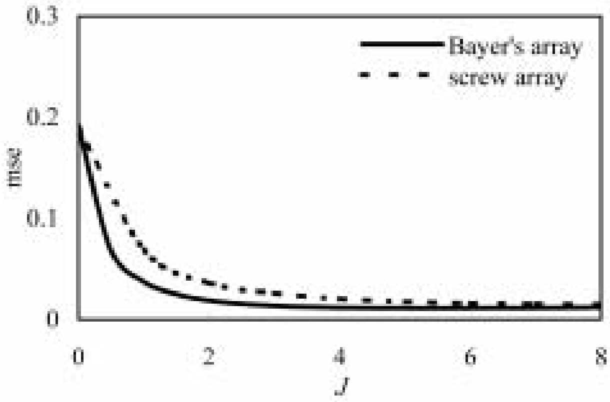

Fig. 5 The mean square error as a function of the parameter *J* obtained by the MPM estimate for a halftone image generated by a standard image "girl" using the Bayer's threshold mask method.

error which is averaged over a set of the snapshots of the Q-Ising model. This result means that the halftone images is obtained when *J*=0, and that the performance is monotonously improved with the increase in the parameter *J* and then that the optimal performance is obtained at a finite value of the parameter J. Then, the optimal performance is obtained in *J*>3. parameter *J*, although it is not clearly seen that the optimal performance is obtained in the large *J* limit due to the statistical uncertainty. This result also indicates that the MPM estimate realizes inverse-halftoning more precisely for halftone images obtained using the Bayer's array than that using the screw array.

Next, as are shown in Fig. 4 (a) and (b), we investigate the gray-level distribution of the gray-level images which are snapshots of the Q-Ising model. As is shown in Fig. 4(a), the reconstruct image is located around the both sides (Q=0 and Q=15) of the gray-level, if the parameter *J* takes smaller value than the optimal one. On the other hand, as is shown in Fig. 4, the reconstructed image is located in the middle range of the gray-level, if the parameter *J* takes a larger value than the optimal one. These results suggest that the Boltzmann factor of the Q-Ising model is available to shift images from both sides to the middle range of the gray-level.

Next, we investigate the performance of the MPM estimate for several kinds of halftone images generated from a standard image "girl" by the threshold mask method using the Bayer's and screw arrays. Here we evaluate how the mean square error depends on the parameter *J*. We obtain the result in Fig. 5 that the optimal performance is obtained by the MPM estimate, if we set the appropriate finite value of the parameter *J*. However, if we set larger *J* value than the optimal one, the MPM estimate realizes inverse-halftoning with less precision due to the over-smoothing by the model prior.

Next, we investigate the performance of the MPM estimate in terms of the histogram of the gray-level for a 256-level standard image "girl" whose histogram is given in Fig. 6(a). The histograms obtained by the MPM estimate are shown in Figs. 6 (b), (c) and (d). These results indicate that the MPM estimate reconstructs the gray-level distribution which has a similar structure with the original image given in Fig.6(a), if we set the parameter J appropriately. However, as is shown in Fig. 6(b),

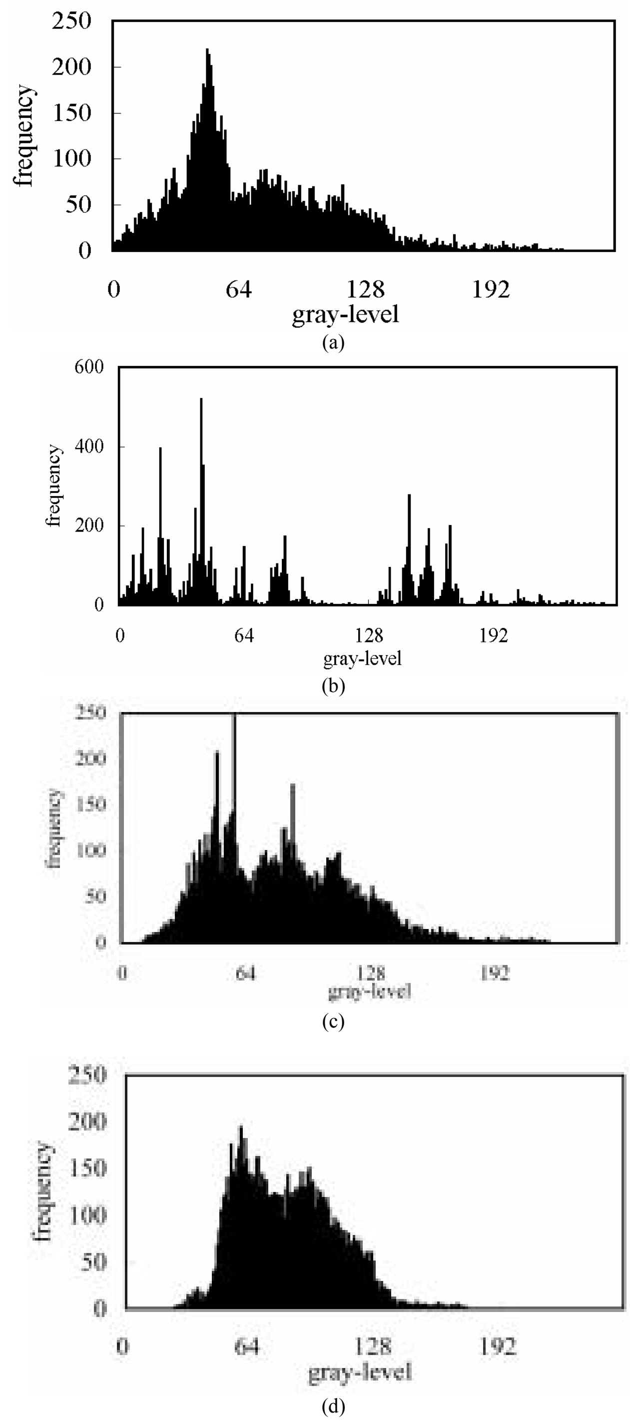

Fig. 6 (a) The histogram of a 256-level standard image "girl" given in Fig.2(b), (b)the histogram of the gray-level image obtained by the MPM estimate with *h*=1, *J*=1 from the halftone image in Fig.2(g), (c) the histogram of the gray-level image obtained by the MPM estimate with *h*=1, *J*=6.0 from the halftone image in Fig. 2(g), (d) the histogram of the gray-level image obtained by the MPM estimate with *h*=1, *J*=20.0 from the halftone image in Fig. 2(g).

the reconstructed image is distributed all over the range of the gray-level, if the parameter *J* is smaller than the optimal one. Then the reconstructed image in Fig. 7 has a property of the

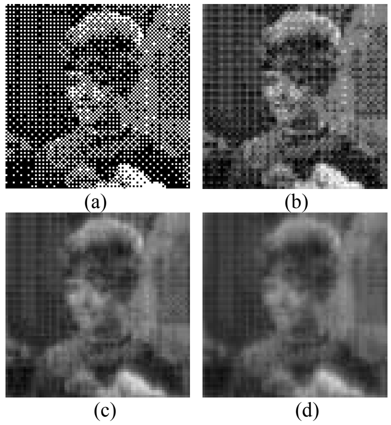

Fig.7 Gray-level images obtained by the MPM estimate for a halftone image generated using the Bayer's array when h=1, T=0.1. (a) J=0, (b) J=1, (c)J=5, (d) J=8.

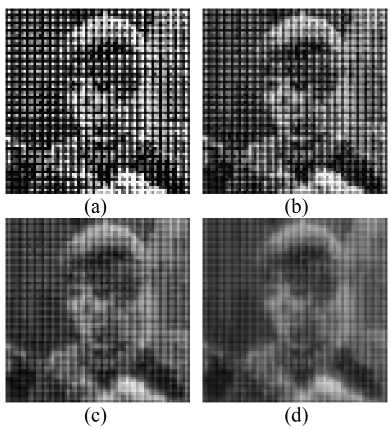

Fig. 8 Gray-level images obtained by the MPM estimate for a halftone image generated using the screw array when h=1, T=0.1. (a) J=0, (b) J=1, (c) J=5, (d) J=8.

halftone image which is shown in Fig. 2(b). Then, as is shown in Fig. 6(d), the reconstructed image is distributed in the intermediate region of the gray-level, if the parameter J is larger than the optimal one. Further, as is shown in Fig. 7(d), we can see that the reconstructed image is over-smoothed due to the model prior expressed by the Boltzmann factor of the Q-Ising model.

IV. Summary and Discussion

In the previous chapters, on the basis of statistical mechanics of the Q-Ising model corresponding to the MPM estimate in the field of information sciences, we formulate the problem of inverse-halftoning for halftone images generated by the threshold mask method using the uniform threshold, the Bayer's mask and the screw mask. Then, we estimate the performance of the MPM estimate in terms of the mean square error and the histogram of the gray-level using the Markov-chain Monte Carlo simulation both for the standard image "girl" and the set of original images which are generated by the assumed true prior expressed by the Boltzmann factor of the Q-Ising model. From the performance estimation in terms of the mean square error, we obtain the result that the model prior expressed by the Boltzmann factor of the Q-Ising model works effectively to reconstruct the gray-level image from the halftone image obtained by the threshold mask method. Then, from the performance estimation in terms of the histogram of the gray-level, we clarify that our method works more effectively for a gray-level image which is located in the intermediate range of the gray-level than for a standard natural image.

As a future problem, we would like to construct the formulation for the problems of inverse-halftoning using the probabilistic information processing based on statistical mechanics of the spin system.

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References

[1] G. Winkler, "Image analysis, random fields, and Markov chain Monte Carlo methods: a Mathematical Introduction", Springer, 2002.

[2] H. Nishimori, "Statistical Physics of Spin Glasses and Information Processing; An Introduction", Oxford, London, 2001.

[3] K. Tanaka, Statistical-mechanical approaches to image processing, J. Phys. A: Math. Gen., **35**, R81-R150, 2002.

[4] S. Geman and D. Geman, IEEE, Trans. Pattern Anal. and Math. Intell. **PAMI-6-6**, 721-742 (1984).

[5] N. Sourlas, Nature **339**, 693 (1993).

[6] J. M. Pryce and A. D. Bruce, J. Phys. A Math. Gen. **28** 511 (1995).

[7] Y. Kabashima and D. Saad, Europhys. Lett. **45**, 98 (1998).

[8] T. Tanaka, Europhys. Lett. **54**, 540 (2001).

[9] R. Ulichney, "Digital Halftoning", the MIT Press, Cambridge, 1987.

[10] B. E. Bayer, "An optimum method for two-level rendition of continuous-tone pictures", ICC CONF. RECORD 26 pp. 11-15, 1973.

[11] R. L. Stevenson, "Inverse-Halftoning via MAP estimation", IEEE, vol.6, pp.574-583, 1997.