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Multi-criteria Set Partitioning for Portfolio Management: A Visual Interactive Method

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Abstract—**A visual interactive multi-criteria decision-making method for partitioning a portfolio of assets into mutually exclusive categories is presented. The two principal decision categories are *hold* and *sell*—portfolio assets in the *sell* category are considered as potential sale prospects, and the other assets in the portfolio are considered as potential retention prospects. The problem may be mathematically formulated as a multi-criteria 0/1 knapsack problem with multiple constraints. The decision-making method centers on the utilization of several coupled 2-D projections of the portfolio in the multi-dimensional criterion space. The decision-maker interacts with these projections in a variety of ways to express and record multi-category (*hold, hold-bias, sell-bias, and sell*) set partitioning preferences. The decision-maker may also set an aggregated preference threshold that is utilized for partitioning the portfolio into the two principal *hold* and *sell* categories. The decision-maker may further fine-tune their preferences and threshold settings so as to achieve a multitude of financial targets.**

Index Terms — **Portfolio management, visual interactive method, multi-criteria, decision-support, set partitioning, knapsack problem.**

I. Introduction

A visual interactive multi-criteria decision-making method for portfolio management is presented. The method supports the partitioning of a portfolio of physical or financial assets into mutually exclusive *hold* and *sell* categories—assets in the *sell* category are considered as potential sale prospects, and the other assets in the portfolio are considered as potential retention prospects. The method utilizes several coupled 2-D projections of the portfolio in the multi-dimensional criterion space. The decision-maker interacts with the projections to express and record preferences. The decision-maker's goal is the partitioning of the portfolio such that multiple criteria are simultaneously optimized and multiple constraints are satisfied. The decision-maker receives immediate feedback on the portfolio-level performance of the preferences through global aggregation of the preferences. The decision-maker may fine-tune their preferences and the preference aggregation method so as to achieve a multitude of financial targets.

The approach described in this paper was motivated by the desire to develop a complementary portfolio management technique—to the state-of-the-art methods being used at General Electric—which rely primarily on human intelligence for solving a particular problem while providing data input, data manipulation, and data visualization support to the human decision-maker.

The portfolio-partitioning problem may be mathematically formulated as a multi-criteria 0/1 knapsack problem, wherein the assets in the *sell* (or alternatively *hold*) category must meet performance criteria and knapsack membership constraints. To our best knowledge, we have not come across prior work that discusses portfolio partitioning with such a multi-criteria formulation. Formally, (1) defines the portfolio partitioning problem given n assets $(a_1, a_2, \cdots, a_n)$, a state-variable x_i associated with each asset a_i, where each x_i may assume a state of 0 (*hold*) or 1 (*sell*), each asset has a set of payoffs $(p_i^1, p_i^2, \cdots, p_i^\alpha)$, weights $(w_i^1, w_i^2, \cdots, w_i^\beta)$, and observations $(z_i^1, z_i^2, \cdots, z_i^\gamma)$. While payoffs and observations correspond to financial performance-related measures, weights correspond to financial burden-related measures.

$$\textbf{Maximize} \left[\sum_{i=1}^{n} p_i^1 x_i, \sum_{i=1}^{n} p_i^2 x_i, \cdots \sum_{i=1}^{n} p_i^\alpha x_i \right] \quad (1)$$

$$\textbf{Such that} \left[\sum_{i=1}^{n} w_i^1 x_i \le c^1, \sum_{i=1}^{n} w_i^2 x_i \le c^2, \cdots \sum_{i=1}^{n} w_i^\beta x_i \le c^\beta \right]$$

Where $(c^1, c^2, c^3 \cdots, c^\beta)$ is the set of constraints. Observations support the overall payoff maximization goals by providing a richer view of the diverse economic performance of assets.

The above description reveals a combinatorial decision-making problem. When considered with a single maximization goal and a single constraint, the problem is known to belong to the *NP-Hard* computational complexity class. By extension, it may be shown that the above multi-criteria version of the problem also belongs to the same computational complexity class. While algorithmic methods could be devised for the solution of (1), our goal for this work was to devise a method

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wherein the decision-maker actively participates in the decision-making process, and not the design of an algorithm that can solve (1) with minimal decision-maker intervention.

In this interactive approach, the decision-maker drives the exploration for the portfolio partition that satisfies the performance criteria and constraints in (1) by expressing their preferences in a systematic manner consistent with their mental representations. Mental representations corresponding to suitable preferences are aided and refined by visualization of 2-D tradeoffs in the multi-dimensional criterion space, making the use of 2-D projections invaluable in this mode of decision-making.

While it may be argued that this interactive method does not guarantee Pareto optimality of the derived portfolio partition in payoffs space, such a decision-maker-driven preference-based portfolio partitioning serves as a flexible “open-box” approach to optimized decision-making wherein the decision-maker is in complete control of partition outcomes and is able to exercise and incorporate progressive judgment, more naturally supporting real-world business decision-making processes. Importantly, the constraints in (1) are somewhat fluid; in that the decision-maker interactively settles on constraint levels they are comfortable with as the decision-making progresses. An algorithmic solution for (1) on the other hand would require the decision-maker to pre-specify all constraint levels, which is not always practical in a real-world situation.

The rest of this paper is organized as follows: Section II presents background on interactive multi-criteria decision-making methods; Section III presents the interactive multi-criteria portfolio-partitioning framework; Section IV presents the portfolio-partitioning decision-support interface and its capabilities. We conclude in Section V.

II. Review of Interactive Decision-making Methods

In this section, we present a short chronological review of interactive multi-criteria decision-making methods. Our goal is not to present an exhaustive literature survey, but instead to review a select sample of relevant prior work.

Angehrn and Lüthi [1] present a conceptual foundation for interactive decision-support systems, wherein they make the case for the evolution of decision-making: from data-driven processing to visual information processing; from formal analysis to interactive modeling; and from algorithmic manipulation to direct manipulation (pp. 18), and further suggest the need for decision-support systems to help decision-makers understand, express, and structure their problems. In addition, they present a survey of the pre-1990 literature in interactive decision-support systems.

Antunes and Clímaco [2] present a mathematical foundation for ordering a finite set of alternatives subject to multiple criteria that may be mutually conflicting. They present weight-based methods to achieve this partial ordering based on the elicitation of decision-maker preferences corresponding to each criterion.

Tan and Fraser [3] introduce star graphs and petal diagrams as visual tools to support interactive selection from a finite set of alternatives considered with respect to multiple criteria. Miettinen [4] also reviews several visual diagrams that support multi-criteria decision-making.

Klimberg and Cohen [5] present GRADS, a visual representation tool that captures the tradeoffs within a finite set of alternatives in a high-dimensional space through the use of 2-D projections. Each alternative in a 2-D criterion space has an associated star or spokes emanating from it on demand, representing the magnitude of the other criteria associated with the alternative.

Köksalan and Rizi [6] consider the problem of selecting the best alternative from a finite set of alternatives in a high-dimensional space given a monotone utility function, by a method that involves the decomposition of the criterion space into multiple non-overlapping cells. The cell partitioning method supports the rapid elimination of alternatives in a cell compared to the ideal solution in that cell. The exponential complexity of the method becomes apparent (r^p cells) when there are p criteria and r partitions per criterion, which may restrict the utility of this method to high-dimensional datasets.

Trinkaus and Hanne [7] present a decision-support tool concept, knowCube, for design and plan optimization—the principal motivation being the design of a user interface for non-expert decision makers. Packham et al. [8] present an interactive decision-support system for engineering design that first generates several design alternatives using evolutionary search methods, and then supports the decision-maker in selecting robust design alternatives through visual clustering.

Subbu et al. [9] present an interactive decision-support method for financial portfolio management. The portfolio management system integrates hybrid multi-objective optimization and interactive Pareto frontier decision-making techniques to optimally allocate financial assets while considering multiple measures of return and risk, and numerous regulatory constraints. The hybrid multi-objective optimization approach combines evolutionary computation with linear programming to simultaneously maximize these return measures, minimize these risk measures, and identify the efficient frontier of portfolios that satisfy all constraints. The method combines a novel interactive graphical decision-making method based on coupled 2-D projections in the criterion space that allows the decision-maker to specify inclusion/exclusion constraints in multi-dimensional criterion space[1] to quickly down-select to a small subset of efficient portfolios.

[1] Graphical tool licensed from Aetion Technologies, LLC (based on [10]).

While the interactive constraint-based down-selection method in [9] could be utilized to partition a portfolio, it would not be straightforward to partition a portfolio considering aggregate payoff maximization and constraint satisfaction goals using the method. Firstly, the aggregate payoff and constraint satisfaction properties of a given sub-set are not expressible as attributes of individuals in the set. Secondly, the constraint-based down-selection method is based on the specification of *binary-valued* inclusion/exclusion preferences, while what is needed is a method that supports *multi-valued* preferences that ultimately leads to binary (or multi-valued) decisions. Therefore, we designed a decision-support framework that was more matched to the mathematical and mental representation of the constrained portfolio-partitioning problem (1). Regardless, we have borrowed concepts on the use of coupled 2-D projections for interactive preference specifications and constraint specifications from this earlier work.

III. Multi-criteria Portfolio-partitioning Framework

In this section, we present the algorithm underlying the interactive framework for multi-criteria portfolio partitioning.

Referring to the problem formulation in (1), a portfolio dataset therefore is of size $n \times \delta$, where $\delta = \alpha + \beta + \gamma$. In practice, α is much lower than γ (number of payoffs associated with an asset is much lower than the number of associated observations). Further, there might be more criteria associated with assets in the form of text-coded identifiers such as geographic zones or linguistic rating labels. We address in the next section how text-coded identifiers may be utilized in the decision-making process.

Given δ criteria, we may visualize $C(\delta, 2)$ 2-D projections in the criterion space[2]. Each j^{th} projection is assigned a normalized importance factor $\lambda_j \in [0, 1] \subset \Re$, whereby an importance of 0 eliminates the corresponding projection's influence on the decision-making process, and its importance increases as it approaches 1. All possible 2-D projections have a default importance of 0.5, which the decision-maker may interactively change.

In each 2-D projection of the portfolio, the decision-maker may interactively associate with an asset a unique label from the set $\mathrm{L} = \{L^-, 0, L^+\} \subset \Re$, where $L^- \subset \Re^-$, and $L^+ \subset \Re^+$. The labels from L^- capture the decision-maker's preference to certainly hold the asset through to marginally hold the asset. The lower the value of L^-, the stronger the preference. The labels from L^+ capture the decision-maker's preference to marginally sell the asset through to certainly sell the asset. The higher the value of L^+, the stronger the preference. The 0 label is the default label associated with each asset in a given 2-D projection, and represents the "no-preference" state. A decision-maker specifies these preferences projection-wise in a manner consistent with their mental representations. This association is achieved by the interactive specification of rectangular regions in a 2-D projection—the assets whose projected criteria fall within the specified rectangular region are associated with the selected label. The decision-maker is not required to specify preferences in all $C(\delta, 2)$ projections, and is required to specify preferences in only one projection to partition a portfolio. However, specification of preferences in multiple 2-D projections is desirable in order to capture the inherent tradeoffs implicit in the portfolio. At any time, the decision-maker may go back to a 2-D projection of choice and change the preferences associated with assets in that view.

Preferences aggregation and performance with respect to the goals and constraints in (1) is an important real-time step in the decision-support process, and is necessary to give immediate feedback to the decision-maker on the consequences of the preferences specified. The aggregated preference associated with each asset a_i is computed as:

$$\sum_{j=1}^{C(\delta,2)} \lambda_j \ \mathrm{L}(a_i, j) \tag{2}$$

where $\mathrm{L}(a_i, j)$ is the preference label associated with asset a_i in the j^{th} projection. During the preference specification process, an asset may receive both negative and positive preferences. The preference aggregation method of (2) supports the incorporation of divergent preferences associated with an asset. Moreover, an asset that did not receive any preference label during the decision-making process would be indistinguishable aggregated-label-wise from an asset that received exactly balanced negative and positive preference labels, in a manner consistent with reason.

Aggregated preference normalization and decomposition into the *hold* and *sell* categories is the next step in the preference aggregation in order to determine the 0/1 decision state associated with each asset. The normalization is performed considering the numeric bounds of aggregated preferences for all n assets. Further, to eliminate the skewing of the final decisions, we do an independent *bi-directional* normalization, wherein the negative aggregated preferences associated with the n assets are mapped to the normalized range [0, 0.5], and the positive aggregated preferences associated with the n assets are mapped to the normalized range (0.5, 1]. The independent bi-directional normalization step is important for the consistency of the decision-making process. Let us consider the following example: assume a decision-maker initiates the decision-making process by interacting with a certain 2-D projection, and specifies that a set of assets in that criterion space should be considered *holds*. Then, without the bi-directional normalization, those assets

[2] $C(\delta, 2)$ is the number of ways of choosing two criteria from δ criteria.

that were not in the *hold* preference category would automatically be considered as belonging to the *sell* category, which is non-intuitive. The decision-maker further has control over a split threshold t in the range $[0, t, 1]$ that makes the final decomposition of the portfolio into the 0/1 *hold*/*sell* categories. The decision-maker may be more aggressive (move t closer to 0) or more conservative (move t closer to 1) in order to meet the goals and constraints in (1). Selection of this threshold is driven not only by the aggressiveness of the decision-maker, but also by considering the tradeoffs between the financial goals for the *hold*/*sell* categories. The decision-maker may also select additional 2-D projections from the available set of projections to specify more preferences that better capture their mental representations.

While we have emphasized two-way portfolio partitioning in our development, a multi-way partitioning (e.g. *hold / further inspection required / sell*) is also easily feasible based upon this foundation. Multi-way partitioning may be achieved by specification of an additional split threshold in the range $[0, t, 1]$.

IV. Decision-support Interface

In this section, we present the graphical interface that supports multi-criteria portfolio partitioning, and discuss its features. The graphical interface represents one implementation of the technical concepts presented in this paper. It is however not the only way to visually organize information and interact with it.

Figure 1 shows the portfolio-partitioning decision-support interface. First, an asset portfolio dataset may be specified and loaded. The three views on the upper portion of the interface support interactive commands within their axes. The ordinate and abscissa of each of these three views may be configured to project all possible 2-D projections of the multi-dimensional portfolio performance space. The decision-maker may apply a *filter* function within each projection—the *filter* function filters the portfolio based on text-coded strings and text-coded string intersections, allowing the projection of subsets of the portfolio. For example, it is possible to project assets of a certain type across all geographical regions, and also project assets of a certain type within a given geographical region. The filter function does not place a limit on the number of text-coded string intersections, as long as a meaningful portfolio subset can be projected. On the left-hand-side of each interactive view is a label palette corresponding to the set L, which the decision-maker may utilize to express preferences consistent with their mental representations. Above each interactive view is a slider (called *Discount* in the interface) that may be utilized to adjust that view's importance factor λ.

The large multi-colored slider in the lower portion of the interface allows the decision-maker to interactively adjust the split threshold t. Just below the slider is a set of portfolio performance metrics that give feedback to the decision-maker on satisfaction of the aggregate goals and constraints in (1). The view on the lower left of the interface is a bubble-chart within a configurable 2-D projection of the portfolio. The bubble-chart displays the portfolio decomposition into four categories: *hold* (blue bubble), *hold-bias* (cyan bubble), *sell-bias* (yellow bubble), and *sell* (red bubble). The bubble-chart transforms a 2-D scatter into a corresponding bubble by creating a circle with its center at the midpoint of the scatter, and a radius corresponding to the aggregate, for the assets in the corresponding partition category, of a configurable third measure. Such a bubble-based display is able to give the decision-maker relative feedback on consequences of the preferences and split threshold they have specified. While four decomposed categories are shown, only the sell assets fall in the decision category coded as 1, and the rest of the assets fall in the decision category coded as 0. Figure 2 shows the portfolio as it appears in Figure 1 and with the same decision-maker specified preferences, but now with a more aggressive sell split threshold t. Finally, the decision-maker may view an online history of their decision actions, and may save the results of their interactive session.

The three interactive axes in the upper portion of the interface are 2-D views of the higher dimensional portfolio performance space. They are coupled to the degree that any view rendered and operated upon via any of these three axes works on the same global portfolio dataset. The 2-D axis in the lower left of the interface is a dependent interface, and reacts to user preference specifications in the three interactive axes, *Discount* adjustments for each interactive view, and split threshold adjustments.

V. Conclusions

In this paper, we have presented a visual interactive multi-criteria decision-support method and system for partitioning a portfolio of assets into two mutually exclusive categories. The two decision categories are *hold* and *sell*— assets in the *sell* category are considered as potential sale prospects, and the other assets in the portfolio are considered as potential retention prospects.

We formulated this decision problem as a constrained multi-criteria knapsack problem, and highlighted its inherent computational complexity. Next, we presented the algorithm underlying the interactive framework for multi-criteria portfolio partitioning. Finally, we presented the interactive decision-support system developed to facilitate portfolio partitioning.

The principal motivation for this work was to devise a method wherein the decision-maker actively participates in the decision-making process, and not the design of an algorithm that can solve the multi-criteria portfolio-partitioning knapsack problem without decision-maker intervention. In this interactive approach, the decision-maker drives the exploration for the portfolio partition that satisfies the performance criteria and constraints by expressing their preferences in a systematic manner consistent with their mental representations. Such an interactive decision-maker-driven preference-based portfolio partitioning more naturally supports real-world business decision-making processes.

We are developing an extension to this work wherein preferences from a group of decision-makers rather than from an individual decision-maker will drive the decision-making process. Such an approach will facilitate an automatic consensus-based decision-making process potentially eliminating the requirement for gathering consensus as a post-processing step to the decision-support process we have described in this paper. Such a group-based decision-making process may also result in more robust decisions [11].

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REFERENCES

[1] A. A. Angehrn, and H-J. Lüthi, "Intelligent Decision Support Systems: A Visual Interactive Approach," *Interfaces*, 20(6):17—28, 1990.

[2] C. H. Antunes, and J. Clímaco, "Decision Aid for Discrete Alternative Multiple Criteria Problems: A Visual Interactive Approach," Information and Decision Technologies, 19(3):185—193, 1994.

[3] Y. S. Tan, and N. M. Fraser, "The Modified Star Graph and the Petal Diagram: Two New Visual Aids for Discrete Alternative Multicriteria Decision Making," *Journal of Multi-criteria Decision Analysis*, 7(1):20—33, 1998.

[4] K. M. Miettinen, *Nonlinear Multiobjective Optimization*, Kluwer Academic, Boston, MA, 1998.

[5] R. Klimberg and R. M. Cohen, "Experimental Evaluation of a Graphical Display System to Visualizing Multiple Criteria Solutions," *European Journal of Operational Research*, 119(1):191—208, 1999.

[6] M. Köksalan, and O. Rizi, "A Visual Interactive Approach for Multicriteria Decision Making with Monotone Utility Functions," Journal of the Operational Research Society, 52(6):665—672, 2001.

[7] H. L. Trinkaus, and T. Hanne, "knowCube: A Visual and Interactive Support for Multicriteria Decision Making," *Computers and Operations Research*, 32(5):1289—1309, 2005.

[8] I. S. J. Packham, M. Y. Rafiq, M. F. Borthwick, and S. L. Denham, "Interactive Visualisation for Decision Support and Evaluation of Robustness—in Theory and in Practice," Advanced Engineering Informatics, 19(4):263—280, 2005.

[9] R. Subbu, P. Bonissone, N. Eklund, S. Bollapragada, and K. Chalermkraivuth, "Multiobjective Financial Portfolio Design: A Hybrid Evolutionary Approach," *Proceedings of the 2005 IEEE International Congress on Evolutionary Computation*, Sep. 2-5, 2005, Edinburgh, UK.

[10] J. R. Josephson, B. Chandrasekaran, M. Carroll, N. Iyer, B. Wasacz, G. Rizzoni, Q. Li, and D. A. Erb, "An Architecture for Exploring Large Design Spaces," Proceedings of the 15th National Conference on Artificial Intelligence/Innovative Applications of Artificial Intelligence, pp. 143—150, 1998.

[11] J. Surowiecki, *The Wisdom of Crowds*. Doubleday Press, Random House Publishers, USA, 2004.

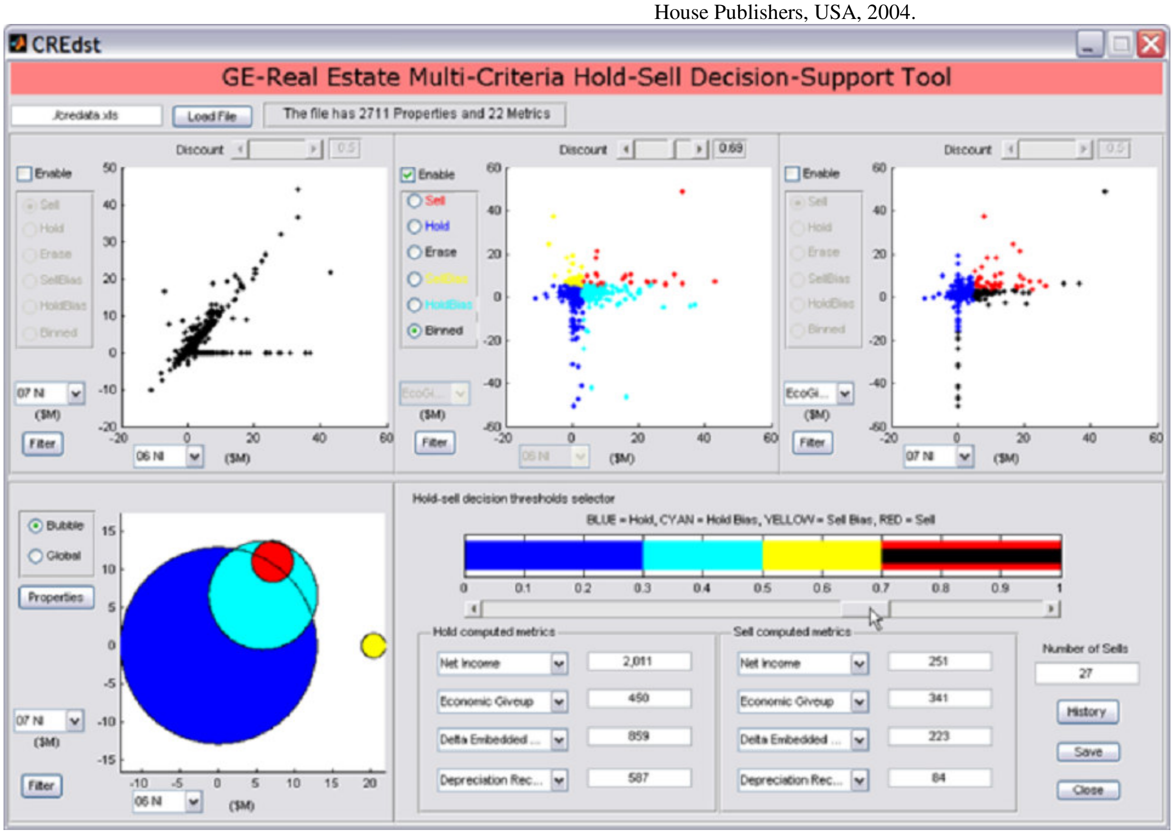

Figure 1: Portfolio-partitioning decision-support interface showing a default partition state.

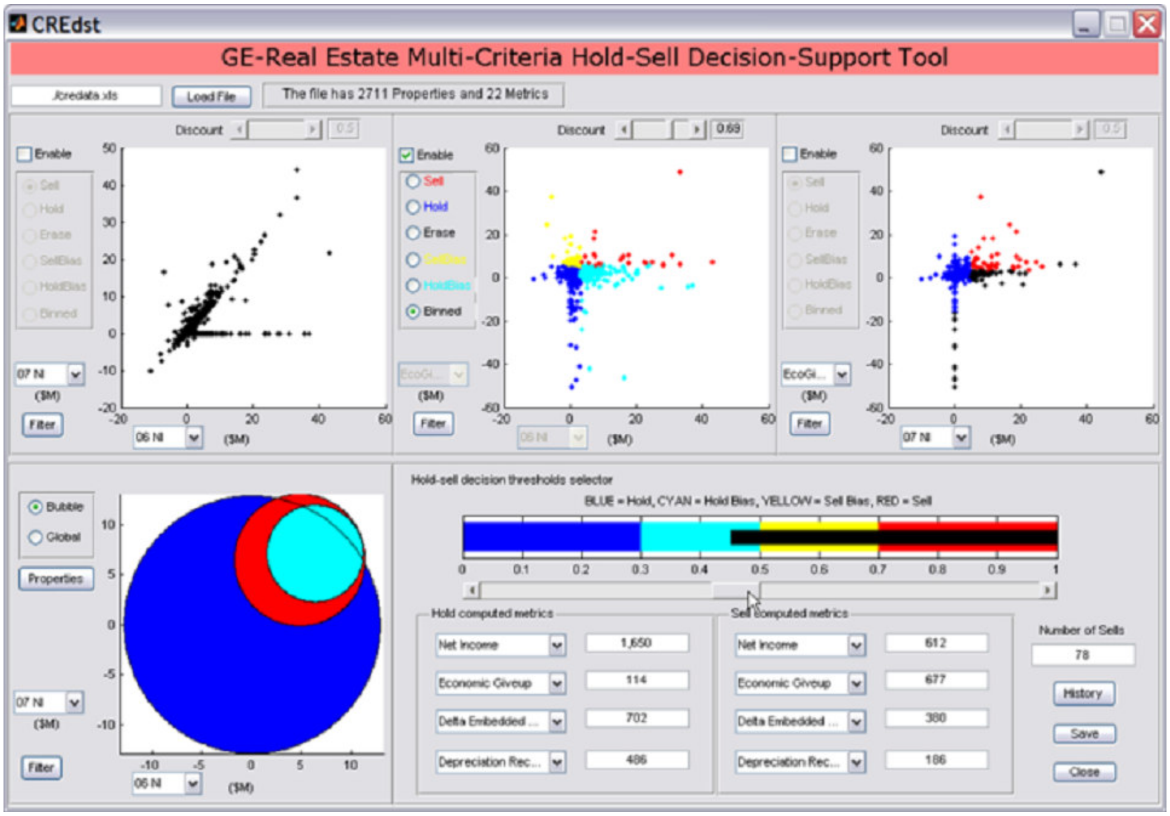

Figure 2: Portfolio-partitioning decision-support interface showing an aggressive partition state.