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Efficient Decision Making with Interactions Between Goals

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Abstract **– A model of interactions between goals based on fuzzy relations for multiple goal decision making (attribute decision making) is presented. In contrast to other approaches, the interactive structure of goals for each decision situation is calculated explicitly based on fuzzy types of interaction. No preference relation defined on the power set of the decision alternatives is required. This helps not only to work with less complex initial information about the decision situation but also provides for a more efficient representation of the decision knowledge and for more efficient decision making procedures. Several real world applications based on the model are used in industry and finance.**

I. INTRODUCTION

In real world applications the understanding of decision making requires much more flexibility than the strict understanding of previously existing decision making models, even if they use fuzzy set theory as framework [12], [10]. As discussed in section V and in [9], the models are not flexible enough to reflect the tension between interacting goals in a way human decision makers do. The key issue of flexible decision making systems is to calculate the interaction between decision making goals in each decision situation.

Compared to the different aggregation operators [15], [17], [19], [21] the decision making procedure based on the interaction between goals is different. The difference is that no preference relation in the sense of a preference relation defined on the power set of the set of decision alternatives is required.
This means that there is both less information required about the decision situation and more efficient calculation of the resulting decisions. The latter is the case because of the smaller cardinality of the sets to be handled.

II. BASIC DEFINITIONS

Before we define interactions between goals as fuzzy relations, we introduce the notion of the positive impact set and the negative impact set of a goal. A more detailed discussion can be found in [7] and [8].

Def. 1)

Let A be a nonempty and finite set of potential alternatives, G a nonempty and finite set of goals,
$A \cap G = \varnothing, a \in A, g \in G, \delta \in [0,1]$.
For each goal g we define the two fuzzy sets S_g and D_g each from A into [0, 1] by:

1. ***positive impact function of the goal g***

$$S_g(a) := \begin{cases} \delta, & a \text{ affects positively } g \text{ with degree } \delta \\ 0, & \text{else} \end{cases}$$

2. ***negative impact function of the goal g***

$$D_g(a) := \begin{cases} \delta, & a \text{ affects negatively } g \text{ with degree } \delta \\ 0, & \text{else} \end{cases}$$

Def. 2)
Let S_g and D_g be defined as in Def.1). S_g is called the *positive impact set of g* and D_g the *negative impact set of g*.
The set S_g contains alternatives with a positive impact on the goal g and δ is the degree of the positive impact. The set D_g contains alternatives with a negative impact on the goal g and δ is the degree of the negative impact.

Def. 3)
Let A be a finite nonempty set of alternatives. Let $P(A)$ be the set of all fuzzy subsets of A. Let $X, Y \in P(A)$, x and y the membership functions of X and Y respectively. The *fuzzy inclusion* **I** is defined as follows:

$$\mathrm{I} : \mathrm{P}(A) \times \mathrm{P}(A) \to [0,1]$$

$$\mathrm{I}(X,Y) =: \begin{cases} \dfrac{\sum_{a \in A} \min\left(x(a), y(a)\right)}{\sum_{a \in A} x(a)}, & \text{for } X \neq \varnothing \\ 1, & \text{for } X = \varnothing \end{cases}$$

with $x(a) \in X$ and $y(a) \in Y$.

The *fuzzy noninclusion* **N** is defined as:

$$\mathrm{N} : \mathrm{P}(A) \times \mathrm{P}(A) \to [0,1]$$
$$\mathrm{N}(X,Y) := 1 - \mathrm{I}(X,Y)$$

The inclusions indicate the existence of interaction between two goals. The higher the degree of inclusion between the positive impact sets of two goals, the more cooperative the interaction between them. The higher the degree of inclusion between the positive impact set of one goal and the negative impact set of the second, the more competitive the interaction. The noninclusions are evaluated in a similar way. The higher the degree of noninclusion between the positive impact sets of two goals, the less cooperative the interaction between them. The higher the degree of noninclusion between the positive impact set of one goal and the negative impact set of the second, the less competitive the relationship.

Note that the pair (S_g, D_g) represents the whole known impact of alternatives on the goal g. [6] show that for (S_g, D_g) the so-called twofold fuzzy sets can be taken. Then S_g is the set of alternatives which more or less certainly satisfy the goal g. D_g is the fuzzy set of alternatives which are rather less possible, tolerable according to the decision maker. Please note that from the technical point of view conditions like: if $S_g(a) > 0$ then $D_g(a) = 0$ could be posted. However, applications have shown that such conditions rather do not help to solve practical problems. Even more: When in real world applications the input information is provided by different, partly or totally independent sources (for instance different departments, different expert decision makers, different input systems) then it is better to accept such possibly inconsistent positive and negative information. Please observe later (see Def. 4), that the types of interactions between decision goals will help to recognize possible inconsistencies, because the interaction between decision goals defined in Def. 4 will reflect them.

Note also that the number of the positive and negative impact sets corresponds to the number of decision goals that are relevant in the decision situation. It can be expected that in real world applications this number is significantly smaller than the number of subsets of decision alternatives (being elements of the power set of decision alternatives) for which preferences have to be specified in the sense of a preference relation as required for the application of average operators.

III. Interactions Between Goals

Based on the inclusion and noninclusion defined above, 8 basic fuzzy types of interaction between goals are defined. The interactions cover the spectrum from very high confluence (analogy) between goals to a strict competition (trade-off). Independence of goals and cases of unspecified dependence are also considered.

Def. 4)

Let $S_{g_1}, D_{g_1}, S_{g_2}$ and D_{g_2} be fuzzy sets given by the corresponding membership functions as defined in Def. 2). For simplicity we write S_1 instead of S_{g_1} etc. Let $g_1, g_2 \in G$ where G is a set of goals.

The types of interaction between two goals are defined as relations which are fuzzy subsets of $G \times G$ as follows:

1. **g_1 is independent of g_2:** $\Leftrightarrow$

$$\textit{IS-INDEPENDENT-OF}\ (g_1, g_2) := \min(\mathrm{N}(S_1, S_2), \mathrm{N}(S_1, D_2), \mathrm{N}(S_2, D_1), \mathrm{N}(D_1, D_2))$$

2. **g_1 assists g_2:** $\Leftrightarrow$

$$\textit{ASSISTS}\ (g_1, g_2) := \min(\mathrm{I}(S_1 S_2), \mathrm{N}(S_1, D_2))$$

3. **g_1 cooperates with g_2:** $\Leftrightarrow$

$$\textit{COOPERATES-WITH}\ (g_1, g_2) := \min(\mathrm{I}(S_1, S_2), \mathrm{N}(S_1, D_2), \mathrm{N}(S_2, D_1))$$

4. **g_1 is analogous to g_2:** $\Leftrightarrow$

$$\textit{IS-ANALOGOUS-TO}\ (g_1, g_2) := \min(\mathrm{I}(S_1, S_2), \mathrm{N}(S_1, D_2), \mathrm{N}(S_2, D_1), \mathrm{I}(D_1, D_2))$$

5. **g_1 hinders g_2:** $\Leftrightarrow$

$$\textit{HINDERS}\ (g_1, g_2) := \min(\mathrm{N}(S_1, S_2), \mathrm{I}(S_1, D_2))$$

6. **g_1 competes with g_2:** $\Leftrightarrow$

$$\textit{COMPETES-WITH}\ (g_1, g_2) := \min(\mathrm{N}(S_1, S_2), \mathrm{I}(S_1, D_2), \mathrm{I}(S_2, D_1))$$

7. **g_1 is in trade-off to g_2:** $\Leftrightarrow$

$$\textit{IS-IN-TRADE-OFF}\ (g_1, g_2) := \min(\mathrm{N}(S_1, S_2), \mathrm{I}(S_1, D_2), \mathrm{I}(S_2, D_1), \mathrm{N}(D_1, D_2))$$

8. **g_1 is unspecified dependent from g_2:** $\Leftrightarrow$

$$\textit{IS-UNSPECIFIED-DEPENDENT-FROM}\ (g_1, g_2) := \min(\mathrm{I}(S_1, S_2), \mathrm{I}(S_1, D_2), \mathrm{I}(S_2, D_1), \mathrm{I}(D_1, D_2))$$

Interactions between goals have subsumption relations [8]. Please also note that it is necessary to consider both the inclusions between S_1, S_2, D_1, D_2 and the corresponding noninclusions in order to better distinguish conflicts and confluences between goals. For instance, one might assume that the interaction 4. g_1 is analogous to g_2 could alternatively be defined as $min(I(S_1, S_2), I(D_1, D_2))$ without using the noninclusion condition. However, in such a case for instance the total analogy g_1 to g_2 in the sense of equality of the goals could not be distinguished from the analogy in the sense of a subgoal relationship g_1 to g_2 [7]. Similar effects would occur too in case of simplifying the definition of g_1 is in trade-off to g_2.

Also the nonassociativity of the types of interaction is better expressed by using both the S_i and D_i , i ∈ {1, 2}.

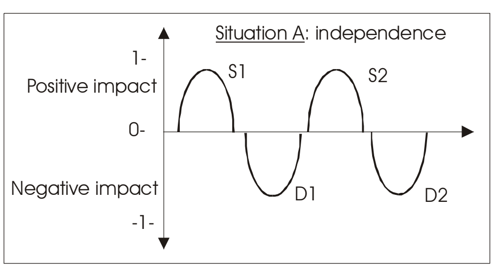
The interactions between goals are substantial for an adequate supervision of the decision making process because they reflect the way the goals depend on each other and describe the pros and cons of the decision alternatives with respect to the goals. Together with information about goal priorities the types of interaction between goals are the basic aggregation guidelines for the decision making process and help to navigate in the decision space. For example, for cooperative goals (see for instance the interaction type 2, 3 and 4) a conjunctive aggregation is appropriate. If the goals are rather competitive (see for instance interactions of type 7), then an aggregation based on an exclusive disjunction is appropriate. In section IV it is shown, based on the types of interaction 1 and 7, how types of interaction imply the way of aggregation.

IV. TYPES OF INTERACTION IMPLY WAY OF AGGREGATION

The assumption that cooperative goals imply conjunctive aggregation and conflicting goals rather lead to exclusive disjunctive aggregation is easy to accept from the intuitive point of view.

The fact that the types of interaction between goals are defined as fuzzy relations based on both positive and negative impacts of alternatives on the goals provides for the information about the confluence and competition between the goals: The negative impact functions reflect the negative aspects of the decision alternatives with respect to each goal and, compared with other approaches, represent additional information which allows distinguishing the nonpresence of confluence between two goals from an effective competition between them.

Figure 1 shows two different representative situations which can be distinguished appropriately only if besides the positive impact additionally the negative impact of decision alternatives on goals is represented.



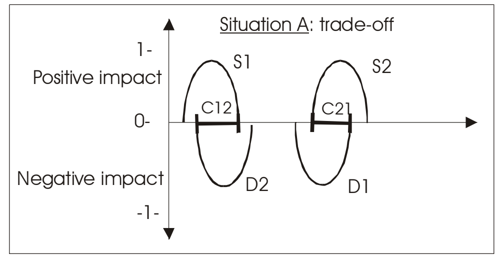


Fig. 1: Distinguishing independence and trade-off based on positive and negative impact functions of goals

In case the goals were represented only by the positive impact of alternatives on them, *Situation A* and *Situation B* could not be distinguished and a disjunctive aggregation would be the recommended in both cases. However, in *Situation B* a decision set $S_1 \cup S_2$ would not be appropriate because of the conflicts indicated by C_{12} and C_{21} (see Fig. 1, Situation B). In this situation the set $(S_1 \;/\; D_2) \cup (S_2 \;/\; D_1)$ could be recommended in case that the priorities of both goals are similar (where X / Y is defined as the difference between the sets X and Y, that means $X \;/\; Y = X \cap \overline{Y}$, where X and Y are fuzzy sets). In case that one of both goals, for instance goal 1, is significantly more important than the other one, the appropriate decision set would be S_1. The aggregation used in that case has not to be a disjunction, but an <u>exclusive</u> disjunction between S_1 and S_2 with emphasis on S_1.

This aspect can easily be integrated into decision making models by integrating the calculation of the types of interaction between goals as defined in Def.4.). The information about the interaction between goals in connection with goal priorities is used in order to apply interaction dependent decision policies which describe the way of aggregation for each type of interaction. For conflicting goals, for instance, the following decision policy, which deduces the appropriate decision set, is given:

If (g_1 is in trade-off to g_2) and (g_1 is significantly more important than g_2) then S_1 .

Another policy for conflicting goals is the following:

If (g_1 is in trade-off to g_2) and (g_1 is insignificantly more important than g_2) then S_1 / D_2.

In this way for every pair of goals g_i and g_j , i,j ∈ {1,…,4} decision sets are calculated. The results of the policies are called local decision sets. For each pair of goals there is a local decision set $D_{Li,j.}$

Note that both policies use priority information which in case of a conflictive interaction between goals is substantial for a correct decision. Note also that the priority information can only by adequately used if the knowledge about whether or not the goals are conflictive is explicitly modeled [11].

Note furthermore that priority information cannot be considered as explicit preference information in the sense of a preference relation used by average operators. On the other hand, the priority information can be used to partly sort the local decision sets which are obtained through the application of the decision policies. The sorting process sorts the local decision sets with respect to the priorities of the goals (for more details see [7]) and induces a preference relation on the local decision sets.

Since the local decision sets are elements of the power sets of the decision alternatives, the decision policies and the sorting process of the local decision sets induce a preference relation on the power set $\mathcal{P}(A)$ of the set of the decision alternatives A (now in the sense of the aggregation operators like OWA, WOWA and others). This is, in opinion of the author, a very interesting observation which may help to close the gap between the decision making based on interactions between goals and the existing literature on aggregation operators and their way of dealing with interactions between goals.

Please note also that priority information is input information and is not explicitly derived from the interaction between decision goals. However, the interactions themselves between goals are derived explicitly using Def. 4 and implicitly provide for the information which pairs of goals have to have contrary priorities: namely goals with a negative type of interaction (for instance trade-off). In this way, the explicitly derived information about the interactions between the goals implicitly helps to find consistent goal priorities which are compatible with the interactions between the goals. That means that the explicitly derived interactions between the decision goals provide for the information about consistent goal priorities and therefore imply a consistent way of aggregation.

Interactions between decision goals as defined in Def. 4 are related to pairs of goals. From the point of view of many real world applications the consideration of such pairwise interactions turned out to be sufficient. However, from the theoretical point of view an extension of the concept to groups of goals is interesting. One idea how to do it is to use the local decision sets calculated for the pairs of goals g_1 / g_2 and to consider them as new impact sets, now standing for the impact sets S_{ij} and D_{ij} that express the common impacts of the goals g_i and g_j.

The recursive application of Def. 4, now to the S_{ij} and D_{ij} for every pair of goals g_i and g_j will provide for the interaction of pairs of pairs of goals. The recursive application stops when no new local decision sets are generated.

V. RELATED APPROACHES, A BRIEF COMPARISON

Since fuzzy set theory has been suggested as a suitable conceptual framework of decision making [2], two directions in the field of fuzzy decision making can be observed. The first direction reflects the fuzzification of established approaches like linear programming or dynamic programming [22]. The basic aspect of this approaches is that decision making is reduced to modeling the goals as linear functions and the decision as a linear combination of the linear goal functions. That means that the aggregation is basically modeled as a weighted sum of linear functions. In case that the goal functions are not linear the methods move towards the so called mathematical programming. If the search for solutions is not analytical but random, the field of models like evolutionary computation is entered. The weighted sum then is called fitness function.

The second direction is based on the assumption that the process of decision making can be modeled by axiomatically specified aggregation operators [5]. None of the related approaches sufficiently addresses one of the most important aspects of decision making, namely the explicit and nonhardwired modeling of the interaction between goals. Related approaches like [5], [3] either require a mathematically strict way of describing the goals or postulate that decision making shall be performed based on a few, very general mathematical characteristics of aggregation like commutativity or associativity.

Compared to aggregation operators defined based on preference relations like OWA, WOWA and similar [15], [17], [19], [20], [21] the difference is that the approach presented in this paper does not require a preference relation but rather induces (generates) one based on the interaction between the goals and the goal priorities. The latter is required but its complexity is linear to the number of goals. This means that the input information required is much less complex and therefore more efficient.

Other approaches are based on fixed hierarchies of goals [18] or on the modeling of the decision situations as probabilistic or possibilistic graphs [13]. In contrast to that, human decision makers usually proceed in a different way. They concentrate on the question, which goals are positively or negatively affected by which alternatives. Furthermore, they evaluate this information in order to understand how the goals interact with each other and ask for the priorities of the goals. In the sense that the decision making approach presented in this contribution explicitly refers to the interaction between goals, it significantly differs from other related approaches [9] and is closer to the human way of decision making.

Surprisingly, although the MCDM literature has significantly grown [1], [14] in the last 10 years, the interaction between goals has not been explicitly considered and the statements made in [7],

[8], [9] still hold. Instead of this, considerable effort has been made in investigating other aspects like different properties [16] and the behavior of existing aggregation operators (for application examples see for instance [4]) which basically are linear combinations of utility functions and weights [1]. However, being linear combinations the aggregation operators cannot adequately model decision situations within which goal conflicts occur since goal conflicts by nature involve nonlinearity into the decision process.

VI. Examples of Applications of Decision Making Based on Interactions Between Goals

In the subsequent subsection two of several examples of applications of the decision making model are briefly described. It is indicated why the positive and negative impacts of decision alternatives on decision goals are needed, and how the positive and the negative impacts imply the interaction between the decision goals. It is indicated what is the gain of efficiency provided by the decision making model presented in this paper.

A. Application to the Optimization of Production Sequences in Car Manufacturing

The assembly of cars is a production process, whose regulation has to respond to the disturbances and fluctuations that often arise in such processes. The blocking and release of different models and special equipment require flexible reactions by the staff supervising the control system, in order to guarantee that utilization of the assembling capacity is as regular as possible. To achieve this, the supervisor often adjusts the production process manually by modifying the order of introduction of the car bodies into the assembly area and the mixture of special equipment required. Due to the complexity of the positive and negative impacts of different mixes of equipment on the utilization of capacity, adequate manual supervision of the assembly process is very complex. If all restrictions are to be incorporated into the supervisor's decision making process, an automated decision system is required. The principle of the optimization presented in this section is used at more than 20 different factories of more than 30 different car models. It is used for both the pre-calculation of the sequences and the online-execution of the sequence during the production process. The planning mode is used for the calculation of sequences of a package of cars scheduled for a period of approximately one day and is performed two or three days before the production. The execution mode is continuously re-optimizing the sequence during the assembly process [10]. The number of decision goals (optimization criteria) in this application is approximately between 60 and 100. The number of decision alternatives corresponds to the number of possible sequences out of the cars to be produced in the optimization period. This number is 1000 and more. Compared to modeling via a preference relation where we have the complexity of approximately 2^{1000} preference statements, our approach requires approximately less than 1000^2 x 100 statements.

B. Application to the Selection of Test Tools

Software testing is an important aspect of the quality assurance in software development projects and in the maintenance of software. Software testing is a very complex process and there is a variety of different software test tools which help the software tester to accelerate the testing process. Since there is no all-purpose testing tool, the test tool selection is an important issue and significantly influences both success and costs of the subsequent testing activities. The test tool selection is based upon a set of selection criteria corresponding to the decision goals. The decision alternatives are the test tools themselves. None of the test tools covers all criteria, some of the criteria are partly covered and some of them are partly or totally uncovered. That means that together with the fact that there are more than forty different test tools and that the selection is based on more than seventy criteria, there are a lot of positive and negative impacts to be considered when the selection ranking is calculated. Compared to preference relation based approaches where we have a space of approximately 2^{40} subsets of tools for which preferences have to be expressed, in our approach we need approximately 70 x 40 statements in order to express the knowledge of an expert decision maker: a significant gain in efficiency when designing the decision making system. The test tool selection is a service provided to the customers of a leading German company that offers services in the context of software quality assurance.

VII. Conclusions

The importance of an appropriate analysis of decision situations with their types of interaction between goals in decision making has been discussed. The interactions reflect the different types of cooperation and conflicts between the decision goals. The explicit representation of both positive and negative impacts of decision alternatives on goals provides for the ability of situation dependent calculation of interactions between goals and for an appropriate way of aggregation. Compared to other approaches the representation of the decision knowledge is significantly more efficient. The way of calculating interactions between decision goals presented above has been used in various application fields, two of them have been briefly described.

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