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Obtaining robust decisions under uncertainty by sensitivity analysis on OWA operator

Mahdi Zarghaami (1), Reza Ardakanian (2), and Ferenc Szidarovszky (3)

(1) Civil Engineering Dept., Sharif Univ. of Tech., Tehran 11365-9313, Iran and Visiting Scholar, Systems and Industrial Engineering Dept., Univ. of Arizona, Tucson, AZ 85721-0020, USA. E-mail: zarghaami@mehr.sharif.edu
(2) Civil Engineering Dept., Sharif Univ. of Tech., Tehran 11365-9313 Iran.
(3) Systems and Industrial Engineering Dept., Univ. of Arizona, Tucson, AZ 85721-0020, USA.

Abstract-The successful design and application of the Ordered Weighted Averaging (OWA) method as a decision making tool depends on the efficient computation of its order weights. The most popular methods for determining the order weights are the Fuzzy Linguistic Quantifiers approach and the Minimal Variability methods which give different behavior patterns for OWA. These methods will be compared by using Sensitivity Analysis on the outputs of OWA with respect to the optimism degree of the decision maker.

The theoretical results are illustrated in a water resources management problem. The Fuzzy Linguistic Quantifiers approach gives more information about the behavior of the OWA outputs in comparison to the Minimal Variability method. However, in using the Minimal Variability method, the OWA has a linear behavior with respect to the optimism degree and therefore it has better computation efficiency. A simulation study is also reported in this paper, where the dependence of the optimal decision on the uncertainty level is examined. Also based on obtained sensitivity measure, a new combined measure of goodness has been defined to have more reliability in obtaining optimal solutions.

Index Terms: OWA operator; Optimal solution; Sensitivity analysis; Optimism degree; Fuzzy linguistic quantifiers; Minimal variability

1. INTRODUCTION

This paper introduces a new sensitivity measure of OWA with respect to the optimism degree. Sensitivity Analysis (SA) is a very important tool in gaining deeper insight into the mathematical models and their solutions. OWA as an aggregation operator was initiated by Yager [1] and has been applied in many fields such as neural networks, data base systems, fuzzy logic controllers, and group decision making [2]. The focus of this paper is the examination of the behavior of OWA. An n-dimensional OWA operator is a mapping $F : I^n \mapsto I$, where I= [0, 1], that has an associated n-dimensional vector $w_j = (w_1, w_2, ..., w_n)$ of order weights with $w_j \geq 0$ for all j and $\sum_{j=1}^{n} w_j = 1$, if it is defined as follows:

$$F(a_1, a_2, ..., a_n) = \sum_{j=1}^{n} w_j b_j = w_1 b_1 + w_2 b_2 + ... + w_n b_n \qquad (1)$$

where b_j is the j th largest element of the set of the aggregated objects $\{a_1, a_2, ..., a_n\}$ and n is the number of the inputs. Notice that the input vector has been ordered before multiplying it by the order weights. As an important characteristic of the OWA, it has a large variety by the different selections of the order weights [3]. Order weights depend on the optimism degree (well known as Orness degree) of the decision maker (DM). The greater the weights at the beginning of the vector are, the higher is the optimism degree (risk acceptance). Yager [1] has defined the optimism degree, θ, as:

$$\theta = \frac{1}{n-1} \sum_{j=1}^{n} (n-j) w_j \qquad (2)$$

The uncertainty in the input data, especially in the value of θ must be taken into account in multi-criteria decision making models. A brief review of the SA literature can be found for example, in [4]. Torra [5] has developed an SA model for OWA with respect to the weights of the criteria and to the evaluations of the alternatives according to the criteria, but not in regard to the optimism degree and order weights. Ben-Arieh [6] explored the effect that the type of linguistic quantifiers and aggregation method used have on the ranking the alternatives. In section 2, after describing the methodology, the SA model for two important approaches (Fuzzy Linguistic Quantifiers and Minimal Variability) will be introduced. These measures clarify which alternatives are more sensitive not only to the evaluations of the project with respect to the criteria but also to the optimistic degree of the DM. In section 3 a case study will be introduced, and in section 4 the models will be compared based on the case study. In section 5, a simulation study will be presented and effect of the level of uncertainty on the optimal solution will be examined. Section 6 as a summary of the paper and motivation of the study gives a new combined measure of goodness to have more reliability in selecting the optimal projects. Robust decision depends on the combined measure of goodness of alternatives and also on the variations of these measures under uncertainty. In order to combine these two characteristics a new combined measure of goodness will be defined. The theoretical results will be illustrated in a watershed management problem. By using this measure will give more sensitive decisions to the stakeholders whose optimism degrees are different than that of the decision maker.

2. Methodology

The order weights have an important role in OWA. They reflect the optimism or pessimism of the DM in an aggregated form. Xu [7] gives a general overview of the methods for determining order weights but no comparison of the models is presented. In this paper two methods for determining the order weights will be described and used to develop SA model.

2.1. Method 1: Fuzzy Linguistic Quantifiers (FLQ) method

In this method quantifiers are used to characterize aggregation imperatives, in which, the more objects are included, the higher is the satisfaction [8]. Some examples of these quantifiers are: all, at least about half, few of them. In this paper these linguistic inputs are modeled by Regular Increasing Monotonic (RIM) quantifiers. RIM quantifiers satisfy the following conditions:

$$Q(0)=0 \;,\quad Q(1)=1 \text{ and } Q(p_1)\geq Q(p_2) \quad \text{if} \quad p_1 \geq p_2 \,. \tag{3}$$

Function Q is usually called the fuzzy membership function. In particular, any RIM quantifier can be associated to an n-dimensional OWA operator whose weighting vector is obtained as follows [1]:

$$w_j = Q(\frac{j}{n}) - Q(\frac{j-1}{n}) \quad j = 1\,,\,2\,,\,...,n. \tag{4}$$

The particular form of Q has been chosen as $Q(p) = p^{\alpha}$ with a positive parameter α. For this type of Q, Malczewski and Rinner [9] have defined seven linguistic quantifiers to aggregate n inputs. They are shown in Figure 1 and Table 1. The optimism degree [10] has been calculated as:

$$\theta = \int_0^1 Q(p).dp = \int_0^1 p^{\alpha}.dp = \frac{1}{1+\alpha} \,. \tag{5}$$

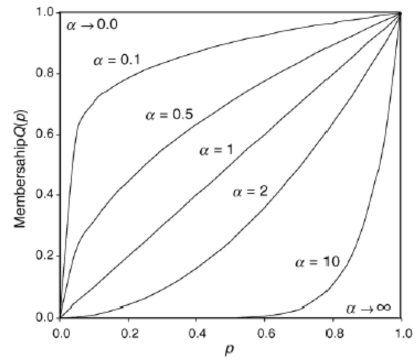

Fig. 1. Membership functions of different RIM quantifiers [9]

Table 1
Equivalent optimism degrees for linguistic quantifiers

Linguistic quantifier	index of quantifier, α	optimism degree, θ
At least one of them	α→0.0	0.999
Few of them	0.1	0.909
Some of them	0.5	0.667
Half of them	1.0	0.500
Many of them	2.0	0.333
Most of them	10.0	0.091
All of them	α→∞	0.001

From equation (5) it is clear that $\alpha = \frac{1}{\theta} - 1$ and by combining equations (4) and (1), we have the following revised form for the combined measure of goodness:

$$F = \sum_{j=1}^{n}\left[(\frac{j}{n})^{\frac{1}{\theta}-1} - (\frac{j-1}{n})^{\frac{1}{\theta}-1} \right] b_j \,. \tag{6}$$

The sensitivity of OWA vs. optimism degree is obtained by differentiation:

$$S = \frac{\partial F}{\partial \theta} = \frac{-1}{\theta^2}\left[\begin{array}{l} (\frac{1}{n})^{\frac{1}{\theta}-1}\ln(\frac{1}{n})b_1 - (\frac{n-1}{n})^{\frac{1}{\theta}-1}\ln(\frac{n-1}{n})b_n \\ + \sum_{j=2}^{n-1}[(\frac{j}{n})^{\frac{1}{\theta}-1}\ln(\frac{j}{n}) - (\frac{j-1}{n})^{\frac{1}{\theta}-1}\ln(\frac{j-1}{n})]b_j \end{array} \right] \tag{7}$$

where S is the sensitivity measure for the change of F by changing the optimism degree θ. It is necessary to mention that the limit of S as $\theta \to 0$ is zero but it is not defined at $\theta = 0$.

2.2. Method 2: Minimal Variability (MV) method

The MV method [11] is based on a nonlinear optimization model to determine order weights as the entropy model [12]. This method gives more attention to all of the orders in the OWA and minimizes the variability of the order weights subject to a constant optimism degree:

$$Minimize.D^2(w) = \sum_{j=1}^{n} \frac{1}{n}(w_j - E(w))^2 = \frac{1}{n}\sum_{j=1}^{n} w^2{}_j - \frac{1}{n^2} \tag{8}$$

subject to

$$\frac{1}{n-1}\sum_{j=1}^{n}(n-j)w_j = \theta$$

$$\sum_{j=1}^{n} w_j = 1$$

$$w_j \geq 0$$

This nonlinear optimization problem can be solved by using the Kuhn-Tucker second-order conditions. The results are as follows:

$$\begin{cases} w_1^* = \dfrac{2(2n-1) - 6(n-1)(1-\theta)}{n(n+1)}, \\ w_n^* = \dfrac{6(n-1)(1-\theta) - 2(n-2)}{n(n+1)}, \\ w_j^* = \dfrac{n-j}{n-1}w_1 + \dfrac{j-1}{n-1}w_n \quad if\ j \in \{2,\dots,n-2\}. \end{cases} \tag{9}$$

By substituting these weights into equation (1)we have:

$$F = \frac{2(2n-1) - 6(n-1)(1-\theta)}{n(n+1)} b_1 + \dots$$
$$+ [\frac{2(2n-1) - 6(n-1)(1-\theta)}{n(n+1)} \cdot \frac{n-j}{n-1} +$$
$$\frac{6(n-1)(1-\theta) - 2(n-2)}{n(n+1)} \cdot \frac{j-1}{n-1}] b_j +$$
$$\dots + \frac{6(n-1)(1-\theta) - 2(n-2)}{n(n+1)} b_n \tag{10}$$

and then by differentiating F with respect to θ we obtain:

$$S = \frac{\partial F}{\partial \theta} = \frac{6}{n(n+1)}\sum_{j=1}^{n}(n-2j+1)b_j \tag{11}$$

In the next section these new sensitivity measures will be applied in a case study concerning water resources projects.

3. Case study

There are thirteen water resources projects under construction in the Sefidrud watershed in the Northwestern region of Iran. These projects are concerned with reservoirs and their water distribution networks.

The DM in the watershed governing board has requested to find the best choice among these projects with respect to seven criteria. The evaluations of these projects with respect to the criteria were done by a group of experts. The evaluation numbers are multiplied by the weights of the criteria and then normalized. The resulted decision matrix of these projects is shown in Table 2.

TABLE 2
Evaluations of water resources projects with respect to seven criteria

	Projects/Criteria	Allocation of water to prior usages	Number of beneficiaries	Supporting other projects	Benefit/ Cost	Range of environmental impacts	Public participation	Job creation
1	Sahand	0.64	0.10	0.59	0.49	0.85	0.78	0.78
2	Shahriar	0.48	0.70	0.59	0.59	0.85	0.65	0.78
3	Ghalechai	0.95	0.30	0.24	0.50	0.42	0.13	0.44
4	Kalghan	0.48	0.20	0.47	0.55	0.42	0.65	0.44
5	Germichai	0.95	0.40	0.71	0.47	0.42	0.52	0.67
6	Givi	0.48	0.10	0.12	0.49	0.85	0.78	0.67
7	Taleghan	0.80	0.70	0.82	0.46	0.28	0.39	0.56
8	Talvar	0.80	0.70	0.12	0.55	0.85	0.13	0.56
9	Galabar	0.80	0.40	0.12	0.65	0.85	0.13	0.78
10	Sanghsiah	0.48	0.20	0.59	0.46	0.85	0.13	0.78
11	Soral	0.48	0.20	0.47	0.46	0.85	0.13	0.78
12	Siazakh	0.48	0.70	0.71	0.46	0.85	0.13	0.78
13	Bijar	0.95	0.70	0.82	0.37	0.42	0.13	0.78

In the application of OWA, the optimism degree of the DM has to be determined. In this study linguistic quantifiers have been used in questioning the DM how many criteria he/she wants to consider. If the DM wants to consider the evaluations of the projects with respect to more criteria then he/she is considered to be more pessimistic. DM has selected the quantifier of 'many of them' from Table 1 then the optimism degree become 0.333. The sensitivity measures of the thirteen projects are calculated using the two approaches, FLQ (θ =0.333) and MV, according to equations (7) and (11) which are shown in Figure 2. The correlation coefficient between the two results is 0.960.

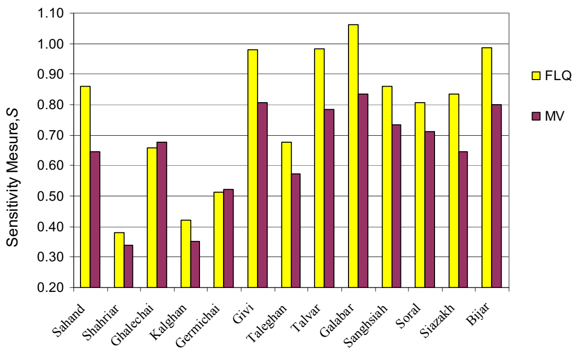

Fig. 2. Sensitivity measures of projects using the two approaches, FLQ (θ =0.333) and MV

The sensitivity measures clarify which projects are very sensitive not only to the evaluations of the project with respect to the criteria but also to the optimistic character of the DM. If a sensitivity measure of a project is high then its combined measure of goodness will change conspicuously for different DMs having different optimism degrees. It is not a good property of a project especially if it has several stakeholders which is the case in water resources projects. Actually if the combined measure of goodness is the same for two projects, then the less sensitive project with respect to the optimism degree is preferred. Section 6 deals with how we can consider this problem in revising the combined measure of goodness. According to Figure 2, the Galabr, Givi, and Talvar projects have the highest sensitivity measures while the Shahriar project has the lowest sensitivity to the optimism degree in using both methods (FLQ and MV).

4. Comparison of the two SA methods: FLQ and MV

It is interesting to see the behavior of the two methods under changing optimism degree. For this reason the sensitivity measures (7) and (11) have been calculated for all thirteen projects when θ varies from 0 to 1. For example, in Figure 3, the sensitivity measures of the Sahand project as function of θ according to the two methods are illustrated. In the appendix the sensitivity measures of the other projects are also shown.

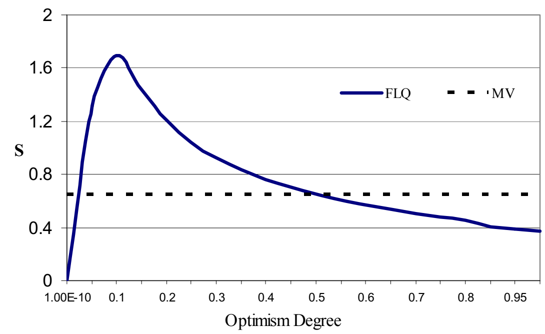

Fig. 3. The sensitivity measures of the Sahand project for the two methods

From these results the following observations can be made:

- As S is constant if the order weights are obtained by the MV method then F (OWA model) has a linear behavior by using MV approach.
- OWA has a nonlinear behavior if the order weights are obtained by the FLQ method.
- If the optimism degree is difficult to obtain, then the MV method can be used for SA since it is independent of the optimism degree.
- The sensitivity measure of OWA with respect to the optimism degree is a context based model if the order weights determined by MV method. Indeed this method is only sensitive to the evaluations of the project with respect to the criteria and not to the optimistic character of the DM.
- OWA is more sensitive with the FLQ method in the pessimistic region (θ<0.5) rather than in the optimistic region (θ>0.5).
- The FLQ method has higher resolution in describing OWA behavior rather than MV. However it cannot be concluded that OWA by the FLQ method is better than OWA by the MV method, since the correlation coefficient between the evaluations of the projects by using the two methods is very high.

Table 3 shows the summary of the comparison of the two methods with respect to some selected criteria, which can be used as a decision matrix for method choice in examining other case studies.

Table 3
Comparison between two methods for sensitivity analysis on OWA (FLQ and MV)

Evaluation criteria	Giving more information and detail	Independence of optimism degree	Computation efficiency	Linear behavior	Learning simplicity	Robustness of answers	Context based
Weights of the criteria	very high	high	fairly high	medium	low	high	medium
Fuzzy Linguistic Quantifiers	high	no	low	no	low	low	partially
Minimal Variability	low	yes	high	yes	medium	very high (solid)	entirely

Based on Table 3 it is clear that the selection between the FLQ and MV methods is dependent on the characteristics of the case study under consideration.

5. Optimal solution

In this study the combined measures of goodness of the projects have been obtained by OWA for finding the optimal solution. By applying the optimism degree described in section 3 (θ =0.333) and using n=7, numbers of criteria, the order weights are calculated as [0.020, 0.061, 0.102, 0.143, 0.184, 0.225, 0.266] by the FLQ method and [0.034, 0.070, 0.106, 0.143, 0.179, 0.216, 0.252] by the MV method. By using these order weights the combined measures of goodness based on equation (1) are obtained and the results are shown in Figure 4. The correlation coefficient between the combined measures is 0.999 and the optimal solution is the Shahriar project for both cases.

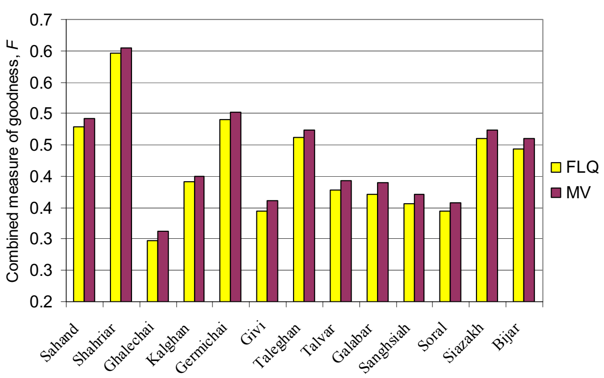

Fig. 4. Combined measures of goodness using OWA by two approaches

The results shown in Figure 5 are based on the deterministic evaluation of the projects (Table 2) and the constant optimism degree. These inputs are however uncertain and the optimal decision may differ in various situations and also might be shifted by the various DMs. Therefore it is important to simulate the combined measure of goodness with uncertain inputs. Three cases considered and examined:

- **Scenario 1:** The evaluation of projects with respect to the criteria, b_j is uncertain but the optimism degree, θ is deterministic.
- **Scenario 2**: Set of $b_j s$ is certain but the θ is uncertain.
- **Scenario 3**: Both the $b_j s$ and θ are uncertain. This scenario is the most realistic.

In each case, 100 random values of the uncertain parameters were generated with expectations being the estimated (known) values and with varying uncertainty levels. We selected α=0.05, 0.1, 0.3 and 0.5 relative standard deviation (that is, the standard deviations were assumed to be the α-multiplies of the expected values). For each case we applied the FLQ method (since MV sensitivity does not depend on the value of θ). The relative frequencies (in %) for each project to be the first, second and third are shown in Table 4 for different values of α. Table 5 presents the singular frequencies for three scenarios with α=0.3.

TABLE 4
Optimal solutions in the scenario 3 by four cases of α

Projects	α=0.05			α =0.1			α =0.3			α =0.5		
	First	Second	Third	First	Second	Third	First	Second	Third	First	Second	Third
Sahand	0	2	100	0	8	99	0	22	76	4	36	39
Shahriar	100	0	0	100	0	0	100	0	0	95	0	0
Ghalechai	0	0	0	0	0	0	0	0	0	0	0	0
Kalghan	0	0	0	0	0	0	0	0	0	0	0	0
Germichai	0	98	0	0	92	1	0	67	0	0	53	18
Givi	0	0	0	0	0	0	0	0	0	0	0	0
Taleghan	0	0	0	0	0	0	0	0	24	1	0	35
Talvar	0	0	0	0	0	0	0	0	0	0	0	0
Galabar	0	0	0	0	0	0	0	0	0	0	0	0
Sanghsiah	0	0	0	0	0	0	0	0	0	0	0	0
Soral	0	0	0	0	0	0	0	0	0	0	0	0
Siazakh	0	0	0	0	0	0	0	0	0	0	0	0
Bijar	0	0	0	0	0	0	0	1	0	1	11	8

TABLE 5
The most frequent projects in best three rankings

Projects	Rank	Frequency (%), α=0.3		
		Scenario 1	Scenario 2	Scenario 3
Shahriar	First	100	100	100
Germichai	Second	100	60	67
Sahand	Third	100	38	76

According to the results, Shahriar is the most preferred project. Germichai and Sahand are the second and third most preferred projects, respectively. As these results are the same for all selected values of *α,* it shows high robustness in the results.

6. Introduction a new combined measure of goodness

In real decisions in water resources management, not only the combined measure of goodness but also the sensitivity measure with respect to the optimism degree is important. Then we define a new combined measure of goodness, F^* which has direct relation with previous combined measure of goodness, F and inverse relation with sensitivity measure, S as:

$$F^* = \beta \frac{F - F_{\min}}{F_{\max} - F_{\min}} + (1-\beta)\frac{S_{\max} - S}{S_{\max} - S_{\min}} \qquad (12)$$

$$0 \le \beta \le 1$$

where $F_{\max}$ and $F_{\min}$ are maximum and minimum of combined measures of goodness of thirteen projects by MV method as in Figure 4. Also $S_{\max}$ and $S_{\min}$ are maximum and minimum of sensitivity measures of thirteen projects by MV method as in Figure 2. New combined measures of goodness have been performed for five cases of β as 0.00, 0.25, 0.50, 0.75, and 1.00 and the results are shown in Table 6.

TABLE 6
New combined measure of goodness by various β

	Projects	F	S_{MV}	F^*				
				β =0.0	β =0.25	β =0.5	β =0.75	β =1.0
1	Sahand	0.493	0.646	0.381	0.440	0.500	0.560	0.620
2	Shahriar	0.604	0.34	1.000	1.000	1.000	1.000	1.000
3	Ghalechai	0.312	0.675	0.322	0.241	0.161	0.080	0.000
4	Kalghan	0.399	0.353	0.974	0.805	0.636	0.467	0.298
5	Germichai	0.502	0.521	0.634	0.638	0.642	0.646	0.651
6	Givi	0.361	0.805	0.059	0.086	0.113	0.141	0.168
7	Taleghan	0.474	0.572	0.530	0.536	0.543	0.549	0.555
8	Talvar	0.394	0.784	0.101	0.146	0.191	0.236	0.281
9	Galabar	0.389	0.834	0.000	0.066	0.132	0.198	0.264
10	Sanghsiah	0.371	0.735	0.200	0.201	0.201	0.202	0.202
11	Soral	0.359	0.711	0.249	0.227	0.205	0.183	0.161
12	Siazakh	0.474	0.645	0.383	0.426	0.469	0.512	0.555
13	Bijar	0.459	0.801	0.067	0.176	0.285	0.394	0.503
	maximum	0.604	0.834					
	minimum	0.312	0.34					

Actually using the new measure of goodness will reduce the risk of rejecting the results by other stakeholders in governing board of the watershed which their optimism degrees are different than the decision maker in this case study. Using β =0.75 the revised measures of thirteen projects have been shown in Figures 5.

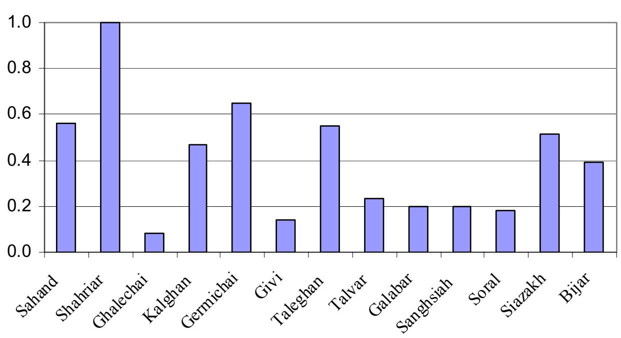

Fig. 5. Revised combined measure of goodness by β=0.75

According to the results of Figure 5 the Shariar, Germichai and Sahand projects are the first, second and third most preferred projects. However these results are as the same as the results in Table 5, but the results at here have more reliability as they comprise both the combined measure of goodness and the sensitivity measure.

7. CONCLUSIONS

In this paper the introduction of new SA models with respect to the optimism degree showed different behavior of OWA when the order weights obtained by the FLQ or the MV method. In using the MV method OWA has a linear relation to the optimism degree of the DM. The FLQ method however has higher resolution in describing OWA behavior rather than MV but MV has better computation efficiency. The comparison of SA models in a real case study showed that the OWA is very sensitive in pessimistic region in comparing to the optimistic region. Also this study introduced a decision matrix for comparing the two methods (FLQ and MV) in sensitivity analysis. The robustness of the results could also be examined by simulation. By merging the combined measure of goodness and the sensitivity measure a new reliable measure for selecting the optimal projects in water resources management has been made.

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