Proceedings of the 2007 IEEE Symposium on Computational Intelligence in Multicriteria Decision Making (MCDM 2007)

A Multi-Objective Genetic Algorithm for Optimizing Highway Alignments

Manoj K. Jha and Avijit Maji
Department of Civil Engineering
Morgan State University
1700 East Cold Spring Lane
Baltimore, MD 21251 USA

***Abstract*—We develop a multi-objective approach to optimize 3-Dimensional (3D) highway alignments using a genetic algorithm. Multi-objective genetic algorithms have been very popular in recent years for handling trade-offs among various objectives. The concept of pareto optimally has been introduced in recent works and multi-objective genetic algorithms have been developed for this purpose. What we have found is that every problem is unique and there is no black box approach to implement multi-objective genetic algorithms in all problems. We implement the pareto-optimality concept to develop a multi-objective genetic algorithm for the 3D highway alignment optimization problem on which we have worked for the last 10 years. We apply the multi-objective optimization approach to an example problem on which we had previously worked. The results suggest that the multi-objective approach has great promise for obtaining the best trade-off among various objectives to reach an optimal solution.**

I. INTRODUCTION

OUR research team has worked for over last eight years [1-7] to develop a 3-Dimensional (3D) highway alignment optimization (HAO) model. A state-of-the-art integrated genetic algorithms and geographic information systems-based approach was developed for optimizing 3D highway alignments by comprehensively formulating [7] alignment sensitive user and operator costs, such as pavement and constructions costs, earthwork cost, right-of-way cost, vehicle operating cost, accident cost, and travel time delay cost.

One of the weaknesses of the HAO model that prevented its widespread applicability in real-world projects was its inability to handle different user preferences and a trade-off analysis, since cost minimization was the only criteria used for optimization. After extensive industry feedback we have decided to expand the model capability to incorporate a multiobjective approach in optimization so that different objectives reflecting diverse user preferences can be incorporated into evaluating several alignments. This paper reports preliminary findings of the multiobjective approach to 3D highway alignment optimization that has the ability to incorporate various user preferences. It can thus prove to be an effective and versatile tool for decision-making and can be used by highway agencies while allowing quick evaluation of several competing alternatives, and performing a trade-off analysis. Before proceeding to the discussion of the multiobjective approach and to put things into perspective we present an overview of our previously completed work in Highway Alignment Optimization (HAO)

II. HIGHWAY ALIGNMENT OPTIMIZATION (HAO) MODEL

As noted above a single objective 3D Highway Optimization Model (HAO) that uses genetic algorithms (GAs) and Geographic Information Systems (GISs) was developed by our team in previous works [1-7]. It was shown that GAs were able to search in a continuous space without getting trapped in local optima. Before our research efforts several studies were reported in the literature for highway alignment optimization [1] but they did not realistically solve the HAO problem due to algorithmic limitations or imprecise and incomplete cost formulations. In our work integration of GAs with GIS [6] allowed working with real maps and performing a trade-off analysis with varying degree of environmental impacts, although with the limitation of a single objective minimization. The unique aspect of GA application [3, 5] is to formulate the encoded string of decision variable and develop problem-specific operators. While GAs do not guarantee optimal solution the extensive research conducted by our research team [1, 3, 5] suggest that a reasonably good solution (usually within 1% tolerance interval) can be found when searched through sufficient number of generations depending on the problem size. For the HAO problem it should be noted that computational burden increases as the size of the search space increases. A stepwise GAs can be developed for reducing the computation burden as suggested in one of our recent works [8].

Since the objective function for HAO cannot be represented as explicit function of the decision variables application of traditional search algorithms (such as, linear programming, numerical search, and gradient-based search) are either not suited or the problem has to be oversimplified before such algorithms could be applied [1]. A suitable objective function can be developed by assuming that the proposed alignment can be sufficiently described by a set of intermediate points P_i's between given start and end points. In order to describe an alignment, first a straight line is drawn connecting given start and end points. Next, orthogonal planes at random intervals are drawn across that line (Figure 1). P_i's are randomly placed along those orthogonal planes. If a P_i falls along the straight line then a tangent section is obtained, otherwise a curved section is obtained. Appropriate curves can be fitted using American Association of State Highway and Transportation Officials (AASHTO) [9] design criteria. Thus, an alignment can be sufficiently described by: (1) random location of cutting planes; (2) random location of P_i's along the planes. Once an alignment is described its associated costs (such as, right-of-way, pavement, construction, environmental impact, accident, travel-time delay, and vehicle operating costs) can be calculated by developing cost functions in sufficient detail [5, 7]. Note that in Figure 1 each intersection point in the 3D space can be determined by only two decision variables (the abscissa and ordinate on the vertical cutting plane). This helps reduce the dimension of the search space. Then by trigonometry, we can further transform these intersection points into the Cartesian coordinate system. Let O_i be the point at which the line segment $\overline{SE}$ intersects the i^{th} cutting plane, where S and E are the start and end points of the alignment. Then the X and Y coordinates of O_i can be obtained by simple trigonometric transformation. On each vertical cutting plane, the abscissa, denoted by d_i, is defined as the axis that passes through O_i and parallels the XY plane, with O_i as its origin. The ordinate on the i^{th} cutting plane is simply defined as the Z coordinate in the Cartesian coordinate system to reduce coordinate transformation requirements.

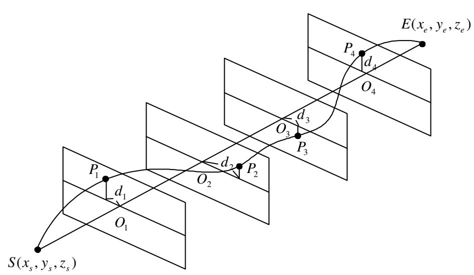

Fig. 1. Representation of a 3-D alignment for optimization formulation

Let P_i be the intersection point on the i^{th} cutting plane, whose coordinates are (d_i, z_i). Then the Cartesian coordinates of P_i, denoted by $(x_{P_i}, y_{P_i}, z_{P_i})$ can be obtained by:

$$\begin{bmatrix} x_{P_i} \\ y_{P_i} \\ z_{P_i} \end{bmatrix} = \begin{bmatrix} x_{O_i} \\ y_{O_i} \\ 0 \end{bmatrix} + \begin{bmatrix} d_i \cos\theta \\ d_i \sin\theta \\ z_i \end{bmatrix} \quad (1)$$

where: (x_{O_i}, y_{O_i}) are the coordinates of the origin of the abscissa on the i^{th} cutting plane and θ = the angle of cutting planes on the XY plane

III. SINGLE OBJECTIVE OPTIMIZATION

The single objective optimization included formulation of a single objective function, and a set of constraints. The objective function consists of alignment sensitive costs, such as user cost (C_U), right-of-way cost (C_R), pavement cost (C_P), earthwork cost (C_R) and structure cost (C_S) as shown in Eq. (2a). These costs are formulated as functions of decision variables, which are reported in our previous works [5, 7]. Additional cost functions can be formulated as desired.

$$\underset{x_{P_1}, y_{P_1}, z_{P_1}, \ldots\ldots, x_{P_n}, y_{P_n}, z_{P_n}}{\text{Minimize}} \quad C_T = C_U + C_R + C_P + C_E + C_S \quad (2a)$$

$$\text{subject to} \quad x_O \le x_{P_i} \le x_{\max}, \quad \forall\, i = 1, \ldots.., n \quad (2b)$$

$$y_O \le y_{P_i} \le y_{\max}, \quad \forall\, i = 1, \ldots.., n \quad (2c)$$

$$z_O \le z_{P_i} \le z_{\max}, \quad \forall\, i = 1, \ldots.., n \quad (2d)$$

where (x_0, y_0, z_0) and $(x_{\max}, y_{\max}, z_{\max})$ are upper and lower limits of the search space, respectively [1-3].

The user cost consists of travel–time cost, vehicle operating cost, and accident cost. The right-of-way cost consists of the land area taken by the alignment and damage to the properties and is calculated directly from a GIS and transmitted to GA during optimal search. The detailed formulations of alignment sensitive costs shown in Eq. (2a) are provided in previously published works [5, 7] and have been skipped here for brevity. To eliminate any confusion it is pointed out here that effects of traffic congestion, travel-time delay, and topography are all considered in the developed cost functions. There are also many design and operational constraints to be met in alignment optimization. Among those, minimum length of vertical curves, gradient constraint, sight-distance constraint, and environmental constraints are important ones, which have been sufficiently formulated in previous works.

A. *Genetic Encoding of Decision Variables*

In GAs the decision variables are encoded as binary or real numbers, called chromosomes. In our HAO approach real numbers are used to represent decision variables within the specified bounds. For an alignment represented by n points of intersections, the encoded chromosome is composed of $3n$ genes. Thus, the chromosome is defined as:

$$\Lambda = [\lambda_1, \lambda_2, \lambda_3, \ldots\ldots, \lambda_{3n-2}, \lambda_{3n-1}, \lambda_{3n}] = [x_{P_1}, y_{P_1}, z_{P_1}, \ldots\ldots x_{P_n}, y_{P_n}, z_{P_n}] \quad (3)$$

where: Λ = chromosome

λ_i = the i^{th} gene, for all $i = 1,.......,3n$

$\left(x_{P_i}, y_{P_i}, z_{P_i}\right)$ = the coordinates of the i^{th} point of intersection, for all $i = 1,.......,n$

B. Genetic Operators

Eight problem-specific genetic operators are designed [3, 5] to work on the encoded points of intersection rather than on individual genes. Those are: uniform mutation, straight mutation, non-uniform mutation, whole non-uniform mutation, simple crossover, two-point crossover, arithmetic crossover, and heuristic crossover. Extensive tests are conducted to ensure that these operators assist in obtaining efficient solution while exploiting the entire search space and without getting trapped in local optima.

IV. MULTI-OBJECTIVE (MO) APPROACH

In highway route planning there are different golas and objectives that need to be satisfied before reaching at a consesus. These goals and objectives can often be conflicting and building a concesus can take exhaustive amout of time. For example, there may be a situation where impact to a historic site may take domimance over impct to parklands requiring several iterations of reoptimization to ensure that there is absolutely no impact to the historic site while also satisfying other objectives (to the extent possible), such as minimizing impact to parklands and expensive rights-of-way.

A. A MO Optimization Scenario

In one of the Maryland State Highway Administration funded research projects [10] on which we recently worked, two relevant issues were noted in searching for the best highway alignment. The first issue was to find the best alignment that minimized total cost. The other issue was to find the best alignment that minimized impacts to environment and other sensitive areas, such as parkland and historic site. The proposed alignment should have simultaneously satisfied these two key issues.

Using the single objective HAO model the only way to control the impacts to environmentally and socio-economically sensitive areas is to impose a very high penalty when such areas are affected. The penalty function acts as a "dummy cost" added to the total cost function such that the optimization algorithm has a tendency to minimize the impact since it's driven by the "single objective cost minimization" principle. In real-world road projects however, there may exist different types of sensitive areas. For example, in the recently complemented project two types of sensitive areas were noted: (1) type 1 area that the proposed alignment must avoid (we call it "hard constraint") and (2) type 2 area to which impacts should be minimized, to the extent possible (we call it "soft constraint") while satisfying the "hard constraints." A multiobjective optimization approach thus becomes inevitable. It should be noted that minimizing varying levels of impacts is frequently encountered in real-world highway projects leading to frequent scope changes and cost overruns.

Let's consider the following case shown in Figure 2. Suppose a highway agency is considering the construction of a new bypass to avoid traffic congestion in the City of Brookeville, and wants to apply the existing single objective HAO model to obtain the best bypass route. Two socio-economically sensitive areas (a Community Center and a historic site) exist within the project area. With the current controlling method for the sensitive region, the HAO model finds the best alignment (alignment B), which detours the Community Center and the Historic site as shown in Figure 2. The alignment B can be considered the optimized alignment only if the Community Center does not want any of its areas to be affected by the new road, i.e., it is treated as a "hard constraint." However, it may be possible that the Community Center allows its site to be taken by the new road up to a very small quantity; for instance, 200 sq. ft (or up to a user-specified upper bound). In other words, impact to the community center is treated as a soft constraint. In that case alignment A may be the preferred alternative. When comparing the optimal costs of Alignments A and B it is found that Alignment B costs more than A. Such multiobjective optimization and trade-off analysis scenarios are currently not handled by the single objective HAO methodology.

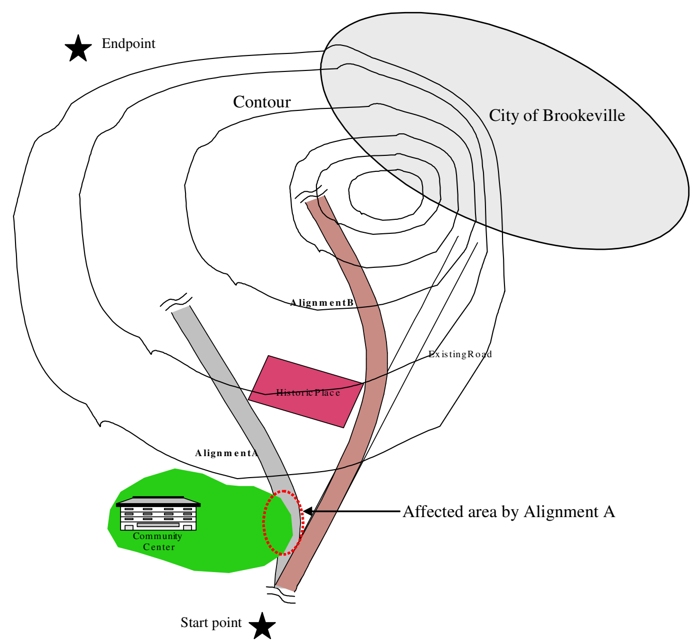

Fig. 2. A Real-World Road Project Requiring Multiobjective Optimization

The need for multi-criteria decision making and multiobjective optimization has long been recognized. A number of articles dealing with these issues can be found in the literature [11-24]. Holguin-Veras [11] performed a comparative assessment of analytic hierarchy process (AHP) and multiattribute value (MAV) functions. Some researchers also performed goal programming [15]. The multiobjective optimization methodology to be developed for HAO is different than AHP, MAV, and goal programming in that these approaches require that the number of alternatives to be analyzed be given a priori. In the multiobjective HAO approach alignments are randomly generated and evaluated. The need for multiobjective optimization arises primarily because one is generally interested to quickly examine changes in different objectives (such as changes in total cost, environmental damage, rights-of-way, and alignment length) as one or more variables are changed.

It is realized that among multi-objective (MO) optimization problems, multiple objectives under consideration are often noncommensurable and cannot be integrated into a single one. With this observation, the notion of Pareto optimality has been introduced in recent years instead of the optimality concept in single-objective optimization [20-22, 24]. However, the Pareto optimal solutions cannot be uniquely determined, i.e. there usually exist a set of solutions that all satisfy Pareto optimality, which form the Pareto Front in the solution space. Hence, decision makers usually cannot find a best solution that dominates all the others. On the contrary, they may prefer to seek a Pareto Front which is formulated by the Pareto optimal solutions, so that they can evaluate the trade-offs among these solutions and make decisions accordingly [20].

B. Pareto Optimum

The notion of Pareto optimum was from Pareto's original work [24]. Consider a k-objective min optimization problem:

$$Min\ \bar{f}(\bar{x}) = [f_1(x_1), f_2(x_2), f_3(x_3), ..., f_k(x_k)]^T \tag{4a}$$

subject to

$$\bar{g}(\bar{x}) \geq 0 \tag{4b}$$

$$\bar{h}(\bar{x}) = 0 \tag{4c}$$

where $\bar{x}$ is the decision vector, $\bar{g}$ is the inequality constraint, and $\bar{h}$ is the equality constraint. A solution vector $\bar{x}^*$ is said to dominate another solution vector $\bar{x}$ if and only if:

$$\forall i, f_i(\bar{x}^*) \leq f_i(\bar{x}) \wedge \exists i, f_i(\bar{x}), i \in \{1,2,3,...,k\} \tag{5}$$

A solution is called Pareto optimal if no other solutions dominate it. In practical MO optimization problems, it is usually impossible to find a unique solution that dominates all other solutions. Instead, it is expected that a number of Pareto optimal (or called non-dominated) solutions can be found. A Pareto Front is formed by those Pareto optimal solutions, where an increase in one objective will surely cause a decrease in another or more in other objectives. The goal of MO optimization is to generate feasible solutions that are on, or close to, the Pareto Front. From those alternatives that are situated on or around the Pareto Front, decision makers can make their final decision.

C. Pareto Ranking

A common measure to convert a MO optimization problem into a single objective one is to use scalarization methods, e.g. weighting method or minimax method. However, these methods cannot efficiently characterize the Pareto Front for a MO optimization problem. To overcome these, Goldberg (see, [24]) first proposed a selection strategy based on Pareto dominance; the strategy has then been modified and improved by other researchers [24].

V. MULTIOBJECTIVE GENETIC ALGORITHM

In order to treat simultaneously several objective functions, it is necessary to substitute the single-fitness-based procedure employed in the single objective GA for comparing two proposals of solution. The comparison of two chromosome-coded solutions with respect to several objectives may be achieved through the introduction of the concepts of Pareto optimality and dominance as described above [20,24], which enable solutions to be compared and ranked without imposing any a priori measure as to the relative importance of individual objectives, neither in the form of subjective weights nor arbitrary constraints. In practice, often, constraints exist, based on experience, which impose restrictions that the candidate solutions have to satisfy.

Such constraints may be handled, just as in the case of single-objective GAs, by testing for the fulfillment of the criteria by the candidate solutions during the population creation and replacement procedures. The introduction of external constraints speeds up the convergence of the algorithm because it reduces the search space. In the multiobjective applications it is suggested not to impose any constraint a priori on the proposed candidate solutions. This is done so as to ascertain that the experience-based constraints are consistent with the set of input data governing the problem. Results quite different from those suggested by experience would imply a deeper investigation to find whether a suboptimal solution is achieved or a more careful analysis on the input data and model consistency.

Let us consider N different objective functions, $f_i(X), i = 1,..,N$ where X represents the vector of independent variables identifying a generic proposal of solution. We say that solution X dominates solution Y if X is better on all objectives [20], i.e., if $f_i(X) > f_i(Y), \forall i = 1,...,N$. If a solution is not dominated by any other in the population, it is said to be a nondominated solution. Using this definition, a ranking of the population can be readily performed. All nondominated individuals in the current population are identified. These solutions are considered the best solutions, and assigned the rank 1. Then, these solutions are virtually removed from the population and the next set of nondominated individuals are identified and assigned rank 2. This process continues until every solution in the population has been ranked. The selection and replacement procedures of the multiobjective GAs are based on this ranking: every solution belonging to the same rank class has to be considered equivalent to any other of the class, i.e. it has the same probability of the others to be selected as a parent and survive the replacement.

During the optimal search, an archive of a given number of nondominated solutions representing the dynamic Pareto optimality surface can be recorded and updated. At the end of each generation, nondominated solutions in the current population can be compared with those already stored in the archive and the following archival rules are implemented:

(1) If the new solution dominates existing members of the archive, those are removed and the new solution is added.

(2) If the new solution is dominated by any member of the archive, it is not stored.

(3) If the new solution neither dominates nor is dominated by any member of the archive then:

- If the archive is not full, the new solution is stored.
- If the archive is full, the new solution replaces the most similar one in the archive.

The setup of an archive of nondominated solutions can also be exploited by introducing an elitist parents' selection procedure which should in principle be more efficient. Every solution in the archive (or a pre-established fraction of the population size N_p, typically $N_p/4$, if the archive's size is too large) is chosen once as a parent in each generation. This should guarantee a better propagation of the genetic code of nondominated solutions, and thus a more efficient evolution of the population towards Pareto optimality. At the end of the search procedure, the result of the optimization is constituted by the archive itself which gives the Pareto optimality region.

A. Multi-Objective Genetic Algorithm for Alignment Optimization

Based on the preceding discussion a multiobjective genetic algorithm is developed for the HAO problem, to handle the objectives of (1) total cost minimization; (2) minimization of floodplain impact area; and (3) minimization of wetland impact area. The approach is similar to that proposed by Busacca et al. [20] in which a multiobjective genetic algorithm is designed to compare the tradeoff between reliability and system safety. We modify the chromosome of the single objective optimization (Eq. (3)) to include two additional genes, one each for floodplain and wetland impacts. Those genes can take up real values between 0-3 where 0 implies no impacts to floodplain and wetland areas whereas 3 implies high impact (within a user-specified range of allowable impact area). The ranking or candidate solutions in the populations leading to selecting replacement scheme is described in the preceding section.

VI. NUMERICAL EXAMPLE

We apply the multiobjective approach to an example study [4] on which we had previously worked using the single objective genetic approach. Two environmental factors were considered: wetlands and floodplains. Impacts to these factors acted as hard constraints, i.e., impacts to these factors must be minimized even though total alignment cost was more than that obtained from single objective cost minimization algorithm.

The study area (Fig. 3) consists of a section of Anne Arundel County, Maryland. It is desired to search for a best alignment alternative connecting existing Interstate Rt. 97 and Reece Rd. (Maryland Rt. 174) between a given start and end points. Existing major highways and environmental factors such as floodplains and wetlands falling in the study section are also shown. The shaded portions represent wetlands. The area of the study section is about 8 square km and the Euclidean distance between the start and end points is 2.68 km. The terrain height in the study section ranges from 40 to 55 meters.

A single objective optimization yielded an optimized alignment with impacts to floodplains at 3 locations. The optimum cost was found to be \$11.005 millions (Table 1). The length of the optimized alignment was 2.89 km, impact to the floodplain was 635 square meters and no wetland impact was noted.

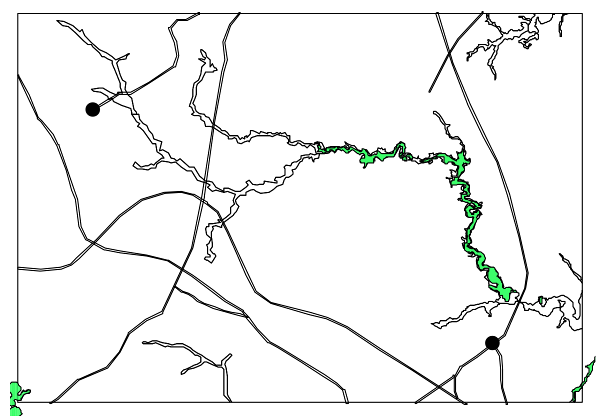

Fig. 3. Study Area for the Example Study

A single objective optimization yielded an optimized alignment with impacts to floodplains at 3 locations. The optimum cost was found to be \$11.005 millions (Table 1). The length of the optimized alignment was 2.89 km, impact to the floodplain was 635 square meters and no wetland impact was noted.

In the multiobjective approach three objectives were imposed: (1) minimize total alignment cost; (2) minimize floodplain impact; and (3) minimize wetland impact. Using the pareto optimality approach with GAs as described above the best solution was obtained with an alignment length of 3.45 km, total cost of \$13.453 millions, impact to floodplain impact= 496 square meters, and no impact to floodplain (see, Table 1). The results are consistent with our previous findings using a criteria-based decision support system that used a single objective optimization [4] for similar analysis.

Table 1. Comparison of Single and Multiobjective Optimization Results

Optimization Scenario	Length (km.)	Cost ($M)	Floodplain Impact (sq. m.)	Wetland Impact (sq. m.)
Single Objective (cost min.)	2.89	11.005	635	0
Multiobjective	3.45	13.453	496	0

VII. RESULTS AND DISCUSSION

We found very interesting results with the multiobjective optimization approach. In our previous works we had saved the candidate solutions in every population of the search generation under the single-objective cost minimization approach. We then manually retrieved those solutions for which cost, floodplain, and wetland impacts were smaller. Since in 100 generations of search nearly 3,200 candidate solutions are explored the manual ranking of the solutions tended to be very time-consuming and did not ensure a best trade-off among the three objectives. The multiobjective approach certainly seems promising since it automates the trade-off procedure eliminating the manual ranking requirement as well as manual approximation in obtaining the best trade-off solution. Additional tests need to be carried out to examine the general applicability of the multiobjective approach.

VIII. ACKNOWLEDGMENTS

The authors would like to thank the Ph.D. candidate Min-Wook Kang at the University of Maryland, College Park who created the Fig. 2 multiobjective scenario and brought the multiobjective idea to the author's attention. This work was completed at the Center for Advanced Transportation and Infrastructure Engineering Research (CATIER; www.eng.morgan.edu/~catier) at the Morgan State University.

REFERENCES

[1] M.K. Jha, P. Schonfeld, J.-C. Jong, and E. Kim, *Intelligent Road Design,* WIT Press, Southampton, UK, 2006.

[2] M.K. Jha and P. Schonfeld, "A Highway Alignment Optimization Model using Geographic Information Systems," *Transportation Research, Part A,* 38(6), pp. 455-481, 2004.

[3] J.-C. Jong and P. Schonfeld, "An Evolutionary Model for Simultaneously Optimizing 3-Dimensional Highway Alignments," *Transportation Research, Part B,* 37(2), pp. 107-128, 2003.

[4] M.K. Jha, Criteria-Based Decision Support System for Selecting Highway Alignments. *Journal of Transportation Engineering,* 129(1), pp. 33-41, 2003.

[5] J.-C. Jong, M.K. Jha, and P. Schonfeld, "Preliminary Highway Design with Genetic Algorithms and Geographic Information Systems," *Computer-Aided Civil and Infrastructure Engineering,* 15 (4), pp. 261-271, 2000.

[6] M.K. Jha and P. Schonfeld, "Integrating Genetic Algorithms and GIS to Optimize Highway Alignments," *Transportation Research Record* 1719, pp. 233-240, 2000.

[7] J.-C. Jong and P. Schonfeld, "Cost Functions for Optimizing Highway Alignments," *Transportation Research Record* 1659, pp. 58-67, 1999.

[8] E. Kim, M.K. Jha, and B. Son, Improving the Computational Efficiency of Highway Alignment Optimization Models through a Stepwise Genetic Algorithms Approach, Transportation Research, Part B, 39(4), pp. 339-360, 2005.

[9] AASHTO, *A Policy on Geometric Design of Highways and Streets,* American Association of State Highway and Transportation Officials, Washington, D. C., 2001.

[10] MSHA (2001). *MD 97: Brookeville Transportation Study,* Maryland State Highway Administration, Baltimore, MD.

[11] J. Holguín-Veras, "Comparative Assessment of the Analytic Hierarchy Process and Multiattribute Value Functions for Highway Evaluation: A Case Study," *Journal of Transportation Engineering,* 121 (2), pp. 191-200, 1995.

[12] T.L. Saaty, *The Analytic Hierarchy Process,* McGraw Hill, Inc., New York, 1980.

[13] M. Chiampi, C. Ragusa, and M. Repetto, "Fuzzy Approach to Multiobjective Optimization in Magnetics," *IEEE Transactions in Magnetics,* 32(3), pp. 1234-1237, 1996.

[14] R.L. Keeney and H. Raiffa, *Decision with Multiple Objectives: Preferences and Value Tradeoffs,* John Wiley & Sons, NY, 1976.

[15] J.T. Taber, "Multiobjective Optimization of Intersection and Roadway Access Design," MPC Report Number 98-90, U.S. Department of Transportation, Washington, D.C., 1998.

[16] J.-B. Yang,, and D. Li, "Normal Vector Identification and Interactive Tradeoff Analysis Using Minimax Formulation in Multiobjective Optimization," IEEE Transactions On Systems, Man, And Cybernetics—Part A: Systems And Humans, 32(3), pp. 305-319, 2002.

[17] H. Zhao, and T.-T. Lee, "Research on Multiobjective Optimization Control for NonLinear Unknown Systems," The IEEE International Conference on Fuzzy Systems, 2003.

[18] Z. Ziyang, K. Hidajat, and A.K. Ray, "Multiobjective Optimization of Simulated Countercurrent Moving Bed Chromatographic Reactor (SCMCR) for MTBE Synthesis," *Ind. Eng. Chem. Res.,* 41, pp. 3213-3232, 2002.

[19] L. Blasi, L. Iuspa, and G.D. Core, "Speed-Sensitivity Analysis by a Genetic Multiobjective Optimization Technique," *Journal of Aircraft,* 39(6), pp. 1076-1079, 2002.

[20] P.G. Busacca, M. Marseguerra, and E. Zio, "Multiobjective Optimization by Genetic Algorithms: Application to Safety Systems," *Reliability Engineering and System Safety,* 72 (1), pp. 59-74, 2001.

[21] F.Y. Cheng and D. Li, "Multiobjective Optimization Design with Pareto Genetic Algorithm," *Journal of Structural Engineering,* 123(9), pp. 1251-1261, 1997.

[22] B. Huang, P. Ferry, and L. Zhang, "Multiobjective Optimization for Hazardous Materials Transportation," *Transportation Research Record* 1906, pp. 64-73, 2005.

[23] S. Obayashi, Y. Yamaguchi, and T. Nakamura, "Multiobjective Genetic Algorithm for Multidisciplinary Design of Transonic Wing Planform," *Journal of Aircraft,* 34(5), pp. 690-693, 1997.

[24] N. Liu, B. Huang, and X. Pan, "Using the Ant Algorithm to Derive Pareto Fronts for Multiobjective Siting of Emergency Service Facilities," *Transportation Research Record* 1935, pp. 120-129, 2005.