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Fuzzy Optimization with Multi-Objective Evolutionary Algorithms: a Case Study

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***Abstract*— This paper outlines a real-world industrial problem for product-mix selection involving 8 decision variables and 21 constraints with fuzzy coefficients. On one hand, a multi-objective optimization approach to solve the fuzzy problem is proposed. Modified S-curve membership functions are considered. On the other hand, an ad hoc Pareto-based multi-objective evolutionary algorithm to capture multiple non dominated solutions in a single run of the algorithm is described. Solutions in the Pareto front corresponds with the fuzzy solution of the former fuzzy problem expressed in terms of the group of three $(\vec{x}, \mu, \alpha)$, i.e., optimal solution - level of satisfaction - vagueness factor. Decision-maker could choose, in a posteriori decision environment, the most convenient optimal solution according to his level of satisfaction and vagueness factor. The proposed algorithm has been evaluated with the existing methodologies in the field and the results have been compared with the well-known multi-objective evolutionary algorithm NSGA-II.**

I. INTRODUCTION

It is well known that optimization problems arise in a variety of situations. Particularly interesting are those concerning management problems as decision makers usually state their data in a vague way: "high benefits", "as low as possible", "important savings", etc. Because of this vagueness, managers prefer to have not just one solution but a set of them, so that the most suitable solution can be applied according to the state of existing decision of the production process at a given time and without increasing delay. In these situations fuzzy optimization is an ideal methodology, since it allows us to represent the underlying uncertainty of the optimization problem, while finding optimal solutions that reflect such uncertainty and then applying them to possible instances, once the uncertainty has been solved. This allows us to obtain a model of the behavior of the solutions based on the uncertainty of the optimization problem.

Fuzzy constrained optimization problems have been extensively studied since the seventies. In the linear case, the first approaches to solve the so-called fuzzy linear programming problem appeared in [2], [15] and [20]. Since then, important contributions solving different linear models have been made and these models have been the subject of a substantial amount of work. In the nonlinear case [1], [6], [13] the situation is quite different, as there is a wide variety of specific and both practically and theoretically relevant nonlinear problems, with each having a different solution method.

In this paper a real-life industrial problem for product-mix selection involving 21 constraints and 8 variables has been considered. This problem occurs in production planning in which a decision-maker plays a pivotal role in making decision under a highly fuzzy environment. Decision-maker should be aware of his/her level of satisfaction as well as degree of vagueness while making the product-mix decision. Thus, the authors have analyzed using the sigmoidal membership function, the fuzziness patterns and fuzzy sensitivity of the solution. In [16], [17], [18] a linear case of the problem is solved by using a linear programming iterative method which is repeatedly applied for different degrees of satisfaction values. In this paper, a non linear case of the problem is considered and we propose a multi-objective optimization approach in order to capture solutions for different degrees of satisfaction and vagueness factors with a simple run of the algorithm. This multi-objective optimization approach has been proposed by authors in [8], [9], [10] within a fuzzy optimization general context.

Given this background, the paper is organized as follows: in section II a non linear case study in chocolate manufacturing firm is described and its mathematical formulation stated. Section III propose a multi-objective optimization approach for this problem and an ad hoc multi-objective evolutionary algorithm. Section IV shows results obtained with the proposed multi-objective evolutionary algorithms and the well-known NSGA-II algorithm. Finally, section V offers the main conclusions and future research direction.

II. NON LINEAR CASE STUDY IN CHOCOLATE MANUFACTURING FIRM

Due to limitations in resources for manufacturing a product and the need to satisfy certain conditions in manufacturing and demand, a problem of fuzziness occurs in industrial systems. This problem occurs also in chocolate manufacturing when deciding a mixed selection of raw materials to produce varieties of products. This is referred here to as the product-mix selection problem [14].

There are a number of products to be manufactured by mixing different raw materials and using several varieties of processing. There are limitations in resources of raw materials and facility usage for the varieties of processing. The raw materials and facilities usage required for manufacturing each product are expressed by means of fuzzy coefficients. There are also some constraints imposed by marketing department such as product-mix requirement, main product line requirement and lower and upper limit of demand for each product. It is necessary to obtain maximum profit with certain degree of satisfaction of the decision-maker.

A. Optimization Problem with fuzzy coefficients

The firm Chocoman Inc. manufactures 8 different kinds of chocolate products. Input variables x_i represent the amount of manufactured product in 10^3 units.

The function to maximize is the total profit obtained calculated as the summation of profit obtained with each product and taken into account the applied discount. Table I shows the profit (c_i) and discount (d_i) for each product i.

There are 8 raw materials to be mixed in different proportions and 9 processes (facilities) to be utilized. Therefore, there are 17 constraints with fuzzy coefficients separated in two sets such as raw material availability and facility capacity. These constraints are inevitable for each material and facility that is based on the material consumption, facility usage and the resource availability. Table II shows fuzzy coefficients $\widetilde{a}_{ij}$ represented by $\left(a^l_{ij}, a^h_{ij}\right)$ for required materials and facility usage j for manufacturing each product i and non fuzzy coefficients b_j for availability of material or facility j.

Additionally, the following constraints were established by the sales department of Chocoman Inc.:

1) Main product line requirement. The total sales from candy and wafer products should not exceed 15% of the total revenues from the chocolate bar products. Table I show the values of sales/revenues (r_i) for each product i.
2) Product mix requirements. Large-sized products (250 g) of each type should not exceed 60% of the small-sized product (100 g).

Finally, the lower limit of demand for each product i is 0 in all cases, while the upper limit (u_i) is shown in table I.

B. Membership Function for Coefficients

We consider the modified S-curve membership function proposed in [16]. For a value x, the degree of satisfaction $\mu_{\widetilde{a}_{ij}}(x)$ for fuzzy coefficient $\widetilde{a}_{ij}$ is given by the membership function given in (1).

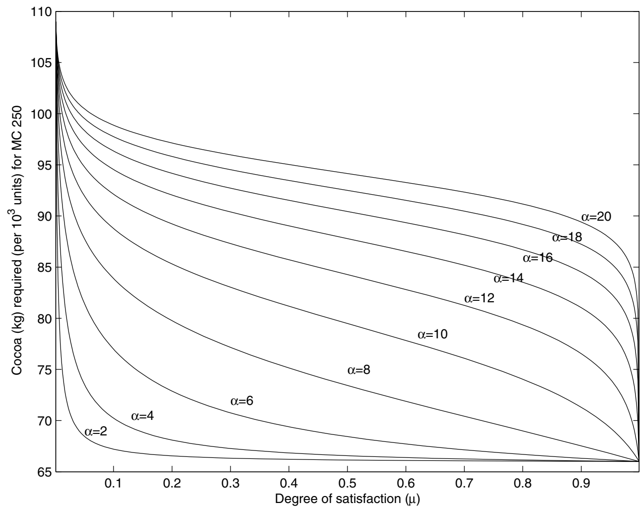

Fig. 1. Membership function for coefficient $\widetilde{a}_{11}$.

$$\mu_{\widetilde{a}_{ij}}(x) = \begin{cases} 1.000 & x < a^l_{ij} \\ 0.999 & x = a^l_{ij} \\ \dfrac{B}{1 + Ce^{\alpha\left(\frac{x - a^l_{ij}}{a^h_{ij} - a^l_{ij}}\right)}} & a^l_{ij} < x < a^h_{ij} \\ 0.001 & x = a^h_{ij} \\ 0.000 & x > a^h_{ij} \end{cases} \quad (1)$$

Given a degree of satisfaction value μ, the crisp value $a_{ij}|_\mu$ for fuzzy coefficient $\widetilde{a}_{ij}$ can be calculated using (2).

$$\widetilde{a}_{ij}|_\mu = a^l_{ij} + \left(\frac{a^h_{ij} - a^l_{ij}}{\alpha}\right) \ln \frac{1}{C}\left(\frac{B}{\mu} - 1\right) \quad (2)$$

The vagueness factor α determines the shape of the membership function, while B and C values can be calculated from α, (3) and (4).

$$C = -\frac{0.998}{(0.999 - 0.001e^{\alpha})} \quad (3)$$

$$B = 0.999\,(1 + C) \quad (4)$$

Figure 1 shows membership functions with different vagueness factors α for cocoa required in manufacturing milk chocolate 250 g (coefficient $\widetilde{a}_{11}$).

C. Problem Formulation

Given a degree of satisfaction value μ, the fuzzy constrained optimization problem can be formulated [10], [17] as the non linear constrained optimization problem shown in table III.

III. A Multi-Objective Evolutionary Approach

In this section, we propose a multi-objective optimization approach to solve the problem shown in table III for all satisfaction degree values and vagueness factors. In the multi-objective problem, the Pareto front represents the fuzzy

TABLE I

PROFIT (c_i), DISCOUNT (d_i), DEMAND (u_i) AND REVENUES/SALES (r_i) IN \$ PER 10^3 UNITS

Product (x_i)	**Synonym**	**Profit** (c_i)	**Discount** (d_i)	**Demand** (u_i)	**Revenues/Sales** (r_i)
$x_1 =$ Milk chocolate, 250 g	MC 250	$c_1 = 180$	$d_1 = 0.18$	$u_1 = 500$	$r_1 = 375$
$x_2 =$ Milk chocolate, 100 g	MC 100	$c_2 = 83$	$d_2 = 0.05$	$u_2 = 800$	$r_2 = 150$
$x_3 =$ Crunchy chocolate, 250 g	CC 250	$c_3 = 153$	$d_3 = 0.15$	$u_3 = 400$	$r_3 = 400$
$x_4 =$ Crunchy chocolate, 100 g	CC 100	$c_4 = 72$	$d_4 = 0.06$	$u_4 = 600$	$r_4 = 160$
$x_5 =$ Chocolate with nuts, 250 g	CN 250	$c_5 = 130$	$d_5 = 0.13$	$u_5 = 300$	$r_5 = 420$
$x_6 =$ Chocolate with nuts, 100 g	CN 100	$c_6 = 70$	$d_6 = 0.14$	$u_6 = 500$	$r_6 = 175$
$x_7 =$ Chocolate candy	CANDY	$c_7 = 208$	$d_7 = 0.21$	$u_7 = 200$	$r_7 = 400$
$x_8 =$ Chocolate wafer	WAFER	$c_8 = 83$	$d_8 = 0.1$	$u_8 = 400$	$r_8 = 150$

TABLE II

RAW MATERIAL AND FACILITY USAGE REQUIRED (PER 10^3 UNITS) (a^l_{ij}, a^h_{ij}) AND AVAILABILITY (b_j).

Material or Facility	**MC 250**	**MC 100**	**CC 250**	**CC 100**	**CN 250**	**CN 100**	**CANDY**	**WAFER**	**Availability**
Cocoa (kg)	$66, 109$	$26, 44$	$56, 94$	$22, 37$	$37, 62$	$15, 25$	$45, 75$	$9, 21$	100000
Milk (kg)	$47, 78$	$19, 31$	$37, 62$	$15, 25$	$37, 62$	$15, 25$	$22, 37$	$9, 21$	120000
Nuts (kg)	$0, 0$	$0, 0$	$28, 47$	$11, 19$	$56, 94$	$22, 37$	$0, 0$	$0, 0$	60000
Cons.sugar (kg)	$75, 125$	$30, 50$	$66, 109$	$26, 44$	$56, 94$	$22, 37$	$157, 262$	$18, 30$	200000
Flour (kg)	$0, 0$	$0, 0$	$0, 0$	$0, 0$	$0, 0$	$0, 0$	$0, 0$	$54, 90$	20000
Alum.foil (ft^2)	$375, 625$	$0, 0$	$375, 625$	$0, 0$	$0, 0$	$0, 0$	$0, 0$	$187, 312$	500000
Paper (ft^2)	$337, 562$	$0, 0$	$337, 563$	$0, 0$	$337, 562$	$0, 0$	$0, 0$	$0, 0$	500000
Plastic (ft^2)	$45, 75$	$95, 150$	$45, 75$	$90, 150$	$45, 75$	$90, 150$	$1200, 2000$	$187, 312$	500000
Cooking (ton-hours)	$0.4, 0.6$	$0.1, 0.2$	$0.3, 0.5$	$0.1, 0.2$	$0.3, 0.4$	$0.1, 0.2$	$0.4, 0.7$	$0.1, 0.12$	1000
Mixing (ton-hours)	$0, 0$	$0, 0$	$0.1, 0.2$	$0.04, 0.07$	$0.2, 0.3$	$0.07, 0.12$	$0, 0$	$0, 0$	200
Forming (ton-hours)	$0.6, 0.9$	$0.2, 0.4$	$0.6, 0.9$	$0.2, 0.4$	$0.6, 0.9$	$0.2, 0.4$	$0.7, 1.1$	$0.3, 0.4$	1500
Grinding (ton-hours)	$0, 0$	$0, 0$	$0.2, 0.3$	$0.07, 0.12$	$0, 0$	$0, 0$	$0, 0$	$0, 0$	200
Wafer making (ton-hours)	$0, 0$	$0, 0$	$0, 0$	$0, 0$	$0, 0$	$0, 0$	$0, 0$	$0.2, 0.4$	100
Cutting (hours)	$0.07, 0.12$	$0.07, 0.12$	$0.07, 0.12$	$0.07, 0.12$	$0.07, 0.12$	$0.07, 0.12$	$0.15, 0.25$	$0, 0$	400
Packaging 1 (hours)	$0.2, 0.3$	$0, 0$	$0.2, 0.3$	$0, 0$	$0.2, 0.3$	$0, 0$	$0, 0$	$0, 0$	400
Packaging 2 (hours)	$0.04, 0.06$	$0.2, 0.4$	$0.04, 0.06$	$0.2, 0.4$	$0.04, 0.06$	$0.2, 0.4$	$1.9, 3.1$	$0.1, 0.2$	1200
Labor (hours)	$0.2, 0.4$	$0.2, 0.4$	$0.2, 0.4$	$0.2, 0.4$	$0.2, 0.4$	$0.2, 0.4$	$1.9, 3.1$	$1.9, 3.1$	1100

TABLE III

CONSTRAINED OPTIMIZATION PROBLEM FOR CHOCOMAN INC. WITH A GIVEN SATISFACTION DEGREE VALUE μ AND A VAGUENESS FACTOR α.

$$
\begin{array}{ll}
Maximize & \displaystyle\sum_{i=1}^{8} \left(c_i x_i - d_i x_i^2\right) \\
Subject\ to: & \\
& \displaystyle\sum_{i=1}^{8} \left[a^l_{ij} + \left(\frac{a^h_{ij} - a^l_{ij}}{\alpha} \right) \ln \frac{1}{C} \left(\frac{B}{\mu} - 1 \right) \right] x_i - b_j \leq 0, \ j = 1, \ldots, 17 \\
& \displaystyle\sum_{i=7}^{8} r_i x_i - 0.15 \sum_{i=1}^{6} r_i x_i \leq 0 \\
& x_1 - 0.6 x_2 \leq 0 \\
& x_3 - 0.6 x_4 \leq 0 \\
& x_5 - 0.6 x_6 \leq 0 \\
& 0 \leq x_i \leq u_i, \ i = 1, \ldots, 8
\end{array}
$$

solution of the former fuzzy optimization problem. Two new input variables and two new objectives are added in order to find the optimal solution for each degree of satisfaction value and vagueness factor [8], [9], [10]. Table IV shows the multi-objective constrained optimization problem for Chocoman Inc. In this problem, x_9 represents the degree of satisfaction value and x_{10} represents the vagueness factor, which must be minimized and maximized respectively to generate the desired Pareto front.

Multi-objective Pareto-based evolutionary algorithms [3], [5] are especially appropriate to solve multi-objective non-linear optimization problems because they can capture a set of Pareto solutions in a single run of the algorithm. We propose an ad hoc multi-objective Pareto-based evolutionary algorithm to solve the Chocoman Inc. problem. The algorithm uses a real-coded representation, uniform and arithmetical cross, uniform, non-uniform and minimal mutation [7]. Diversity among individuals is maintained by using an ad-hoc elitist generational replacement technique.

The algorithm have a population P of N solutions. For each solution i, f^i_j is the value for j-th objective ($j = 1, \ldots, n$) and g^i_j is the value for j-th constraint ($j = 1, \ldots, m$). For Chocoman Inc. problem, $n = 3$ and $m = 21$.

TABLE IV

MULTI-OBJECTIVE CONSTRAINED OPTIMIZATION PROBLEM FOR CHOCOMAN INC.

$$
\begin{array}{ll}
\textit{Maximize} & \sum_{i=1}^{8}\left(c_i x_i - d_i x_i^2\right) \\
\textit{Minimize} & x_9 \\
\textit{Maximize} & x_{10} \\
\textit{Subject to:} & \\
& \sum_{i=1}^{8}\left[a_{ij}^{l} + \left(\frac{a_{ij}^{h}-a_{ij}^{l}}{x_{10}}\right)\ln\frac{1}{C}\left(\frac{B}{x_9}-1\right)\right]x_i - b_j \leq 0,\ j=1,\ldots,17 \\
& \sum_{i=7}^{8} r_i x_i - 0.15\sum_{i=1}^{6} r_i x_i \leq 0 \\
& x_1 - 0.6x_2 \leq 0 \\
& x_3 - 0.6x_4 \leq 0 \\
& x_5 - 0.6x_6 \leq 0 \\
& 0 \leq x_i \leq u_i,\ i=1,\ldots,8 \\
& 0.001 \leq x_9 \leq 0.999 \\
& 2 \leq x_{10} \leq 20
\end{array}
$$

TABLE V

UTOPIA MINIMUM AND MAXIMUM VALUE FOR CHOCOMAN INC. PROBLEM.

	Utopia min.	**Utopia max.**
Objective 1	140000	210000
Objective 2	0.001	0.999
Objective 3	2	20

Given a population P of N individuals, N children are generated by random selection, crossing and mutation. Parents and children are ordered in $\left(\left\lfloor \sqrt[n-1]{N}\right\rfloor + 1\right)^{n-1}$ slots. A solution i belongs to slot s_i such that:

$$
s_i = \sum_{j=2}^{n}\left\lceil \frac{f_j^i - f_j^{umin}}{f_j^{umax} - f_j^{umin}}\left\lfloor \sqrt[n-1]{N}\right\rfloor \right\rceil \left(\left\lfloor \sqrt[n-1]{N}\right\rfloor\right)^{n-j}
$$

where f_j^{umax} and f_j^{umin} are the utopia maximum and minimum values for the j-th objective. For Chocoman Inc. problem, utopia minimum and maximum values are shown in table V.

The order inside slots is established with the following criteria. Position p_i of solution i is lower than position p_j of solution j in slot if:

- i is feasible and j is unfeasible, or
- i and j are unfeasible and $g_{max}^{i} \leq g_{max}^{j}$, or
- i and j are feasible and i dominates j
- i and j are feasible and non dominated and $cd_i > cd_j$.

where $g_{max}^{i} = \max_{j=1,\ldots,m}\left\{g_j^i\right\}$

and cd_i is a metric for the crowding distance of solution i:

$$
cd_i = \begin{cases} \infty,\ if\ f_j^i = f_i^{max}\ or\ f_j^i = f_i^{min}\ for\ any\ j \\ \sum_{j=1}^{n}\frac{f_j^{sup_i} - f_j^{inf_i}}{f_j^{max} - f_j^{min}},\ in\ other\ case \end{cases}
$$

where $f_j^{max} = \max_{i=1,\ldots,N}\left\{f_j^i\right\}$ and $f_j^{min} = \min_{i=1,\ldots,N}\left\{f_j^i\right\}$

$f_j^{sup_j^i}$ is the value of the j-th objective for the solution higher adjacent in the j-th objective to i,

$f_j^{inf_j^i}$ is the value of the j-th objective for the solution lower adjacent in the j-th objective to i.

The new population is obtained by selecting the N best individual from the parent and children. The following heuristic rule is considered to establish an order. Solution i is best than solution j if:

- $p_i < p_j$, or
- $p_i = p_j$ and $cd_i > cd_j$

where p_i is the position of solution i in its slot.

IV. EXPERIMENTS AND RESULTS

To compare the algorithms performance in multi-objective optimization, we have followed an empirical methodology similar to the proposed in [11] and [12]. It has been used a measure ν that calculates the fraction of the space which is not dominated by any of the solutions obtained by the algorithm ([11], [21]). The aim is to minimize the value of ν. This measure estimates both the distance of solutions to the real Pareto front and the spread. Value ν can be calculated as shown in (5) where f_j^i is the value for the j-th objective of the i-th solution in population P' which is composed by the N' non dominated solutions of P and f_j^{umax} and f_j^{umin} are the utopia maximum and minimum value for the j-th objective.

$$
\nu = 1 - \frac{\sum_{i=1}^{N'}\left[\left(f_n^{umax} - f_n^i\right)\prod_{j=1}^{n-1}\left(f_j^{sup_j^i} - f_j^i\right)\right]}{\prod_{j=1}^{n}\left(f_j^{umax} - f_j^{umin}\right)} \tag{5}
$$

The parameters were set up using a previous process using a methodology similar to the one proposed in [11]. Table VI shows the parameters obtained.

TABLE VI

PARAMETERS IN THE RUN OF THE PROPOSED ALGORITHM AND NSGA-II FOR CHOCOMAN INC. PROBLEM.

Number of iterations	$T = 10000$
Population size	$N = 1000$
Cross probability	$pCross = 0.8$
Mutation probability	$pMutate = 0.5$
Uniform cross probability	$pUniformCross = 0.7$
Uniform mutation probability	$pUniformMutate = 0.7$
Parameter c for non uniform mutation	$c = 2.0$

Various metrics for both convergence and diversity of the populations obtained have been proposed for a more exact evaluation of the effectiveness of the evolutionary algorithms. In his book, Deb [5] assembles a wide range of the metrics which figure in the literature. For this paper we propose the use of two metrics to evaluate the goodness of the algorithm. The first metric, the generational distance (Υ) proposed by Veldhuizen [19], evaluates the proximity of the population to the Pareto optimal front by calculating the average distance of the population from an ideal population P^* made up of N^* solutions distributed uniformly along the Pareto front. This metric is shown in (6).

$$\Upsilon = \frac{\left(\sum_{i=1}^{N'} d_i^v\right)^{1/v}}{N'} \tag{6}$$

We use $v = 1$, and parameter d_i is the Euclidean distance (in the objective space) between the solution i and the nearest solution in P^*:

$$d_i = \min_{k=1}^{N^*} \sqrt{\sum_{j=1}^{n} \left(f_j{}^i - f_j^{*k}\right)^2}$$

where f_j^{*k} is the value of the j-th objective for the k-th solution in P^*. For our problem, we use the profits in table VII as the ideal population P^*.

The second metric we use is the spread (Δ) put forward by Deb et al. [5] to evaluate the diversity of the population. Equation (7) shows this measure.

$$\Delta = \frac{\sum_{j=1}^{n} d_j^e + \sum_{i=1}^{N} |d_i - \overline{d}|}{\sum_{j=1}^{n} d_j^e + N\overline{d}} \tag{7}$$

where d_i may be any metric of the distance between adjacent solutions, and $\overline{d}$ is the mean value of such measurements. In our case, d_i has been calculated using the Euclidean distance. Parameter d_j^e is the distance between the extreme solutions in P^* and P corresponding to the j-th objective.

Figures 2 and 3 show the non dominated solutions obtained in the best of 10 executions of the proposed algorithm and NSGA-II respectively for Chocoman Inc. problem. Table VIII shows the best, worst, medium and variance values for the ν, Υ and Δ measures obtained in 10 executions of both algorithms.

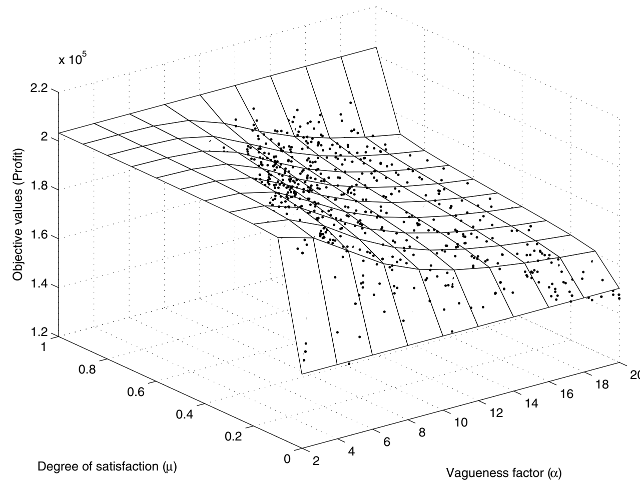

Fig. 2. Non dominated solutions obtained with NSGA-II for Chocoman Inc. problem.

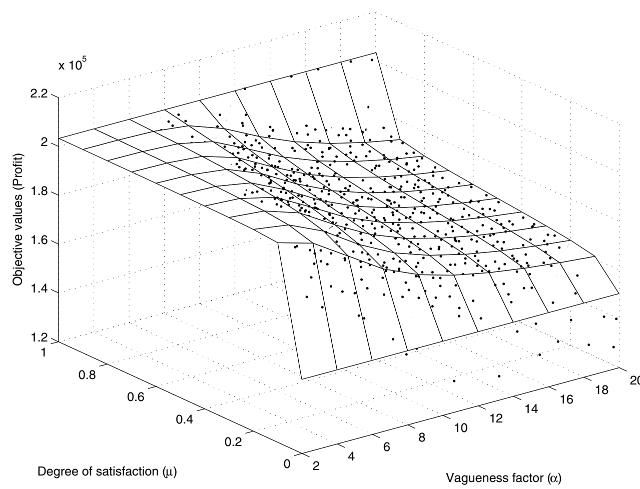

Fig. 3. Non dominated solutions obtained with the proposed algorithm for Chocoman Inc. problem.

As we can see from Table VIII, the proposed algorithm perform better than NSGA-II in all measures ν, Υ and Δ. Although both algorithms use similar heuristic search, but the niche of the good performance of the proposed algorithm is very specific to the problem in which it has been designed. For the kind of multi-objective problems we are considering, Pareto search based on the space search partition in linear slots is most efficient than general search strategies exclusively based on diversity functions, as in NSGA-II.

V. CONCLUSIONS AND FUTURE WORKS

A. Conclusions

Fuzzy nonlinear optimization problems are, in general, difficult to solve. In this paper we describe a multi-objective approach to solve a fuzzy nonlinear constrained optimization

TABLE VII

OPTIMAL PROFITS FOR UNIFORMLY DISTRIBUTED VALUES IN μ AND α OBTAINED WITH GRADIENT FOR CHOCOMAN INC. PROBLEM.

μ	$\alpha = 2$	$\alpha = 4$	$\alpha = 6$	$\alpha = 8$	$\alpha = 10$	$\alpha = 12$	$\alpha = 14$	$\alpha = 16$	$\alpha = 18$	$\alpha = 20$
0.001	150325.3	150325.3	150325.3	150325.3	150325.3	150325.3	150325.3	150325.3	150325.3	150325.3
0.100	201766.7	197697.5	188504.0	179299.1	173060.6	168939.6	166067.1	163958.5	162346.6	161074.8
0.200	202636.3	200588.1	194094.9	184750.6	177448.8	172496.5	169038.7	166505.7	164573.9	163052.7
0.300	202930.4	201681.5	197001.1	188269.9	180416.9	174914.4	171056.1	168231.8	166080.4	164388.6
0.400	203078.2	202257.0	198847.1	191010.4	182866.0	176926.2	172734.7	169666.3	167331.1	165496.4
0.500	203167.2	202612.2	200140.9	193347.9	185112.3	178793.7	174294.5	170998.3	168491.3	166523.3
0.600	203226.6	202853.4	201104.1	195459.0	187340.3	180679.0	175872.4	172345.1	169663.6	167560.0
0.700	203269.1	203027.7	201851.6	197447.4	189720.8	182749.5	177611.8	173829.8	170955.0	168701.3
0.800	203301.0	203159.7	202449.7	199386.3	192503.5	185288.5	179761.0	175665.4	172550.6	170110.3
0.900	203325.8	203263.1	202939.9	201337.9	196253.0	189096.9	183047.4	178479.3	174996.0	172267.9
0.999	203345.5	203345.5	203345.5	203345.5	203345.5	203345.5	203345.5	203345.5	203345.5	203345.5

TABLE VIII

RESULTS OF 10 RUNS OF THE PROPOSED ALGORITHM AND NSGA-II FOR CHOCOMAN INC. PROBLEM.

Algorithm	ν_{best}	ν_{worst}	ν_{mean}	$\nu_{variance}$
Proposed Algorithm	0.4616	0.4713	0.4665	1.0774×10^{-6}
NSGA-II	0.4938	0.5021	0.4986	4.6962×10^{-7}
	Υ_{best}	Υ_{worst}	Υ_{mean}	$\Upsilon_{variance}$
Proposed Algorithm	299.0841	489.703	378.0084	403.0047
NSGA-II	415.2723	483.2596	451.0228	43.1426
	Δ_{best}	Δ_{worst}	Δ_{mean}	$\Delta_{variance}$
Proposed Algorithm	1.1752	1.1957	1.1865	4.562×10^{-6}
NSGA-II	1.2677	1.3067	1.288	1.3429×10^{-5}

problem which appears in production planning for chocolate manufacturing. A Pareto-based multi-objective evolutionary algorithm is proposed to capture the (fuzzy) solution in a single run of the algorithm. Optimality and diversity metrics have been used for the evaluation of the effectiveness of the proposed multi-objective evolutionary algorithm compared with the well known algorithm NSGA-II. We show the values obtained using these metrics for the solutions generated by both algorithms. The results clearly show a real ability and effectiveness of the proposed approach to solve fuzzy problems in production planning for chocolate manufacturing.

B. *Future Works*

Multi-objectives with several other objective functions can be considered for future research work, as well as fuzzy costs and fuzzy right-side coefficients in constraints. There is a possibility of designing a productive computational intelligence self-organized evolutionary fuzzy system. It's also possible to do a comparative study with other evolutionary computational approach for the Industrial production planning in near future.

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