Proceedings of the 2007 IEEE Symposium on
Foundations of Computational Intelligence (FOCI 2007)

Effect of fin number and position on thermal behavior of natural convection in enclosed cavity using fuzzy controller algorithm

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Abstract
Laminar natural convection in an enclosed cavity with differentially heated walls and having fins on the cold wall is investigated. The effect of the number of fins and their position on the cold wall on heat transfer characteristics is analyzed. Governing equations of continuity, momentum and energy are solved using SIMPLER algorithm. A fuzzy controller with pre-defined set of fuzzy rules is implemented as a guiding mechanism for faster convergence. Relaxation factors are adjusted using fuzzy rules which guide the iterative scheme towards convergence in comparatively fewer numbers of iterations. A total of 16 cases with different fin aspect ratios (0.1 and 0.4) and their position on the cold wall are considered for a range of Grashof numbers (10^3, 10^5 and 10^7).

Keywords
Fuzzy logic, enclosed cavity, conjugate heat transfer, fin, SIMPLER

1. Introduction

Natural convection in enclosed cavities embedded with a fin or a series of fins on one wall has many industrial applications including reactor design, cryogenic systems, cooling of radioactive waste containers, and solar collectors [1]-[3]. With the presence of the protruding bodies in one or more walls the complexity of the problem increases and issue such as convergence of solution variables arise. Iterative methods form the basis of solving simultaneously the continuity, momentum, and energy equations in fluid flow and heat transfer associated with it. The SIMPLER algorithm [4], the basis of the present work, uses simple substitution in order to solve the discretized governing equations of fluid motion, energy, and scalar transport. However as stated in [5], the success of the iterative method in most CFD problems relies on the relaxation of state variables. The optimum relaxation factor depends on the nature of the problem, number of grid points used for discretization, grid spacing, iterative procedure used and other parameters. The optimum relaxation factor cannot be analytically determined. In relaxation methods, the value of the variable to be used for obtaining the solution in the next iteration is the value in the current iteration plus a fraction of the difference between the current value and the predicted value.

2. Cognitive Computing

Research concerned with using cognitive computing methods such as fuzzy logic or neural networks to aid CFD simulations are limited in number in the literature. Cort et al. [6] used a simple feedback control method to adjust the relaxation factors in one-dimensional finite element heat transfer simulation. Iida et al. [7] published a study in which wobbling adaptive control was applied to a CFD simulation of the Benard problem. Studies to improve the convergence of genetic algorithms using fuzzy control have been reported in the literature [8]-[10]. Xunliang et. al. [11] controlled the convergence criteria using fuzzy logic based on the residual ratio of momentum or energy equation. A fuzzy logic algorithm for solving turbulent flow conditions is reported by Dragojlovic et. al. [12].

3. Methodology

The relaxation method discussed in the present work enables and improves convergence by slowing down the update rate of the system matrix coefficients. The iterative scheme used in this work is dependent upon the relaxation factor according to the following equation:

$$\phi_p = \phi_p^* + \alpha \left(\frac{\sum a_{nb}\, \phi_{nb} + b}{a_p} - \phi_p^* \right)$$

where $0 < \alpha < 1$ is the relaxation factor, φ_p is the value of the state variable at node P to be used for the next iteration, ϕ_p^* is the value of the state variable at node P in the previous iteration, ϕ_{nb} are the values of the variables at the surrounding nodes and a_p, a_{nb}, and b are the constants from the discretized equation.

The present work deals with the computational fluid dynamics (CFD) simulation of laminar natural convection in an enclosed cavity with a single fin and series of fins attached to the cold wall using fuzzy logic guidance methodology. Conjugate heat transfer condition is applied on the hot wall. Based on the history of the solution curve the membership functions are adjusted using a pre-defined set of fuzzy rules. The objective of the present work is to study the heat transfer behavior due to the presence of fins by varying the number, position and aspect ratio of the fins, thereby adding complexity for the fuzzy controller. In all the cases analyzed the controller algorithm with a heuristic set of fuzzy rules was able to find a converged solution in comparatively lesser number of iterations than the constant relaxation factor. The trend in the Nusselt number for the above mentioned cases is analyzed for three different Grashof numbers and the optimum fin number and position on the cold wall is reported.

4. Problem Analyzed - Rectangular cavity with conjugate heat transfer in one wall and fin on the cold wall

A schematic of the problem considered is shown in Fig. 1. The problem investigated consists of a rectangular enclosure having width W and height H, ($H = W$) with conjugate heat transfer imposed in the wall of thickness t. Isothermal boundary conditions are applied at the extreme sides, i.e. cold (T_c) and hot wall (T_h) conditions at the left and right sides respectively. The cavity is filled with a constant property fluid (air, $Pr = 0.71$) and the horizontal sides are insulated. Three walls of the enclosure are assumed to be of zero wall thickness while the fourth, the right vertical wall, has a thickness t. Because of the temperature gradient along the x direction, a buoyancy-driven recirculation pattern appears in the cavity. The solid wall at the right side is simulated by substituting a very high value of the dynamic viscosity in the algorithm. The problem uses the domain distribution of 40 X 34, i.e. 40 grid lines along the x-axis and 34 along the y-axis. Out of the 40 grid lines used for discretizing x-axis a disproportionate share of 10 grid lines were used for simulating the solid wall conditions. The grid was packed close to the solid walls and the solid-fluid interface so that the boundary layer could be well resolved.

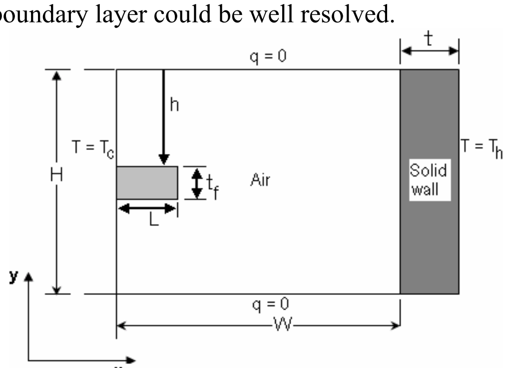


Figure 1. Fin attached to the cold wall of the enclosed cavity

A fin of thickness t_f and length L is mounted on the cold wall. Two aspect ratios of $L/W = 0.1$ and 0.4 are studied. The number and the position of the fins considered are schematically shown in Fig. 2. Table 1 defines the 8 cases investigated; in all the cases fin thickness is kept constant.

Natural convection in an enclosed cavity due to thin fin is considered in [13]-[14]; however, the fins were attached to the hot wall. Numerical analysis of natural convection in an enclosure with fins attached to the cold wall is investigated in [15]-[16]; however, multiple fins were taken into account and conjugate effects were not considered.

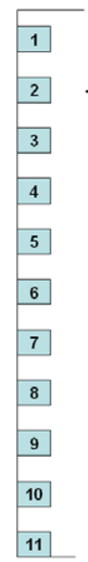


Table 1. Different cases investigated

Case	Fin
1	6
2	1,11
3	1,6,11
4	3,6,9
5	5,6,7
6	4,6,8
7	1,3,5,7,9,11
8	2,4,6,8,10

Figure 2. Fin position

The flow was assumed to be Newtonian, incompressible, laminar, two-dimensional and steady. Viscous dissipation was neglected. All thermophysical properties were assumed constant and independent of the pressure and temperature fluctuations. The fluid density was treated using the Boussinesq approximation. The buoyancy force is in the y-direction. The conservation equations for continuity, momentum, and energy are given in Patankar [4] as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu\nabla^2 u\ ,$$

$$\rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + g\beta\rho\left(T - T_a\right) + \mu\nabla^2 v$$

$$\rho c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\nabla^2 T$$

where u, v are the velocity components in x and y directions, μ, ρ, k and c_p are the fluid viscosity, density, thermal conductivity and specific heat respectively, β is the expansion coefficient and T_a is the ambient temperature. The velocity components at the boundaries are taken as zero. At the solid-liquid interface, the temperature and the heat flux must be continuous; this condition is mathematically expressed as:

$$\left(\frac{\partial\theta}{\partial x}\right)_{fluid} = \frac{k_w}{k_f}\left(\frac{\partial\theta}{\partial x}\right)_{wall}$$

where θ is the non-dimensional temperature given by:

$$\theta = \frac{T - T_c}{T_h - T_c}$$

and k_w and k_f represent the thermal conductivities of wall and fluid respectively. The problem was solved for a range of Grashof numbers from 10^3 to 10^7; Grashof number is defined as:

$$Gr = \frac{\rho^2 g\beta\left(T_H - T_C\right)L^3}{\mu^2}$$

5. Fuzzy logic approach

The conservation equations listed above were discretized by a finite volume approach as defined by the following equation:

$$a_p\phi_p = \sum a_{nb}\phi_{nb} + b$$

where a_p is the coefficient for the point P under consideration, a_{nb}'s are the coefficients of neighboring grid points, ϕ_p is the value of the dependent variable for the equation under consideration, ϕ_{nb} 's are the values of the neighboring grid points and b is the source term. The generic variable ϕ is used to represent *u, v,* and *T.* Each of the velocity components, temperature and pressure are relaxed by a separate relaxation factors. The membership functions are adjusted using a pre-defined set of fuzzy rules. These rules are designed to adjust the relaxation factors during the iterative process. To supply more information to the decision making system the controller algorithm takes a larger set of characteristic values. The number of iterations the new algorithm considers is N where

$$N = n \quad \text{if} \quad n < 50$$
$$N = 50 \quad \text{if} \quad n \geq 50$$

and n is the number of iterations, this includes the current iteration and a moving window of up to 50 earlier iterations. The algorithm is based on iterative oscillations and basically consists of two subroutines: one evaluated the nature of 'solution history curve' and the other controls the features of this curve during iteration in order to produce and preserve those features that bring the fastest convergence. The characteristic quantity that represents the solution at the n^{th} iteration is the square norm of the solution, also known as the magnitude of the solution vector:

$$S^{\varphi}\left(n\right) = \sqrt{\sum_{i=1}^{l}\sum_{j=1}^{m}\left[\varphi_n\left(i,j\right)\right]^2}$$

where i and j are the node numbers in x and y directions respectively, φ_n is the nodal value of the state variable φ at the iteration n, l is the total number of nodes in the x direction and m is the total number of nodes in the y direction. At every iteration, the assumed values of the solution vector were updated with under-relaxed values according to the following equation:

$$\phi_n^* = \phi_{n-1} + \alpha\left(\phi_n + \phi_{n-1}\right)$$

where α is a relaxation factor which varies between 0 and 1. The details of the present algorithm are given in Dragojlovic et. al. [17]. The degrees of the membership of the input and output variables in a given fuzzy set are based on their actual values. The set of rules are governed by the fuzzy membership functions which vary between 0 and 1, as shown in Fig. 3. Here **P** and **N** represent the positive and negative sides and **S** and **B** represent the small and big respectively.

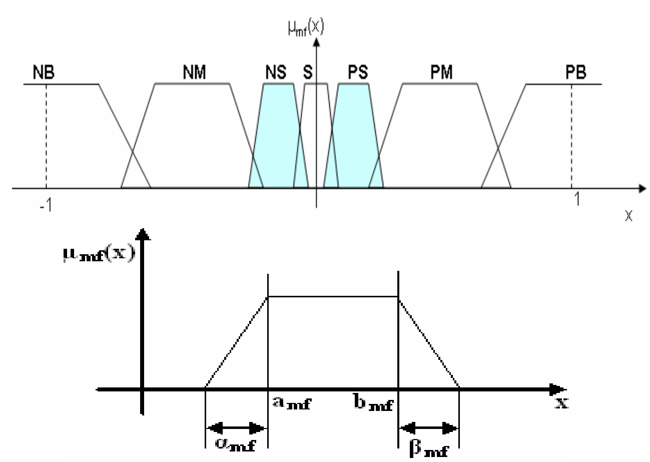


Figure 3. Distribution of fuzzy membership functions

The values of the parameters a_{mf}, b_{mf}, α_{mf} and β_{mf} for each membership function which apply to error and the change in error are listed in Table 2. The fuzzy membership functions which are applied to the increments in the relaxation factors are listed in Table 3. For this set the defining parameters are scaled down in value compared to those applied to error and error differences in order to prevent disturbances in the solution vector during iteration caused by large positive increments in relaxation factor.

Table 2. Fuzzy membership functions for errors and error differences

Membership function	a_{mf}	b_{mf}	α_{mf}	β_{mf}
Negative big (NB)	-1	-0.65	0	0.25
Negative medium (NM)	-0.4	-0.2	0.25	0.125
Negative small (NS)	-0.075	0	0.125	0
Positive small (PS)	0	0.075	0	0.125
Positive medium (PM)	0.2	0.4	0.125	0.25
Positive big (PB)	0.65	1	0.25	0

Table 3. Fuzzy membership functions for relative increments in relaxation factor

	a_{mf}	b_{mf}	α_{mf}	β_{mf}
NB	-0.0025	-0.00162	0	-0.000625
NM	-0.001	-0.0005	-0.00062	-0.000312
NS	-0.000188	0	0.000312	0
PS	0	0.000188	0	0.000312
PM	0.0005	0.001	0.000312	0.000625
PB	0.00162	0.0025	0.000625	0

The membership of the input and output variables in the fuzzy sets is shown in Table 4.

Table 4. Fuzzy rule set

$\Delta\gamma^{\varphi}(n)$ \ $\gamma^{\varphi}(n)$	NB	NM	NS	PS	PM	PB
PB		NS	NM	NM	NB	NB
PM	PS		NS	NM	NM	NB
PS	PM	PS		NS	NM	NM
NS	PM	PM	PS		NM	NM
NM	PB	PM	PM	PS		NS
NB	PB	PB	PM	PM	PS	

Thus the rule set is a sequence of "If—then" type rules which engage the fuzzy linguistic variables in order to mimic the human, qualitative way of making decisions. An example rule used in this algorithm is:

IF *the error* $\gamma^{\varphi}(n)$ *is* ***positive medium*** *and the error difference* $\Delta\gamma^{\varphi}(n)$ *is* ***negative big***

THEN *the change in the relaxation factor* $\delta^{\varphi}(n)$ *is* ***positive small***

Each rule gives a fuzzy set as an output, which is a contribution to the final decision on the increment in the relaxation factor. The output from the example cited above is the fuzzy set "positive small". The membership functions were chosen heuristically based on expert experience with laminar CFD problems. The particular set used here has also successfully solved a wide range of laminar flow problems, including a driven cavity, a backward facing step, and a Dean flow. The algorithm is not necessarily limited to laminar CFD problems, and does not rely on any general features of such problems. In fact it has been applied to a non-linear thermal radiation simulation successfully, showing the potential for solving a wide range of problems. The degree of membership of the input and output variables in a given fuzzy set are based on their actual values. If the variable $\gamma^{\varphi}(n)$ has a degree of membership of 0.75 in the fuzzy set "*positive big*", for example, this value is the degree of truth to which $\gamma^{\varphi}(n)$ can be considered positive and big in magnitude. The value of '0' would mean that $\gamma^{\varphi}(n)$ is not positive big at all while the value of 1 means that $\gamma^{\varphi}(n)$ fully belongs to the set "*positive big*".

6. Comparison with benchmark results

Table 5 shows the comparison of the results obtained from the controller algorithm with multiple fins attached to the cold wall for Rayleigh number of 10^4 with the benchmark results. The grid size used for the comparison was 40 X 150. C_{ratio} was kept at 28.6.

Table 5. Comparison of fuzzy logic algorithm with published solutions (enclosed cavity having fins on the cold wall – no conjugate effects)

Number of fins	3	7	9	11
Yucel [15]	1.542	1.565	1.575	1.592
Scozia [16]	1.528	1.571	1.586	1.576
Present work	1.5378	1.5687	1.591	1.5794

7. Results

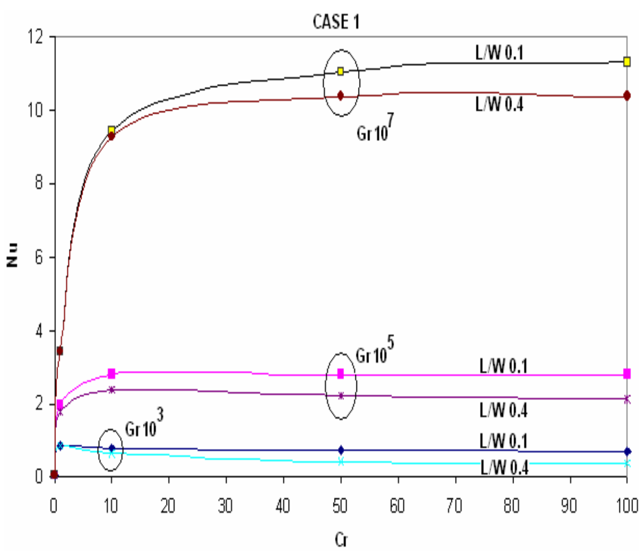
7.1 Effect of fin aspect ratio on Nusselt number

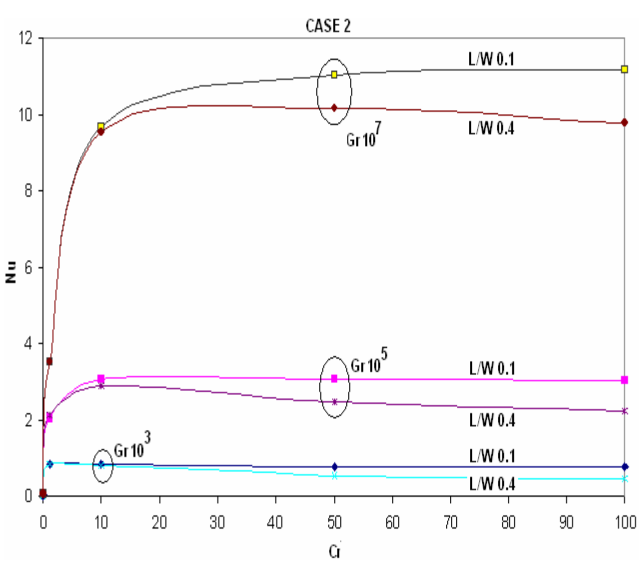
Figure 4 shows the variation of the Nusselt number with different conductance ratios at fin aspect ratios of 0.1 and

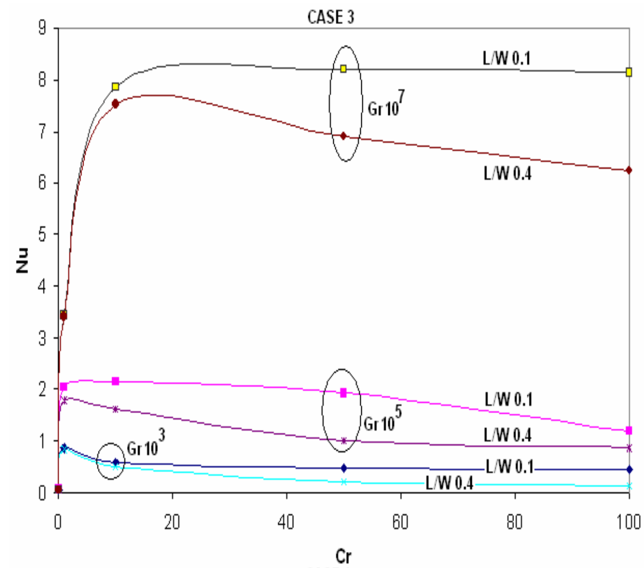
0.4 for different cases. Conductance ratio (Cr) is defined as:

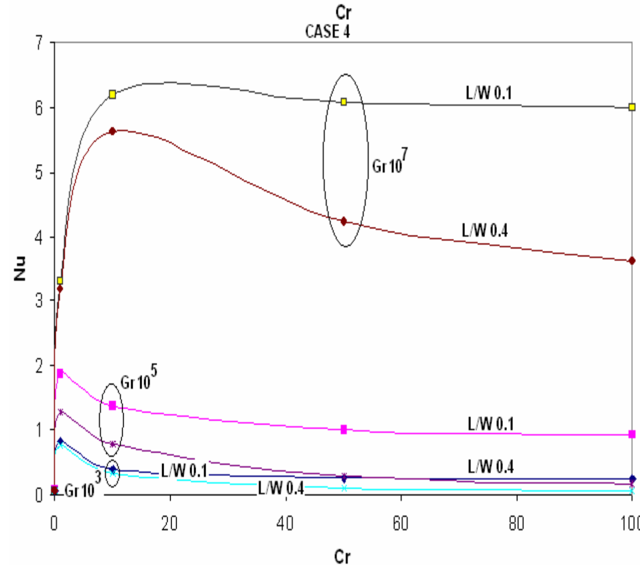
$$Cr = \frac{k_w t}{k_f W}$$

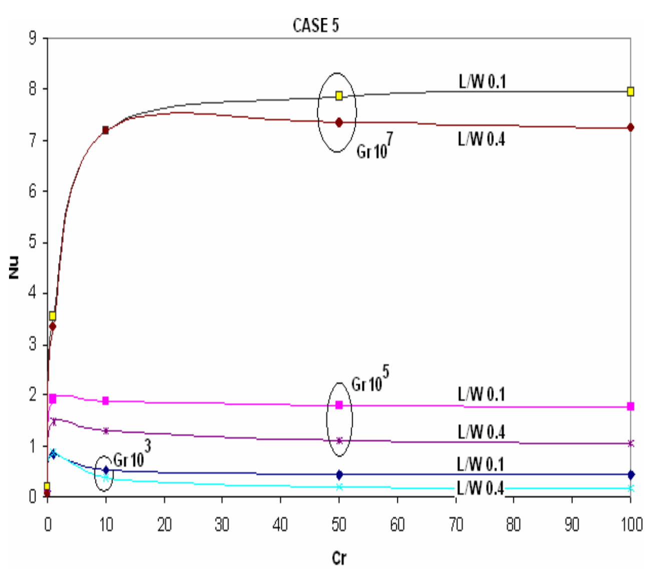
where k_w and k_f are the thermal conductivities of wall and fluid and t and W are the respective lengths. It can be seen from the figure that for all the cases increasing the aspect ratio decreases the Nusselt number and this effect is prominent at higher values of Cr (=50.0 and 100.0). Higher aspect ratio causes the blockage of heat circulation patterns thereby reducing the overall heat transfer area, which reduces the Nusselt number. A higher value of Cr adds to the aforementioned effect. For $Gr = 10^3$, the effect of aspect ratio and the number of fins is insignificant. In this case the flow is mainly governed by the conduction and the flow patterns are minimally disturbed by the length and the number of the fins.











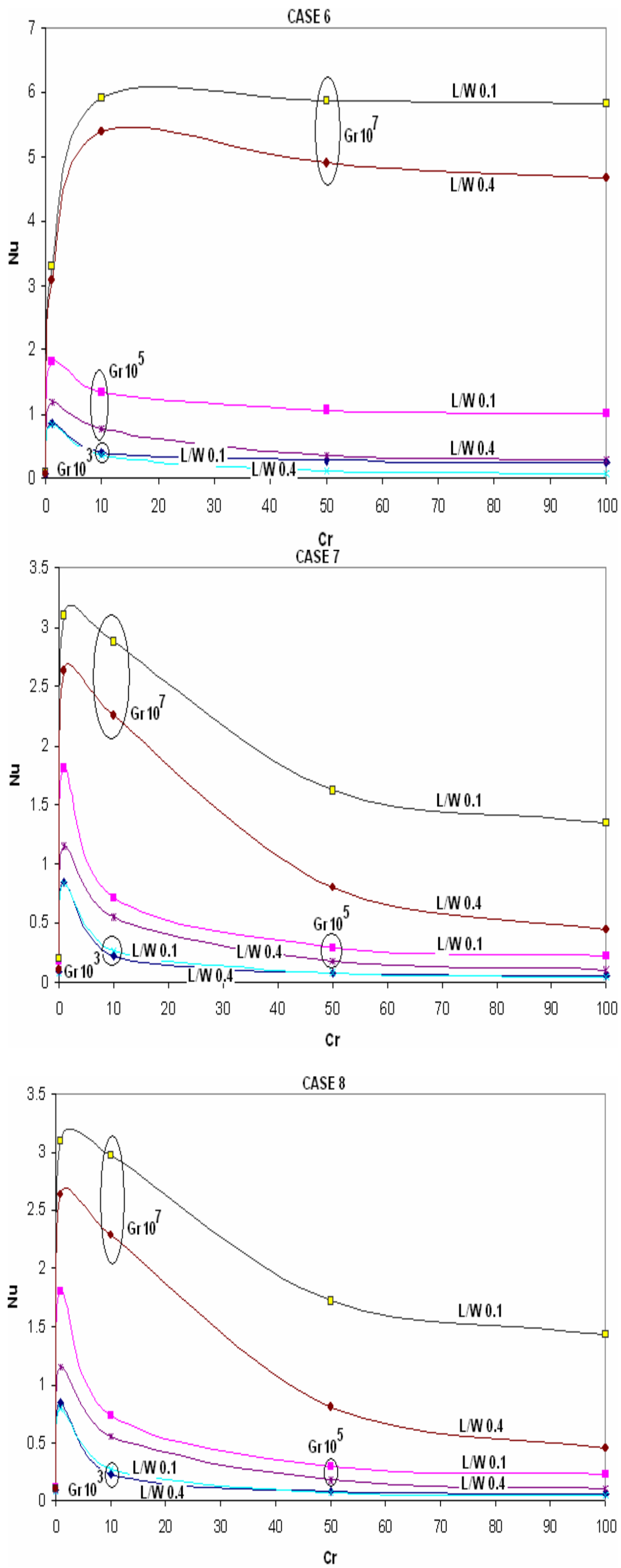


Figure 4. Nusselt number vs. Cr for different cases studied

7.2 Effect of number and position of fin on Nusselt number

Figure 5 compares the Nusselt number at Grashof number of 10^5 for the two aspect ratios. Interestingly, for this case and all the cases considered, Case 2 gives the highest value of Nusselt number indicating an improved heat transfer as compared to the other cases. Keeping the fins at positions 1 and 11 does not affect the circulation flow cells. At higher *Gr* and *Cr* the flow cells are less disturbed when the fin is at the center of the wall which enhances heat transfer. Also for lower values of *Gr* it is found that the fins should be placed near the top and bottom surfaces of the wall while for higher values of *Gr* and *Cr* only one fin at the center of the wall is an intelligent choice for enhancing thermal characteristics of the cavity.

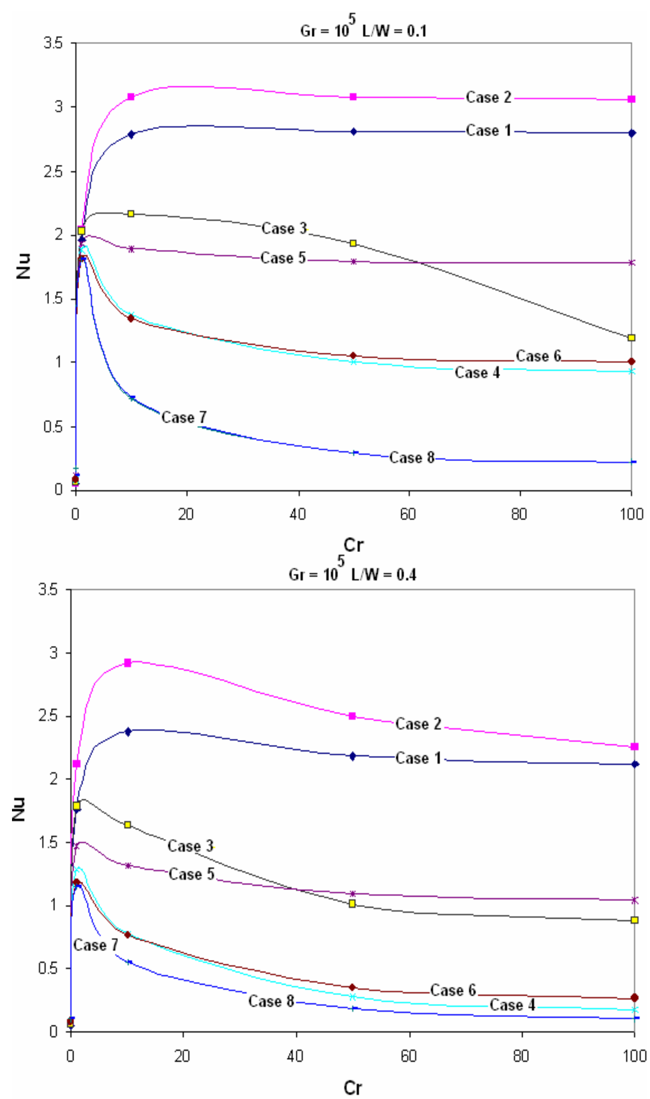


Figure 5. Nusselt number variation with Cr at Gr = 10^5 for different aspect ratios

8. Conclusions

The present work explores the heat transfer characteristics of laminar natural convection in enclosed cavity with fin attached to the cold wall. Fuzzy rules with adjustable membership functions were implemented for faster convergence. The fin position, its length and number were varied to study the effect on the Nusselt number in the domain. The control algorithm was linked with the sequential solver named SIMPLER, which solves the partial different equations of fluid flow and heat transfer. A short fin with aspect ratio of 0.1 is better for improving heat transfer behavior as compared to a longer fin with aspect ratio of 0.4.

9. References

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