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Decision Making under Subjective Uncertainty

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Abstract— **The uncertainty may be classified into two major groups, "objective uncertainty" and "subjective uncertainty". The subject of this article is the decision making under subjective uncertainty. One of the formal models that deal with subjective uncertainty, the Mathematical Theory of Evidence, is extended and its counter-intuitive behavior corrected, allowing the making of correct decisions in a wider range of situations than the original model.**

The Mathematical Theory of Evidence, or Dempster-Shafer Theory, is a popular formalism to model someone's degrees of belief. This theory provides a method for combining evidence from different sources without prior knowledge of their distributions, it is also possible to assign probability values to sets of possibilities rather than to single events only, and it is unnecessary to divide all the probability values among the events, once the remaining probability should be assigned to the environment and not to the remaining events, thus modeling more naturally certain classes of problems. However, it has some pitfalls caused by the non-natural embodiment of the uncertainty in the results.

In this paper we present a method of automatic embodiment of the uncertainty that overcomes the aforementioned pitfalls, allowing the combination of evidence with higher degrees of conflict, and avoiding the excessive tendency toward the common possibility of otherwise disjoint hypotheses. This is accomplished by means of a new rule of combination of bodies of evidence that embodies in the numeric results the unknown belief and conflict among the evidence, naturally modeling the epistemic reasoning.

I. Introduction

The dual nature of the uncertainty was first defined by Helton [1], making its taxonomy into two major groups, "objective uncertainty" and "subjective uncertainty".

Objective uncertainty corresponds to the "variability" that emerges from the stochastic characteristic of an environment, non-homogeneity of the materials, time drifts, space variations, or other kinds of differences among components or individuals. This variability is also known as "Type I Uncertainty", "Type A", "Stochastic", or "Aleatory", emphasizing its relationship to the random aspects of games of chance. Another term attributed to it is "Irreducible Uncertainty", since, at least in principle, it cannot be reduced through additional investigation (although it can be better characterized) [2], [3].

Subjective Uncertainty is the uncertainty that comes from scientific ignorance, uncertainty in measurement, impossibility of confirmation or observation, censorship, or other knowledge deficiency. It is also known as "Uncertainty Type II", "Type B", "Epistemic Uncertainty", "Ignorance", or "Reducible Uncertainty", since it, *a priori*, is able to be reduced through additional empiric efforts [2], [3].

Objective or aleatory uncertainty already has well clarified origins and concepts from the origins and concepts of classic probability itself, being the uncertainty that usually comes to mind when we think in contingencies – by "classic probability" this work refers to the probability described by the Axioms of Kolmogorov and the concepts of Bayesian Theory.

Since objective uncertainty has already been extensively explored in works on classic probability, the decision-making under subjective uncertainty is the subject of this article, which extends one of the formal models that deals with it, the Mathematical Theory of Evidence (or Dempster-Shafer Theory).

A key issue in dealing with knowledge representation is how to combine bodies of evidence from different sources, adequately modeling its subjective uncertainty and conflict.

The Theory of Evidence tries to do this, but exhibits a counter-intuitive behavior when the bodies of evidence to be combined have a high degree of conflict, or when they are disjoint regarding the more believed hypothesis. This counter-intuitive behavior limits the range of application of this theory, and, at the same time, leads to a potential disregard of hypotheses that otherwise could add information to the system.

In this work we present a new rule of combining bodies of evidence, which is able to overcome these flaws, by the means of a meta-probability mass, named "Lateo". With this approach, it becomes possible to eliminate the counter-intuitive behavior of the original theory, therefore extending its range of application and better using the available information.

II. The Theory of Evidence

The Theory of Evidence, or Dempster-Shafer Theory, was introduced in the late seventies based on Dempster's works, extended by Shafer [4].

Unlike the Bayesian Theory, the Theory of Evidence does not need prior knowledge of the probability distribution, and it is able to assign probability values to sets of possibilities rather than to single events only. Another differential is that there is no need to divide all the probability among the events, once the remaining probability is assigned to the environment and not to the remaining events. These two differentials allow this theory to model more precisely the natural reasoning process on evidence accumulation, making it progressively more popular.

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This formalism provides methods for combining the bodies of evidence carried by different sources, being the Dempster's Rule the *de-facto* method [5], although there are other rules differing basically in their normalization part [6], [2]. The procedures adopted by all rules of combination, are independent of evidence order (exchangeability).

A. Frame of Discernment

A Frame of Discernment, or Environment, is a set of primitive hypotheses, denoted by Θ. It must:

- be exhaustive, in the sense of being complete, containing all possible primitive (atomic) solutions.
- have mutually exclusive primitive elements.

B. Mass Function

The basic probability assignment, or Mass Function, assigns some quantity of belief to the elements of the Frame of Discernment.

Considering a given evidence, the Mass Function, m, assigns to each subset of Θ (i.e. to 2^Θ, the powerset of Θ), a number in the interval $[0,1]$, where 0 means no belief, and 1 means certainty. The sum of all assignments is equal to 1, meaning that the right hypothesis is in the Frame of Discernment. Therefore 0 should be assigned to the empty set, once it is the representation of the false hypothesis. The probability non-assigned to any subset of Θ, is named "non-assigned belief", $m(\Theta)$, being in fact assigned to Θ, and not to the negation of the hypothesis that received some belief, as it would be in the Bayesian Theory.

Thus, $m(\mathcal{A})$ is the measure of the belief assigned by a given evidence to $\mathcal{A}$, where $\mathcal{A}$ is any element of 2^Θ. As $m(\mathcal{A})$ deals with the belief assigned to $\mathcal{A}$ only, and not to subsets of $\mathcal{A}$, no belief is forced by the lack of knowledge.

Summarizing:

$$m : 2^\Theta \rightarrow [0,1] \quad (1)$$
$$m(\varnothing) = 0 \quad (2)$$
$$\sum_{\mathcal{A}\in\Theta} m(\mathcal{A}) = 1 \quad (3)$$

Example 1: Mass function and frame of discernment (see Figure 1) for an evidence of Mary's grade in philosophy:

$$\Theta = \{1,2,3,4,5,6,7,8,9,10\}$$

Mass Function (m):

$$m_1(\mathcal{A}) = 0.3$$
$$m_1(\mathcal{B}) = 0.25$$
$$m_1(\mathcal{C}) = 0.35$$
$$m_1(\Theta) = 0.1$$

Note that the belief non-assigned to the subsets is assigned to the environment.

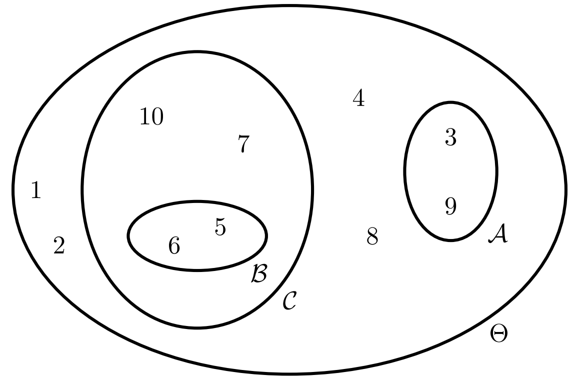

Fig. 1. Frame of discernment for Mary's grade in philosophy

C. Belief Function

The Belief Function, $\mathcal{B}el$, measures how much the information, given by a source, support the belief in a specified element as the right answer. The Belief Function for the element $\mathcal{A}$, $\mathcal{B}el(\mathcal{A})$, is given by:

$$\mathcal{B}el : 2^\Theta \rightarrow [0,1] \quad (4)$$
$$\mathcal{B}el(\mathcal{A}) = \sum_{\mathcal{B}\subseteq\mathcal{A}} m(\mathcal{B}) \quad (5)$$

Example 2: (using the data from Example 1)

$$\mathcal{B}el(\mathcal{A}) = 0.3$$
$$\mathcal{B}el(\mathcal{B}) = 0.25$$
$$\mathcal{B}el(\mathcal{C}) = 0.6$$
$$\mathcal{B}el(\Theta) = 1$$

Note that the belief in $\mathcal{C}$ is the sum of the mass of belief of $\mathcal{B}$, 0.25, with the mass of belief of $\mathcal{C}$, 0.35, given that $\mathcal{C}$ contains $\mathcal{B}$; and the belief in Θ is the sum of the mass of beliefs of the subsets.

D. Plausibility Function

The Upper Probability Function, or Plausibility Function, $\mathcal{P}l$, measures how much the information, given by a source, does not contradict a specified element as the right answer, or in other words, how much we should believe in an element if all unknown belief is assigned to it.

The Plausibility Function for the element $\mathcal{A}$, $\mathcal{P}l(\mathcal{A})$, is defined by (6) and (7) and subjected to properties (8) and (9).

$$\mathcal{P}l : 2^\Theta \rightarrow [0,1] \quad (6)$$
$$\mathcal{P}l(\mathcal{A}) = \sum_{\mathcal{B}\cap\mathcal{A}\neq\varnothing} m(\mathcal{B}) \quad (7)$$
$$\mathcal{B}el(\mathcal{A}) \leq \mathcal{P}l(\mathcal{A}) \mid \mathcal{A} \subseteq \Theta \quad (8)$$
$$\mathcal{P}l(\mathcal{A}) = 1 - \mathcal{B}el(\mathcal{A}') \quad (9)$$

(where $\mathcal{A}'$ is the complement of $\mathcal{A}$)

E. Belief Interval

It is the interval

$$\mathcal{I}(\mathcal{A}) = [\mathcal{B}el(\mathcal{A}), \mathcal{P}l(\mathcal{A})] \tag{10}$$

meaning the range of epistemic relevance, where we can believe in $\mathcal{A}$ without severe errors. The Belief Interval is as larger as the uncertainty in $\mathcal{A}$.

F. Dempster's Rule

The reasoning process over evidence accumulation needs a method for combining the independent evidence from different sources [7]. The method usually used to combine the bodies of evidence is the Dempster's Rule [5], [4]. Although there are other rules of combination, they differ basically in their normalization part [8], [9], being the procedures adopted by all rules independent of the evidence presentation order.

The Dempster's Rule is composed by an orthogonal sum and a normalization:

$$m_1 \oplus m_2(\mathcal{A}) = \mathcal{X} \sum_{\substack{\mathcal{B} \cup \mathcal{C} = \mathcal{A} \\ \mathcal{A} \neq \varnothing}} m_1(\mathcal{B}).m_2(\mathcal{C}) \ , \ \forall \mathcal{A} \subseteq \Theta \tag{11}$$

Where $m_1 \oplus m_2(\mathcal{A})$ denotes the combined effects of the mass functions m_1 and m_2 and $\mathcal{X}$ is the normalization constant, defined as $1/k$, where:

$$k = 1 - \sum_{\mathcal{A}_i \cap \mathcal{B}_j = \varnothing} m_1(\mathcal{A}_i).m_2(\mathcal{B}_j) \tag{12}$$

Or, likewise:

$$k = \sum_{\mathcal{A}_i \cap \mathcal{B}_j \neq \varnothing} m_1(\mathcal{A}_i).m_2(\mathcal{B}_j) \tag{13}$$

Example 3: An examination question has as the possibilities of correct answer $\Theta = \{a, b, c, d, e\}$. Considering $\mathcal{A} = \{a\},\ \mathcal{B} = \{b\},\ \mathcal{C} = \{c\},\ \mathcal{D} = \{d\}$, and $\mathcal{E} = \{e\}$, was asked to two people what was the probability of each answer to be the correct one.

The first person answered:

$$m_1(\mathcal{A}) = 0.23$$
$$m_1(\mathcal{B}) = 0.18$$
$$m_1(\mathcal{C}) = 0.28$$
$$m_1(\mathcal{D}) = 0.18$$
$$m_1(\mathcal{E}) = 0.13$$

Note that 100% of the belief was assigned to the elements of Θ, nothing being assigned to Θ itself.

The second person's opinion became the second evidence:

$$m_2(\mathcal{A}) = 0.27$$
$$m_2(\mathcal{B}) = 0.17$$
$$m_2(\mathcal{C}) = 0.21$$
$$m_2(\mathcal{E}) = 0.21$$
$$m_2(\Theta) = 0.14$$

Note that the second person preferred not stating anything about the possibility "d"; and as he did not divide 100% of his beliefs among the possibilities, the remaining (0.14) was assigned to Θ.

Using Dempster's combination rule, would result in:

$$m_3(\mathcal{A}) = 0.30$$
$$m_3(\mathcal{B}) = 0.17$$
$$m_3(\mathcal{C}) = 0.31$$
$$m_3(\mathcal{D}) = 0.08$$
$$m_3(\mathcal{E}) = 0.14$$

G. Weight of Conflict

It is the logarithm of the normalization constant, denoted by $\mathcal{C}on(\mathcal{B}el_1, \mathcal{B}el_2)$, where:

$$\mathcal{C}on(\mathcal{B}el_1, \mathcal{B}el_2) = log(\mathcal{X}) \tag{14}$$

If there is no conflict between $\mathcal{B}el_1$ and $\mathcal{B}el_2$, the sum of the beliefs will be 1 and then $\mathcal{C}on(\mathcal{B}el_1, \mathcal{B}el_2) = 0$. Same wise, if there is nothing in common between the evidence, $\mathcal{C}on(\mathcal{B}el_1, \mathcal{B}el_2) = \infty$.

Example 4: (using data from Example 3)

$$\mathcal{X} = 3.1368$$

and consequently

$$\mathcal{C}on(\mathcal{B}el_1, \mathcal{B}el_2) = log(\mathcal{X}) = 0.4965$$

The combination of bodies of evidence with a high weight of conflict can lead to counter intuitive, unreasonable, results by the Dempster's Rule.

III. Counter intuitive behavior of the combination rules

A classical problem [4], [10], [11], with the Combination Rules used until now, is a counter intuitive result found when the evidence to be combined have a concentration of belief in elements disjoint among them, and a common element with low degrees of belief assigned to it. Because the rules do not include any intrinsic mean of belief derating, proportionally to the amount of uncertainty, coming from the conflict among them, they can assign 100% of belief to the element less believed, but common to the evidence.

Example 5: Your car has broken and you called two auto mechanics to give their diagnostics.

The mechanic 1 gave his opinion of 99% of belief to a fuel injection problem ({injection}), and 1% of belief in an electronic ignition problem ({ignition}):

$$m_1(\{\text{injection}\}) = 0.99$$

$$m_1(\{\text{ignition}\}) = 0.01$$

The mechanic 2 assigned 99% of certainty to a command belt problem ({belt}), and 1% to an electronic ignition ({ignition}) problem:

$$m_2(\{\text{belt}\}) = 0,99$$

$$m_2(\{\text{ignition}\}) = 0,01$$

By the Dempster's Rule:

$$\Theta = \{\text{injection, ignition, belt}\}$$

$$m_3(\{\text{belt}\}) = 0$$
$$m_3(\{\text{injection}\}) = 0$$
$$m_3(\{\text{ignition}\}) = 1$$

That is, a 100% of belief on an electronic ignition problem, contradicting the intuition, and making some authors as [12] state as not advisable the combination of evidence with weight of conflict bigger than a certain value, as 0.5 (as a rule of thumb).

IV. Analyzing the counter intuitive behavior

It is of great importance to analyze the kind of phenomenon portrayed on Example 5. By an epistemic point of view it should be a "confirmation effect" about the hypothesis upon which the opinions agreed, once both opinions came from specialists with the same degree of reliability. Thus the discordance concerning the hypothesis in which most belief were assigned, must, in fact, decrease the belief on these hypotheses, increasing the uncertainty about them, and at the same time, increasing the belief in the hypothesis in which they assigned a lesser degree of belief, but about which they agreed, although it is exaggerated a belief assignment of a 100% to the less believed, but common, hypothesis.

To make it clear: imagine a problem with a frame of discernment having 11 hypotheses. We then ask to 10 people which one of these hypotheses would be the right answer. Each one of these 10 people assigned most of their belief to an hypothesis disjoint from the choice of the others, and little of their belief to a common hypothesis. Considering all people with the same reliability, the divergence, about the more individually believed hypothesis, increases the uncertainty about them, at the same time increasing also the belief upon the individually lesser believed one, once all the people agreed about it.

Thus, "specialists" agreeing about a hypothesis increase its degree of certainty, although do not make it "totally certain" (i.e. with an assignment of a 100% of belief), given the divergence about the more individually believed one. Corroborating this, the assignment of only a small portion of the individual belief, to the common hypothesis, decreases its intrinsic information value.

We can model this, extending the Theory of Evidence, by using a new rule of evidence combination, that not only corrects this counter intuitive effect, but also embodies in the result the uncertainty coming from the non-assigned belief and conflicting hypotheses by using a "measure" – so to speak – of the subjective uncertainty named "Lateo".

V. Our approach

The proposed rule derates the beliefs according to the degree of conflict between the evidence, assigning the remaining belief to the environment (and not to the common hypothesis) along with the uncertainty that would be assigned to the environment by the original Dempster's Rule [13]. This quantity of belief assigned to the environment constitutes a measure of the subjective uncertainty coming from the lack of knowledge or conflict among the evidence, being named "Lateo", and denoted by Λ, in allusion to its causes, once "Lateo" in Latin means "being hidden", "being out of sight", "be unknown".

This rule makes it possible to combine evidence with most of their belief assigned to disjoint hypotheses, without the side effect of a counter intuitive behavior. It also allows the use of evidence with high values of conflict, making useful evidence otherwise useless.

For two bodies of evidence, this is accomplished by dividing the orthogonal sum, from Dempster's Rule, by $(1+log(1/k))$, that is, $(1 + \mathcal{C}on(\mathcal{B}el_1, \mathcal{B}el_2))$:

$$m_1 \Psi m_2(\mathcal{A}) = \frac{\mathcal{X} \sum_{\substack{\mathcal{B}\cap\mathcal{C}=\mathcal{A} \\ \mathcal{A}\neq\varnothing}} m_1(\mathcal{B}) \cdot m_2(\mathcal{C})}{1 + log(\frac{1}{k})}\ ,\ \forall \mathcal{A} \subset \Theta \quad (15)$$

The additional belief, from the derating of the hypotheses, is added to the initial environment belief, originating the Lateo:

$$\Lambda = (\mathcal{X}.m_1(\Theta).m_2(\Theta)) + 1 - \sum_{\substack{\mathcal{A}\subset\Theta \\ \mathcal{A}\neq\varnothing}} m_1 \Psi m_2(\mathcal{A}) \quad (16)$$

It can be noted that $(\mathcal{X}.m_1(\Theta).m_2(\Theta))$ is equal to $m_1 \oplus m_2(\Theta)$, by the Dempster's Rule, and the proposed rule adds to this belief a value proportional to the conflict and non-assigned belief among the evidence.

The numeric value expressed by the Lateo represents a mobile mass of belief, that, in the absence of unknown belief and conflict among the evidence, could be associated with any element, or combination of elements, of the frame of discernment.

A. Combining evidence with most of their beliefs assigned to disjoints hypotheses

The proposed approach solves the counter intuitive behavior of the original theory when combining evidence whose most belief is assigned to disjoint hypotheses, as it is illustrated by Example 6.

Example 6: Applying our rule to the data from Example 5, we get:

$$k = 0.0001$$
$$\mathcal{X} = 10,000$$
$$log(\mathcal{X}) = 4$$

And thus:

$$m_3(\{belt\}) = 0$$
$$m_3(\{injection\}) = 0$$
$$m_3(\{ignition\}) = 0.2$$
$$\Lambda = 0.8$$

As it can be seen, the reasoning is more naturally modeled once the belief in the command belt and in the fuel injection continue to be disregarded due to their disjunction, but the uncertainty is better represented, since 80% of the belief is assigned to the environment and not to a hypothesis in particular [14].

The Plausibility Function and the Belief Interval would be:

$$\mathcal{B}el(\{ignition\}) = 0.2$$
$$\mathcal{P}l(\{ignition\}) = 1$$
$$\mathcal{I}(\{ignition\}) = [0.2, 1]$$

This shows a much more realistic modeling of the problem, as the plausibility of the electronic ignition hypothesis continue to be 100%, while its belief is decreased to 20%. From an epistemological point of view it would not be appropriate a belief assignment of 100% to the electronic ignition simply because the mechanics disagreed about the most believed hypothesis, and agreed about the one with a low belief.

Note that in which regards a decision making process the original theory would suggest an "ignition" problem without uncertainty (Example 5), while our approach makes clear that the information collected is not enough to allow a reasonable decision, once the subjective uncertainty measure (the Lateo) is bigger than the knowledge available (that is, 80% Lateo against 20% "ignition").

B. Combining evidence with high degree of conflict

It should be noted that the proposed rule shows a better modeling, even if the evidence combined do not show concentration of belief in disjoint elements, once, whatever be the case, it will decrease the beliefs assigned to the hypotheses proportionally to the weight of conflict between them, allowing the combination of evidence with a high degree of conflict, and modeling the uncertainty and/or inconsistence among the specialists/consultants.

Example 7: Using Example 3 data, it can be seen that even a relatively high weight of conflict ($\mathcal{C}on(\mathcal{B}el_1, \mathcal{B}el_2) = 0.4965$), do not make any difference to an evidence combination by the Dempster's Rule, working the same way as if the evidence had no conflict at all:

$$m_3(\mathcal{A}) = 0.30$$
$$m_3(\mathcal{B}) = 0.17$$
$$m_3(\mathcal{C}) = 0.31$$
$$m_3(\mathcal{D}) = 0.08$$
$$m_3(\mathcal{E}) = 0.14$$

However, applying the new rule we get an belief assignment of 33% of belief to the environment, and an accompanying decrease of each hypothesis' belief, denoting the uncertainty from the conflict between the evidence:

$$m_3(\mathcal{A}) = 0.200$$
$$m_3(\mathcal{B}) = 0.114$$
$$m_3(\mathcal{C}) = 0.207$$
$$m_3(\mathcal{D}) = 0.053$$
$$m_3(\mathcal{E}) = 0.094$$
$$\Lambda = 0.332$$

Note that the relative positions among the elements stay intact, but their beliefs are reduced proportionally to the weight of conflict, as happens in the real world when we intuitively process our conflicting evidence.

Regarding a decision making process, by the Dempsters Rule, one would choose hypothesis $\mathcal{A}$ or $\mathcal{C}$ as the correct answer, while our rule makes clear that no hypotheses should be chosen, as the value of Lateo summed to any hypothesis is enough to make this hypothesis the bigger one.

VI. CONCLUSION

Although the Theory of Evidence is able to deal with subjective uncertainty, it shows two major flaws caused by the rules of evidence combination until now used:

1. A counter intuitive behavior when the evidence to be combined have a concentration of belief in elements disjoints among them, and a common element with low degrees of belief assigned to it.

2. A lack of an intrinsic representation of the subjective uncertainty, coming from the unknown or from the conflict among the evidence, becoming non-advisable to combine evidence with a high weight of conflict.

A decision making process can be affected by these flaws leading to erroneous decisions. Nevertheless, it is possible to solve these two flaws, extending the application range of the Theory of Evidence, by the adoption of a proposed new rule of evidence combination. This rule corrects the counter intuitive effect, and embodies in the result the subjective uncertainty. This is accomplished by decreasing the beliefs proportionally to the degree of conflict among the evidence, and assigning the remaining belief to the environment instead of to the common hypothesis, resulting in a measure of the subjective uncertainty named "Lateo".

With the proposed rule it becomes possible to know the degree of subjective uncertainty involved in the combination of evidence, making clear the possibility of making reasonable decisions based in the evidence combined.

Additionally, the implementation of the Lateo introduces a number of interesting possibilities as it represents a measure of the subjective uncertainty, allowing:

- An indication of how much the numerical results obtained, by the Dempster-Shafer Theory, are distant from the numeric results obtained by the theories of precise probability.
- To know how much one can trust in the results for decision making purposes.
- An estimation of the level of confidence that one can have in the sources consulted regarding the solution of the given question.

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