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Recurrent TS Fuzzy Neural Network and Its Application on Controlling Nonlinear Time-Delay Systems

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***Abstract*— This paper proposes a TS recurrent fuzzy neural network (RFNN) and a robust RFNN control of uncertain nonlinear time-delay systems. First, the TS-RFNN is proposed to learn complex functions with delays. Next, the robust adaptive TS-RFNN control is developed for time-delay nonlinear systems. The advantages of the proposed controller includes: i) asymptotic stability independent on the delay; ii) more simple and legible gain design; and iii) simpler structure of FNN (fewer fuzzy rules). Simulation results demonstrate the validity of the proposed control scheme.**

***Index Terms*— FNN, robust control, time-delay system, uncertainty.**

I. Introduction

Since time-delay is a main source of instability and poor performance, the control problem of time-delay systems has received considerable attentions in literature, such as [1]-[7]. Most literatures focus on linear time-delay systems due to the fact that the stability analysis developed in the two methods is usually based on linear matrix inequality (LMI) techniques [8]. Some sliding-mode control (SMC) schemes have been applied to uncertain nonlinear time-delay systems in [4], [5], [6]. However, these SMC schemes still exist some limits as follows: i) specific form of the dynamical model and uncertainties [4], [5]; ii) an exactly known delay time [6]; and iii) a complex gain design [4], [5], [6].

Recently, many fuzzy neural network (FNN) articles are proposed by combining the fuzzy concept and the configuration of neural network, e.g., [9]-[12]. There, the fuzzy logic system is constructed from a collection of fuzzy If-Then rules while the training algorithm adjusts adaptable parameters. Nevertheless, few results using FNN are proposed for time-delay nonlinear systems due to a large computational load and a vast amount of feedback data, for example, see [11], [12]. Moreover, the training algorithm is difficultly found for systems with unknown time-delay.

To cope with uncertain complex systems, a TS recurrent fuzzy neural network (TS-RFNN) is introduced and extended to robust adaptive control in this paper. The proposed TS-RFNN combines the concept of TS fuzzy rules ([13], [14], [15]) and the structure of neural networks to provide high capacity for approximation of complex uncertainty which may contain delays. To apply to robust adaptive control, this paper introduces a novel sliding surface design to keep the sliding motion insensitive to uncertainties and unknown time-delay. The gain condition is transformed in terms of a simple and legible LMI. Based on the asymptotic sliding surface, the ideal and TS-RFNN-based reaching laws are derived. In detail, the TS-RFNN provides a near ideal reaching law by combining TS fuzzy rules and recurrent neural network. The advantages of the proposed TS-RFNN are: i) allowing fewer fuzzy rules for complex systems (since the Then-part of fuzzy rules can be properly chosen); and ii) a small switching gain is used (since the uncertainty is indirectly cancelled by the TS-RFNN). As a result, the adaptive TS-RFNN based sliding mode controller achieves asymptotic stabilization for a class of uncertain nonlinear time-delay systems.

This paper is organized as follows. The problem formulation is given in Section 2. The sliding surface design and ideal sliding mode controller are given in Section 3. In Section 4, the adaptive TS-RFNN control scheme is developed to solve the robust control problem of time-delay systems. Section 5 shows simulation results to verify the validity of the proposed method. Some concluding remarks are finally made in Section 6.

II. TS-RFNN

In control engineering, neural network is usually used as a tool for modeling nonlinear system functions because of their good capabilities in function approximation. In this section, the TS-RFNN (TS recurrent fuzzy neural network) is proposed to approximate a complex nonlinear function $u^*(t,x)$ (which may be an ideal control law) with states x. Indeed, the recurrent FNN is composed of a collection of T-S fuzzy IF-THEN rules as follows:

$Rule\ i$:

IF $\bar{z}_1(t)$ is G_{i1} and $\cdots$ and $\bar{z}_{n_i}(t)$ is G_{in_i} THEN

$$u_N = z_1 v_{i1} + z_2 v_{i2} + \cdots + z_{n_v} v_{in_v} = \mathrm{z}^T \mathrm{v}_i \quad (1)$$

for $i = 1, 2, ..., n_R$, where n_R is the number of fuzzy rules; $\overline{z}_1(t) \sim \bar{z}_{n_i}(t)$ are the premise variables composed of available signals s_j form the inputs; u_N is the fuzzy output; $\mathrm{v}_i = [\ v_{i1} \quad v_{i2} \quad \cdots \quad v_{in_v}\ ]^T \in R^{n_v}$ with tunable weights $v_{i1} \sim v_{in_v}$; $\mathrm{z} = [\ z_1 \quad z_2 \quad \cdots \quad z_{n_v}\ ]^T \in R^{n_v}$ with properly chosen signal $z_1 \sim z_{n_v}$; G_{ij} $(j = 1, 2, \cdots, n_i)$ are the fuzzy sets with Gaussian membership functions which have the form

$$G_{ij}(\bar{z}_j) = exp\left(\frac{-(\bar{z}_j - m_{ij})^2}{\sigma_{ij}^2}\right) \quad (2)$$

for $i = 1,\ 2,\ \cdots,\ n_R$ and $j = 1,\ 2,\ \cdots,\ n_i$, where m_{ij} is the center of the Gaussian function; and σ_{ij} is the variance of the Gaussian function. Note that when the FNN fuzzifies the inputs, the activated level will be back-propagated into the premise variables as

$$\bar{z}_j(N) = s_j(N) + h_{ij} G_{ij}(\bar{z}_j(N-1))$$

where N denotes the number of iteration; and h_{ij} is the recurrent weight associate with the (i,j) membership function. Therefore, the FNN has a recurrent form.

Using the singleton fuzzifier, product fuzzy inference and weighted average defuzzifier, the inferred output of the fuzzy neural network is

$$u_N = \sum_{i=1}^{n_R} \mu_i(\bar{z}(t)) \mathrm{z}^T \mathrm{v}_i$$

where $\mu_i(\bar{z}(t)) = \bar{w}_i(\bar{z}(t)) / \sum_{i=1}^{n_R} \bar{w}_i(\bar{z}(t))$ with $\bar{z}(t) = [\ \bar{z}_1(t) \quad \bar{z}_2(t) \quad \cdots \quad \bar{z}_{n_i}(t)\ ]$ and $\bar{w}_i(\bar{z}(t)) = \prod_{j=1}^{n_i} G_{ji}(\bar{z}_j(t))$. For simplification, define two auxiliary signals

$$\begin{aligned} \xi &= [\ \mathrm{z}^T\mu_1 \quad \mathrm{z}^T\mu_2 \quad \cdots \quad \mathrm{z}^T\mu_{n_R}\ ]^T \in R^{n_v n_R} \\ \theta &= [\ \mathrm{v}_1^T \quad \mathrm{v}_2^T \quad \cdots \quad \mathrm{v}_{n_R}^T\ ]^T \in R^{n_v n_R} \end{aligned}$$

In turn, the output of the recurrent TS-RFNN is rewritten in the form:

$$u_N = \xi^T \theta \quad (3)$$

Thus, the above TS-RFNN has a simple structure, which is easily implemented in comparison of traditional FNN. Moreover, the signal z can be appropriately selected for more complex function approximation. In other words, we can use less fuzzy rules to achieve a better approximation. Moreover, the recurrent neuron can further provides better approximation of complex functions with delays.

According to the uniform approximation theorem, there exists an optimal parametric vector θ^* of the TS-RFNN which arbitrarily accurately approximates the function u^*. This implies that the function can be expressed in terms of an optimal TS-RFNN as

$$u^* = \xi^T \theta^* + \varepsilon(x)$$

where $\varepsilon(x)$ is a minimum approximation error which is assumed to be upper bounded in a compact discussion region. Meanwhile, the output of the TS-RFNN is further rewritten in the following form:

$$\begin{aligned} u_N &= u^* - u^* + \xi^T\theta \\ &= u^* + \xi^T\tilde{\theta} - \varepsilon(x) \end{aligned} \quad (4)$$

where $\tilde{\theta} = \theta - \theta^*$ is the estimation error of the optimal parameter. Thus, the tuning law of the FNN will be derived for some criteria. Since the proposed FNN contains a recurrent loop, we take controlling time-delay systems as its application in the following.

III. APPLICATION PROBLEM DESCRIPTION

Consider a class of nonlinear time-delay systems described by the following differential equation:

$$\begin{aligned} \dot{x}(t) &= (A + \triangle A(t))x(t) + (A_d + \triangle A_d(t))x(t-d) \\ &\quad + Bg^{-1}(x)(u(t) + h(x(t), x(t-d))) \\ x(t) &= \psi(t),\ t \in [\ -d \quad 0\ ] \end{aligned} \quad (5)$$

where $x(t) \in R^n$ and $u(t) \in R$ are the state vector and control input, respectively; d is an unknown constant delay time; A and A_d are nominal system matrices with appropriate dimensions; $\triangle A$ and $\triangle A_d$ are time-varying uncertainties; $h(\cdot)$ is an unknown bounded nonlinear function; B is a known input matrix; $g(x) \in R$ is an unknown function presenting the input uncertainties; and $\psi(t)$ is the initial condition. In the system (5), for simplicity, we assume the input matrix

$$B = [\ 0 \quad 0 \quad \cdots \quad 0 \quad 1\ ]^T$$

and partition the state vector x into $x = [\ x_1^T \quad x_2\ ]^T$ with $x_1 \in R^{n-1}$ and $x_2 \in R$. Accompanying the state partition, the system matrices can be decomposed into the following:

$$\begin{aligned} A &= \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \triangle A(t) = \begin{bmatrix} \triangle A_{11} & \triangle A_{12} \\ \triangle A_{21} & \triangle A_{22} \end{bmatrix} \\ A_d &= \begin{bmatrix} A_{d11} & A_{d12} \\ A_{d21} & A_{d22} \end{bmatrix}, \triangle A_d = \begin{bmatrix} \triangle A_{d11} & \triangle A_{d12} \\ \triangle A_{d21} & \triangle A_{d22} \end{bmatrix} \end{aligned}$$

where A_{ij}, A_{dij}, $\triangle A_{ij}$, and $\triangle A_{dij}$ (for $i, j = 1, 2$) are decomposed components of A, A_d, $\triangle A$, and $\triangle A_d$, respectively; $A_{11}, \triangle A_{11}, A_{d11}, \triangle A_{d11} \in R^{(n-1)\times(n-1)}$;

and $A_{22}, \triangle A_{22}, A_{d22}, \triangle A_{d22} \in R$. Thus, we rewrite the system (5) as two coupling subsystems:

$$\begin{aligned} \dot{x}_1(t) &= (A_{11}+\triangle A_{11}(t))x_1 + (A_{12}+\triangle A_{12}(t))x_2 \\ &\quad +(A_{d11}+\triangle A_{d11}(t))x_1(t-d) \\ &\quad +(A_{d12}+\triangle A_{d12}(t))x_2(t-d) \qquad (6) \\ \dot{x}_2(t) &= f(x(t),x(t-d)) + g^{-1}(x)(u+h) \qquad (7) \\ x(t) &= \psi(t),\ t\in[\ -d \quad 0\ ] \end{aligned}$$

where $f(x(t),x(t-d)) = (A_{21}+\triangle A_{21}(t))x_1 + (A_{22}+\triangle A_{22}(t))x_2(t) + (A_{d21}+\triangle A_{d21}(t))x_1(t-d) + (A_{d22}+\triangle A_{d22}(t))x_2(t-d)$.

Throughout this study we need the following assumptions:

Assumption 1: For controllability, $g(x) > 0$ for all $x \in U_c$, where $U_c \subset R^n$ is a certain controllable region. Moreover, $\dot{g}(x(t),x(t-d)) \in L_\infty$ if $x(t) \in L_\infty$.

Assumption 2: The uncertain matrices $\triangle A_{11}(t)$, $\triangle A_{12}(t)$, $\triangle A_{d11}(t)$, and $\triangle A_{d12}(t)$ satisfy

$$\begin{aligned} [\ \triangle A_{11}(t) \quad \triangle A_{12}(t)\ ] &= D_1C_1(t)[\ E_{11} \quad E_{12}\ ] \\ [\ \triangle A_{d11}(t) \quad \triangle A_{d12}(t)\ ] &= D_2C_2(t)[\ E_{21} \quad E_{22}\ ] \end{aligned}$$

for some known matrices D_1, D_2, E_{11}, E_{12}, E_{21}, E_{22} with proper dimensions and unknown matrices $C_1(t)$, $C_2(t)$ satisfying $\|C_1(t)\| \le 1$ and $\|C_2(t)\| \le 1$.

The control objective is to determine a robust adaptive FNN controller such that the state $x(t)$ converges to zero. Since high uncertainty is considered here, we want to derive a SMC-based design for the control goal.

IV. SMC-BASED TS-RFNN CONTROL

A. Sliding Surface Design

Due to the high uncertainty and nonlinearity in the system (5), we propose a SMC-based TS-RFNN controller to solve the control problem. To this end, an asymptotic stable sliding surface is designed below.

Without loss of generality, let the sliding surface denote

$$s_f(t) = [\ -\Lambda \quad 1\ ]\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \widetilde{\Lambda}x(t) = 0 \qquad (8)$$

where $\Lambda \in R^{1\times(n-1)}$ and $\widetilde{\Lambda} = [\ -\Lambda \quad 1\ ] \in R^{1\times n}$ determined later. In the surface, we have

$$\begin{aligned} \dot{x}_1(t) &= (A_{11}+A_{12}\Lambda + D_1C_1(t)E_{11} \\ &\quad +D_1C_1(t)E_{12}\Lambda)x_1(t) + (A_{d11}+A_{d12}\Lambda \\ &\quad +D_2C_2(t)E_{21}+D_2C_2(t)E_{22}\Lambda)x_1(t-d) \end{aligned} \qquad (9)$$

where the facts $x_2(t) = \Lambda x_1(t)$ and Assumption 2 have been applied to (6). Since the subsystem (9) is not coupling with $x_2(t)$, the asymptotical stability of $x_1(t)$ can be achieved by an appropriate Λ. Thus, the result of sliding surface design is stated in the following theorem.

Theorem 1: Consider the system (5) lie in the sliding surface (8). The sliding motion is asymptotically stable independent of delay, i.e., $\lim_{t\to\infty} x_1(t), x_2(t) = 0$, if there exist positive symmetric matrices $X, \overline{Q}$ and a parameter Λ satisfying the following LMI:

$$\begin{aligned} &Given\ \epsilon > 0 \\ &subject\ to\ X>0, \overline{Q}>0 \end{aligned}$$

$$\begin{bmatrix} M & (*) & (*) & (*) & (*) & (*) \\ XA_{d11}^T + K^TA_{d12}^T & -\overline{Q} & (*) & (*) & (*) & (*) \\ E_{11}X+E_{12}K & 0 & -\epsilon I_a & (*) & (*) & (*) \\ 0 & E_{21}X+E_{22}K & 0 & -\epsilon I_a & (*) & (*) \\ D_1^T & 0 & 0 & 0 & -\epsilon^{-1}I_b & (*) \\ D_2^T & 0 & 0 & 0 & 0 & -\epsilon^{-1}I_b \end{bmatrix} < 0 \qquad (10)$$

where $M = A_{11}X + XA_{11}^T + A_{12}K + K^TA_{12}^T + \overline{Q}$; $K = \Lambda X$; I_a, I_b are identity matrices with proper dimensions; and $(*)$ denotes the transposed elements in the symmetric positions.

Proof: When the system (5) lie in the sliding surface (8), the sliding motion is described by the dynamics (9). To analyzes the stability of the sliding motion, let us define the following Lyapunov-Krasoviskii function

$$V(t) = x_1^T(t)Px_1(t) + \int_{t-d}^{t} x_1^T(\tau)Qx_1(\tau)d\tau$$

where $P > 0$ and $Q > 0$ are symmetric matrices. The time derivative of $V(t)$ along the dynamics (9) is $\dot{V}(t) = \bar{x}^T(t)\Omega\bar{x}(t)$ with $\bar{x}(t) = [\ x_1^T(t) \quad x_1^T(t-d)\ ]^T$,

$$\begin{aligned} \Omega &= \begin{bmatrix} \Psi & (*) \\ \Omega_{21} & -Q \end{bmatrix} \\ \Omega_{21} &= [A_{d11}+A_{d12}\Lambda + D_2C_2(t)(E_{21}+E_{22}\Lambda)]^T P \end{aligned}$$

and $\Psi = [A_{11}+A_{12}\Lambda + D_1C_1(t)(E_{11}+E_{12}\Lambda)]^T P + P[A_{11}+A_{12}\Lambda + D_1C_1(t)(E_{11}+E_{12}\Lambda)]^T + Q$. Thus, if $\Omega < 0$ is satisfied, then $\dot{V}(t) < 0$. In other words, the design is transformed to find matrices P, Q, Λ satisfying $\Omega < 0$. To this end, we decompose the matrix Ω into two parts as follows:

$$\Omega = \Omega_1 + \Omega_2$$

with

$$\begin{aligned} \Omega_1 &= \begin{bmatrix} \begin{array}{c} \{(A_{11}+A_{12}\Lambda)^T P \\ +P(A_{11}+A_{12}\Lambda)+Q\} \end{array} & (*) \\ (A_{d11}+A_{d12}\Lambda)^T P & -Q \end{bmatrix} \\ \Omega_2 &= \begin{bmatrix} \begin{array}{c} \{PD_1C_1(t)(E_{11}+E_{12}\Lambda) \\ +[D_1C_1(t)(E_{11}+E_{12}\Lambda)]^T P\} \end{array} & (*) \\ [D_2C_2(t)(E_{21}+E_{22}\Lambda)]^T P & 0 \end{bmatrix}. \end{aligned}$$

Note that the second term Ω_2 can be further rewritten in the form:

$$\begin{aligned}\Omega_2 \;=\; & \begin{bmatrix} PD_1 & PD_2 \\ 0 & 0 \end{bmatrix} \\ & \cdot\overline{C}(t)\begin{bmatrix} E_{11}+E_{12}\Lambda & 0 \\ 0 & E_{21}+E_{22}\Lambda \end{bmatrix} \\ & + \begin{bmatrix} E_{11}+E_{12}\Lambda & 0 \\ 0 & E_{21}+E_{22}\Lambda \end{bmatrix}^T \\ & \cdot\overline{C}^T(t)\begin{bmatrix} PD_1 & PD_2 \\ 0 & 0 \end{bmatrix}^T \end{aligned}$$

with $\overline{C}(t) = diag\{C_1(t), C_2(t)\}$ satisfying $\overline{C}^T(t)\overline{C}(t) \leq I_d$ for identity matrix I_d from Assumption 2. According to the above decomposition, the stability condition $\Omega < 0$ is equivalent to

$$\Omega_1 + \begin{bmatrix} E_{11}+E_{12}\Lambda & 0 \\ 0 & E_{21}+E_{22}\Lambda \\ (PD_1)^T & 0 \\ (PD_2)^T & 0 \end{bmatrix}^T \cdot I_\epsilon \begin{bmatrix} E_{11}+E_{12}\Lambda & 0 \\ 0 & E_{21}+E_{22}\Lambda \\ (PD_1)^T & 0 \\ (PD_2)^T & 0 \end{bmatrix} < 0$$

for some $\epsilon > 0$ and $I_\epsilon = diag\{\epsilon^{-1}I_a, \epsilon^{-1}I_a, \epsilon I_b, \epsilon I_b\}$ with proper identity matrices I_a, I_b. After applying Schur complement to the above inequality, we further obtain the LMI (10) with $X = P^{-1}$ and $\overline{Q} = XQX$. Therefore, if the LMI problem has a feasible solution, then the sliding dynamical system (9) is asymptotically stable, i.e., $\lim_{t\to\infty} x_1(t) = 0$. In turn, from the fact $x_2(t) = \Lambda x_1(t)$ in the sliding surface, the state $x_2(t)$ will asymptotically converge to zero as $t \to \infty$. Moreover, since the gain condition (10) does not contain the information of the delay time, the stability is independent of the delay. ■

B. TS-RFNN Controller Design

Based on Theorem 1, the control goal becomes to drive the system (5) to the sliding surface defined in (8). To this end, take the time derivative of $s_f(t)$ below

$$\begin{aligned} g(x)\dot{s}_f \;=\; & g(x)\Lambda[(A+\triangle A)x(t) \\ & +(A_d+\triangle A_d)x(t-d)] + u + h(x(t), x(t-d)) \end{aligned}$$

where $g(x) > 0$ from Assumption 1. If the plant dynamics and delay-time are exactly known (i.e., all matrices A, $\triangle A$, A_d, $\triangle A_d$ and the functions $g(\cdot), \dot{g}(\cdot), h(\cdot)$ are exactly known), then the ideal control law u^* is set to

$$\begin{aligned} u^* \;=\; & -\{g(x)\Lambda[(A+\triangle A)x(t) + (A_d+\triangle A_d)x(t-d)] \\ & +\frac{1}{2}\dot{g}(x)s_f + h(x(t), x(t-d)) + k_f s_f\} \end{aligned} \quad (11)$$

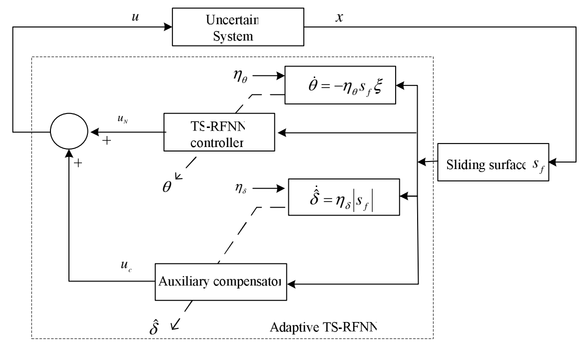

Fig. 1. The configuration of TS-RFNN.

where k_f is a positive control gain. Thus, the error signal s_f converges to zero in an asymptotic manner. Unfortunately, the ideal control law (11) is unrealizable in practice applications. To overcome this difficulty, the TS-FRFNN reaching control law is stated in the following.

Based on the proposed TS-RFNN and sliding surface design, the overall control law is set to

$$u = u_N + u_c \quad (12)$$

where u_N is the TS-RFNN controller part defined in (3); and u_c is an auxiliary compensation controller part determined later. Indeed, the configuration of the adaptive TS-RFNN sliding mode control system is depicted in Fig. 1. The TS-RFNN control u_N is the main tracking controller part that is used to imitate the idea control law u^* due to high uncertainties, while the auxiliary controller part u_c is designed to cope with the difference between the idea control law and the TS-RFNN control. Then, applying the control law (12) and the expression form of u_N in (4), the error dynamics of s_f is obtained as follows:

$$g(x)\dot{s}_f = -k_f s_f - \frac{1}{2}\dot{g}(x)s_f + \xi^T\tilde{\theta} - \varepsilon(x) + u_c \quad (13)$$

where the definition of u^* in (11) has been used. Now, the auxiliary controller part and tuning law of FNN are stated in the following.

Theorem 2: Consider the uncertain time-delay system (5) using the sliding surface designed by Theorem 1 and the control law (12) with the TS-RFNN controller part (4) and the auxiliary controller part

$$u_c = -\hat{\delta}sgn(s_f). \quad (14)$$

The controller is adaptively tuned by

$$\dot{\theta} \;=\; -\eta_\theta s_f \xi \quad (15)$$

$$\dot{\hat{\delta}} \;=\; \eta_\delta |s_f| \quad (16)$$

where η_θ and η_δ are positive constants. The closed-loop error system is guaranteed with asymptotic convergence of $s_f(t), x_1(t)$, and $x_2(t)$, while all adaptation parameters are bounded.

Proof: Consider a Lyapunov function candidate as

$$V_N(t) = \frac{1}{2}\left(g(x)s_f^2(t) + \frac{1}{\eta_\theta}\tilde{\theta}^T\tilde{\theta} + \frac{1}{\eta_\delta}\tilde{\delta}^2\right)$$

where $\tilde{\delta} = \hat{\delta} - \delta$ is the estimation error of the bound of $\varepsilon(x)$ (i.e., $\sup_t |\varepsilon(x(t))| < \delta$). By taking the derivative the Lyapunov $V_N(t)$ along with (13), we have $\dot{V}_N(t) \leq -k_f s_f^2$. Since $V_N(t) > 0$ and $\dot{V}_N(t) \leq 0$, we obtain the fact that $V_N(t) \leq V_N(0)$, which implies all s_f, $\tilde{\theta}$ and $\tilde{\delta}$ are bounded. In turn, $\dot{s}_f \in L_\infty$ due to all bounded terms in the right-hand side of (13). Moreover, integrating both sides of the above inequality, the error signal s_f is L_2-gain stable. As a result, combining the facts that $s_f, \dot{s}_f \in L_\infty$ and $s_f \in L_2$, the error signal $s_f(t)$ asymptotically converges to zero as $t \to \infty$ by Barbalat's lemma. Therefore, according to Theorem 1, the state $x(t)$ will is asymptotically sliding to the origin. ■

To further improve the performance and increase the learning freedom, the fuzzy membership functions are trained on-line by a gradient descent method along with an error function $V_s = \frac{1}{2}g(\bar{x})s^2(t)$. The following theorem is given.

Theorem 3: Consider the uncertain time-delay system (5) using the sliding surface designed by Theorem 1 and the control law (12) with the auxiliary controller part (14) and the TS-RFNN controller part (4) adaptively tuned by (15),

$$\begin{aligned}
\dot{\hat{m}}_{ij} &= -\eta_m s_f \left(\frac{\mathrm{z}^T\mathrm{v}_i - u_N}{\sum_{i=1}^{n_R}\bar{w}_i}\right)\left(\bar{w}_i\frac{2(\bar{z}_j - m_{ij})}{\rho_{ij}^2}\right)\\
\dot{\hat{\rho}}_{ij} &= -\eta_\rho s_f \left(\frac{\mathrm{z}^T\mathrm{v}_i - u_N}{\sum_{i=1}^{n_R}\bar{w}_i}\right)\left(\bar{w}_i\frac{2(\bar{z}_j - m_{ij})^2}{\rho_{ij}^3}\right)\\
\dot{\hat{h}}_{ij} &= -\eta_h s_f \left(\frac{\mathrm{z}^T\mathrm{v}_i - u_N}{\sum_{i=1}^{n_R}\bar{w}_i}\right)\left(\frac{2(\bar{z}_j - m_{ij})}{\sigma_{ij}^2}\right)\\
&\quad \cdot G_{ij}(N-1)
\end{aligned}$$

with $\eta_m, \eta_\rho, \eta_h > 0$. The closed-loop error system is guaranteed with asymptotic convergence of $s_f(t), x_1(t)$, and $x_2(t)$, while all adaptation parameters are bounded.

Note that the above tuning laws have used the property $\frac{\partial V_s}{\partial u_N} = s_f$ from the gradient descent method applied on the adaptive law of the weight θ. The above adaptive law will cope with an inappropriate initial selection of fuzzy membership functions. Moreover, the stability property derived in Theorem 2 is not affected when using the online tuning laws.

V. Simulation Results

In this section, the proposed TS-RFNN sliding mode controller is applied to an uncertain time-delay system. Consider an uncertain time-delay system described by the dynamical equation (5) with $x = [\ x_1 \quad x_2 \quad x_3\ ]^T$, $B = [\ 0 \quad 0 \quad 1\ ]^T$, $g(x) = 1$, $h(x(t), x(t-d)) = \|x(t)\| + \|x(t-d)\| + sin(t)$, and

$$A + \triangle A(t) = \begin{bmatrix} -10 + sin(t) & 1 & 1 + sin(t) \\ 1 & -8 - cos(t) & 1 - cos(t) \\ 5 + cos(t) & 4 + 2sin(t) & 2 + cos(t) \end{bmatrix}$$

$$A_d + \triangle A_d(t) = \begin{bmatrix} 1 + sin(t) & 0 & 1 + sin(t) \\ 0 & 1 + cos(t) & 1 + cos(t) \\ 3 + sin(t) & 4 + cos(t) & 2 + sin(t) \end{bmatrix}.$$

It is easily checked that Assumptions 1∼2 are satisfied for the above system. Moreover, for Assumption 2, the uncertain matrices $\triangle A_{11}(t)$, $\triangle A_{12}(t)$, $\triangle A_{d11}(t)$, and $\triangle A_{d12}(t)$ are decomposed as Assumption 2 with $E_{12} = E_{22} = [\ 1 \quad 1\ ]^T$, $D_1 = D_2 = E_{11} = E_{21} = diag\{1, 1\}$, $C_1(t) = diag\{sin(t), -cos(t)\}$, $C_2(t) = diag\{sin(t), cos(t)\}$.

First, let us design the asymptotic sliding surface according to Theorem 1. By choosing $\epsilon = 0.2$ and solving the LMI problem (10), we obtain $\Lambda = [\ -0.4059 \quad -0.4270\ ]$. The error signal $s_f(x)$ is thus created from (8). Next, the TS-RFNN (1) is constructed with $n_i = 1$, $n_R = 8$, and $n_v = 4$. Since the T-S fuzzy rules are used in the FNN, the number of the input of the TS-RFNN can be reduced by an appropriate choice of THEN part of the fuzzy rules. Here the error signal $s_f(x)$ is taken as the input of the FNN, while the discussion region is characterized by 8 fuzzy sets with Gaussian membership functions as (2). Each membership function is set to the center $m_{ij} = -2 + 4(i-1)/(n_R - 1)$ and variance $\sigma_{ij} = 10$ for $i = 1 \sim n_R$ and $j = 1$. On the other hand, the basis vector of THEN part of fuzzy rules is chosen as $\mathrm{z} = [\ 1 \quad x_1 \quad x_2 \quad x_3\ ]^T$. Then, the fuzzy parameters v_j are tuned by the update law (15) with all zero initial condition (i.e., $\mathrm{v}_j(0) = 0$ for all j).

In this simulation, the update gains are chosen as $\eta_\theta = 0.04$ and $\eta_\delta = 0.01$. When assuming the initial state $x(\tau) = [\ 1 \quad 2 \quad 1\ ]^T$ for $-d \leq \tau \leq 0$ and delay time $d = 0.2$, the TS-RFNN sliding controller (12) designed from Theorem 3 leads to the control results shown in Figs. 2 and 3. The trajectory of the system states and error signal $s_f(x)$ asymptotically converge to zero. Figure 4 illustrates the corresponding control effort.

VI. Conclusion

In this paper, the robust control problem of a class of uncertain nonlinear time-delay systems has been solved by the proposed robust TS-RFNN control scheme. The TS-RFNN provides high capacity to learn complex functions with time-delay states. The sliding surface design using LMI techniques achieves an asymptotic sliding motion in the presence of mismatched uncertainty. Although the system has high uncertainties (here both state and input uncertainties are considered), the adaptive TS-RFNN guarantees the asymptotic convergence. Therefore, the resultant control scheme is suitable for dealing with more general nonlinear time-delay systems with uncertainties.

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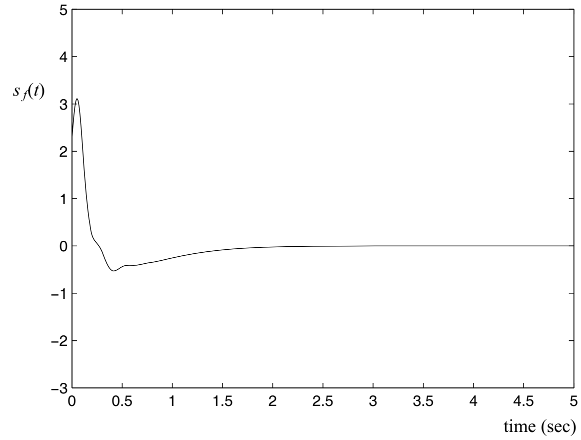

Fig. 2. The response of sliding surface signal $s_f(t)$.

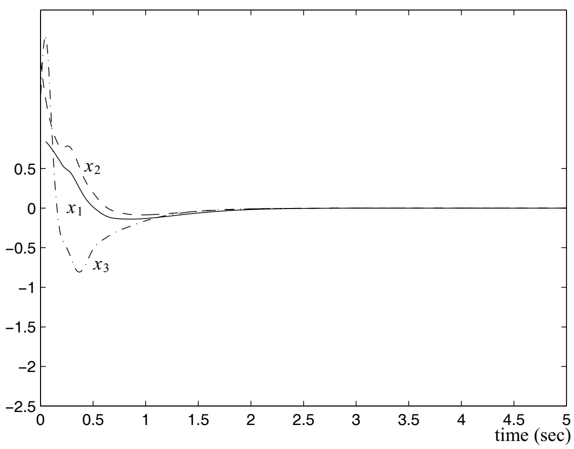

Fig. 3. The control response of the states.

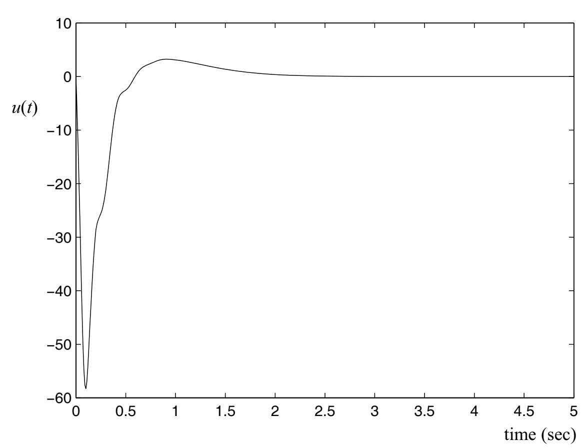

Fig. 4. The control force.