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Fuzzy multiple attribute decision making with eight types of preference information on alternatives

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Abstract - **A new approach is proposed for the fuzzy multiple attribute decision making (MADM) problems with preference information on alternatives. In the approach, multiple decision makers give their preference information on alternatives in different formats. The uniformities and aggregation process with fuzzy majority method are employed to obtain the social fuzzy preference relation on the alternatives. Accordingly, an optimization model is constructed to assess the ranking values of the alternatives.**

Keywords - **Multiple attribute decision making; Preference information; Fuzzy preference relation; Ranking;**

I. INTRODUCTION

In Multiple attribute decision making (MADM) problems, alternatives are always evaluated against some noncommensurate and conflicting attributes. How to rank the alternatives or select the best one has attracted many researches [1, 4]. In MADM problems, the decision makers (DMs)' preference information is often used. However, the DMs' judgments vary in form and depth. Different DMs may use different ways when expressing their preference information on alternatives. The approaches to solving MADM problems can be classified into three categories according to the preference information given by the DMs [1,4]: (1) the approaches without preference information [3], (2) the approaches with information on attributes [7, 11], and (3) the approaches with information on alternatives [1,2,4,8].

This study falls into the third category with DMs' preference information on alternatives, where, eight types of formats of preference information on alternatives are employed: preference orderings, utility values and fuzzy preference relations, linguistic term vector, normal preference relation, selected subset, fuzzy selected subset and pairwise comparison [3][14]. Preference orderings of alternatives can be transformed into fuzzy preference relations [3][10]. Also utility values of alternatives are always converted into fuzzy preference relations for ranking of alternatives [3][9]. So are the other types of preference information. After the preference information from multiple DMs are uniformed, fuzzy majority method with fuzzy quantifier can be used to aggregate these uniformed preference information into a social one and to select the best acceptable alternative [3]. Based on the social preference information, this paper proposes an optimization model to assess the tanking values of the alternatives. It is a new way of reflecting the DMs' subjective preference information based on optimization theory.

This paper is organized as follows: Section 2 describe the MADM problem with preference information on alternatives; Section 3 proposes an approach to the MADM problem, where the eight types of preference information on alternatives are given by multiple decision makers, uniformed and aggregated. An optimization model is proposed for alternative ranking. In section 4, the example in [3] is used to illustrate the proposed approach. Conclusion is given in section 5.

II. PROBLEM DESCRIPTION

The following assumptions or notations are used to represent the MADM problems:

- let $S=\{S_1,S_2,\cdots,S_m\}$ denote a discrete set of $m\,(\geq 2)$ possible alternatives.
- let $R=\{R_1,R_2,\cdots,R_n\}$ denote a set of $n\,(\geq 2)$ attributes.
- let $A=[a_{ij}]_{m\times n}$ denote the decision matrix where $a_{ij}\,(\geq 0)$ is the consequence with a numerical value for alternative S_i with respect to attribute R_j, $i=1,\cdots,m$, $j=1,\cdots,n$.
- let $E = (e_1, e_2, \ldots, e_K)$ denotes the set of DMs. Different DMs can express their preference on the candidate alternatives in different formats, i.e., preference orderings, utility values and fuzzy preference information.

  This paper considers the MADM problems with DMs' preference information on alternatives in following formats.
- preference orderings, or an ordered vector can be used by a DM to express his preference on the alternatives: $O^k=(o^k(1),\cdots,o^k(m))$, where $o^k(\cdot)$ is a permutation function over the index set {1, …, m} and $o^k(i)$ represents the position of alternative S_i in the preference ordering, $i=1,\cdots,m$. The alternatives are ordered from the best to the worst.
- utility values, or an utility vector can be used by a DM to express his preference on the alternatives:

$U^k = (u_1^k, \cdots, u_m^k)$, $u_i^k \in [0,1]$, $1 \le i \le m$, where u_i^k represents the utility evaluation given by the decision maker to alternative S_i.

- fuzzy preference information on alternatives can be given by a DM. The DM's preference relation is described by a binary fuzzy relation P in S, where P is a mapping $S \times S \to [0, 1]$ and p_{ik} denotes the preference degree of alternative S_i over S_k. We assume that P is reciprocal, by definition, (i) $p_{ik} + p_{ki} = 1$ and (ii) $p_{ii} = -$ (symbol '–' means that the decision maker does not need to give any preference information on alternative S_i), $\forall i,k$ [3][5][10].
- Let $L^k = (l_1^k, l_2^k, ..., l_m^k)$ be a linguistic term vector given by a DM e_k. l_i^k is the linguistic evaluation by e_k to S_i, $i = 1,...,m$.
- Let $\overline{S} = \{S_{i_1}, S_{i_2}, ..., S_{i_m}\}$ be a selected subset of S by a DM, to express the preference on part of the alternatives. $\overline{S} \subset S$, $i_m < m$. Alternatives in $\overline{S}$ are equivalent and dominate those in the left of S. The alternatives in $S/\overline{S}$ are also equivalent to each other.
- Let $\widetilde{S} = \{(S_{i_1}, l_{i_1}^k), (S_{i_2}, l_{i_2}^k), ..., (S_{i_n}, l_{i_n}^k)\}$, $i_n < m$, be a fuzzy selected subset of S used by a DM e_k, to express the preference on part of the alternatives using linguistic terms. $l_{i_j}^k$ is a linguistic term, $i_j = 1,...,i_n$.
- normal preference relation on alternatives can be given by a DM. For example, the decision-maker prefers alternative S_i to S_j, and prefers alternative S_c to alternatives S_l and S_h.
- pairwise comparison on alternatives: Let $H = (h_{ij})_{m \times m}$ be a pairwise comparison matrix used by a DM e_k. Where h_{ij} represents the ratio of the preference of alternative S_i to S_j and can be given in Saaty's 1-9 scale [12]. Matrix H represents following characteristics:

$$h_{ij} = \frac{1}{h_{ji}}, \qquad i, j = 1,...,m; i \ne j, \tag{1}$$

$$h_{ii} = 1, \qquad i = 1,...,m, \tag{2}$$

$$h_{ij} > 0, \qquad i, j = 1,...,m. \tag{3}$$

III. THE PROPOSED APPROACH

When multiple DMs are involved in the decision process, usually two phases are needed to find the final solution: aggregation and exploitation. Aggregation is to combine opinions on alternatives from different perspectives; Exploitation is to rank the alternatives or to select the best one based on the social information on the alternatives. In this section, the following types of preference information on alternatives are first converted into fuzzy preference relations, then preference aggregation and exploitation processes will follow.

A. Preference uniformities

DM can use preference orderings or an ordered vector to express his preference on the alternatives. In this paper, the preference orderings would be transformed into fuzzy preference relations as follows [3]:

$$p_{ij}^k = \tfrac{1}{2}\left(1 + \frac{o^k(j)}{m-1} - \frac{o^k(i)}{m-1}\right), \qquad 1 \le i \ne j \le m, \tag{4}$$

Also a DM can use an utility vector to express the preference on the alternatives. The utility vector can be transformed into fuzzy preference relations as follows [3]:

$$p_{ij}^k = \frac{(u_i^k)^2}{(u_i^k)^2 + (u_j^k)^2}, \qquad 1 \le i \ne j \le m, \tag{5}$$

Given a linguistic term vector from e_k, suppose two alternatives S_i and S_j are awarded linguistic terms $\widetilde{A}_i = (u_i, \alpha_i, \beta_i)$ and $\widetilde{A}_j = (u_j, \alpha_j, \beta_j)$ respectively. Following transformations can be used to obtain the fuzzy preference relation between S_i and S_j [6],

$$f(\widetilde{A}_i, \widetilde{A}_j) = \frac{\widetilde{A}_i \times \widetilde{A}_i}{\widetilde{A}_i \times \widetilde{A}_i + \widetilde{A}_j \times \widetilde{A}_j} = \left(\frac{u_i^2}{u_i^2 + u_j^2}, \frac{\alpha_i^2}{\alpha_i^2 + \alpha_j^2}, \frac{\beta_i^2}{\beta_i^2 + \beta_j^2}\right), \tag{6}$$

$$p_{ij}^k = g(f(\widetilde{A}_i, \widetilde{A}_j)) = \frac{u_i^2}{u_i^2 + u_j^2}. \tag{7}$$

With a selected subset of S, e.g., $\overline{S} = \{S_{i_1}, S_{i_2}, ..., S_{i_m}\}$, the fuzzy preference relation on any two alternatives in S can be defined as,

$$p_{ij}^k = \begin{cases} 1, & \text{if } S_i \in \overline{S},\ S_j \in S/\overline{S}, \\ 0.5, & \text{otherwise,} \end{cases} \qquad i, j = 1,...,m; i \ne j \tag{8}$$

When a DM e_k gives fuzzy selected subset $\widetilde{S}$ on S, for any two alternatives S_i and S_j, if they both belong to $\widetilde{S}$, where $l_i^k = (u_i, \alpha_i, \beta_i)$ and $l_j^k = (u_j, \alpha_j, \beta_j)$, the fuzzy preference relation on them is,

$$p_{ij}^k = g(f(l_i^k, l_j^k)) = \frac{u_i^2}{u_i^2 + u_j^2}, \qquad i, j = 1,...,m; i \ne j. \tag{9}$$

If none of S_i and S_j belong to $\widetilde{S}$, then

$$p_{ij}^k = 0.5, \qquad i, j = 1,...,m; i \ne j. \tag{10}$$

If S_i belongs to $\widetilde{S}$ and S_j does not, then

$$p_{ij}^k = u_i, \qquad i, j = 1,...,m; i \ne j. \tag{11}$$

Suppose a DM e_k expresses a pairwise comparison matrix on *S*, *H*= $(h_{ij})_{m\times m}$. Then,

$$r_i = \left[\prod_{j=1}^{m} h_{ij}\right]^{1/m} \Big/ \sum_{r=1}^{m}\left[\prod_{j=1}^{m} h_{rj}\right]^{1/m}, \quad i = 1,...,m. \quad (12)$$

Thus, the fuzzy preference relation on *S* can be obtained,

$$p_{ij}^k = \frac{r_i}{r_i + r_j} = \left[\prod_{r=1}^{m} h_{ir}\right]^{1/m} \Big/ \left(\left[\prod_{r=1}^{m} h_{ir}\right]^{1/m} + \left[\prod_{r=1}^{m} h_{jr}\right]^{1/m}\right), \quad i, j = 1,...,m; i \neq j. \quad (13)$$

With the normal preference relation, if part of the alternatives are involved, the alternatives that are not considered, are equivalent with each other, and are also equivalent with those considered. For the considered alternatives, dominance relationships among them exist.

B. Preference aggregation

After the DMs' preference information has been uniformed into fuzzy preference relations respectively, the next step is to aggregate these uniformed preference information into a social fuzzy preference relation. The social fuzzy preference relation can be obtained by using the ordered weighted averaging (OWA) operator to aggregate individual fuzzy preference relations [13]. An OWA operator of dimension *K* is a function *F* as follows,

$$F : [0,1]^K \rightarrow [0,1]. \quad (14)$$

In this paper, to aggregate $p_{ij}^1, p_{ij}^2, \cdots, p_{ij}^K$, *F* is associated with a weight vector $V = [v_1, v_2, \cdots, v_K]$, where $v_h \in [0,1]$, *h*=1,…, *K*, and $\sum_{h=1}^{K} v_h = 1$. *F* can be expressed as

$$F(p_{ij}^1, p_{ij}^2, \cdots, p_{ij}^K) = V \cdot C^T = \sum_{h=1}^{K} v_h c_h, \quad i, j = 1,...,m; i \neq j, \quad (15)$$

where $C = [c_1, c_2,..., c_K]$ and c_h is the *h*th largest value among the collection of $p_{ij}^1, p_{ij}^2,..., p_{ij}^K$, $h = 1,..., K$. $P^l = (p_{ij}^l)_{m\times m}$ is the matrix of the uniformed fuzzy preference relation on the alternatives from the decision-maker e_l , *l* =1,…, *K*. The weight vector *V* can be obtained by a proportional quantifier *Q* [13], i.e.,

$$v_h = Q(h/K) - Q((h-1)/K), \quad h = 1,..., K, \quad (16)$$

where *Q* is a fuzzy linguistic quantifier, e.g., "at least half" and "as many as possible".

If $p_{ij}^1, p_{ij}^2,..., p_{ij}^K$ are assigned importance $z_1, z_2,..., z_K$, respectively, and t_h is the importance associated with c_h correspondingly, $h = 1,..., K$, then (16) is changed into follows:

$$v_h = Q\left(\frac{\sum_{l=1}^{h} t_l}{\sum_{l=1}^{K} t_l}\right) - Q\left(\frac{\sum_{l=1}^{h-1} t_l}{\sum_{l=1}^{K} t_l}\right), \quad h = 1,..., K. \quad (17)$$

In this paper, semantics "most", involved in the fuzzy linguistic quantifier with a pair (0.3,0.8), is used by the OWA operator to aggregate DMs' individual preference relations, i.e.

$$G = (g_{ij})_{m\times m}, \quad (18)$$

$$g_{ij} = F_Q(p_{ij}^1, p_{ij}^2,..., p_{ij}^K), \quad i, j = 1,...,m; i \neq j, \quad (19)$$

where $F_Q(\cdot)$ is defined in (15) and *Q* is the fuzzy linguistic quantifier with "most" which is used to obtain the weight vector V in (15) and (16).

C. Exploitation

Let the ranking value of alternative S_j be d_j , , where $\sum_{j=1}^{m} d_j = 1$ and $d_i \geq 0$ for $j = 1, \cdots, m$. Consider the element g_{ij} of matrix *G*. It is desirable to determine d_j ' value such that

$$g_{ij} \approx \frac{d_i}{d_i + d_j}, \quad i, j = 1, \cdots, m, i \neq j, \quad (20)$$

From (20), it is clear that the greater p_{ij} , the greater the corresponding d_i and the better alternative S_i . To obtain the ranking values of the alternatives, according to (20), a constrained optimization model is set up as follows:

$$\min z = \sum_{i=1}^{m} \sum_{\substack{j=1 \\ j\neq i}}^{m} [d_i - (d_i + d_j) g_{ij}]^2, \quad (21a)$$

$$\text{s.t.} \quad \sum_{j=1}^{m} d_j = 1, \quad (21b)$$

$$d_i \geq 0. \quad (21c)$$

For the convenience, the above model (21) can be alternatively expressed in the form of matrix as follows:

$$\min z = d^T H d, \quad (22a)$$

$$\text{s.t.} \quad e^T d = 1, \quad (22b)$$

$$d \geq 0. \quad (22c)$$

where $d = (d_1, d_2, \cdots, d_m)^T$, $e = (1,1, \cdots,1)^T$, and $H = [h_{ij}]_{m\times m}$. The elements in matrix *H* are

$$h_{ii} = 2\sum_{\substack{k=1 \\ k\neq i}}^{m} g_{ki}^2 , \qquad i \in \{1,\cdots,m\} , \tag{23a}$$

$$h_{ij} = 2(g_{ij}^2 - g_{ij}) , \qquad j \in \{1,\cdots,m\} ; i \neq j . \tag{23b}$$

To solve model (22), the following Lagrangian function can be set up as:

$$L = d^T Hd + 2\lambda(e^T d - 1) , \tag{24}$$

where λ is the Lagrangian multiplier. Let $\partial L/\partial d = 0$ and $\partial L/\partial \lambda = 0$, then

$$H\, d + \lambda\, e = 1 , \tag{25a}$$

$$e^T d = 1 . \tag{25b}$$

If matrix H is invertible, then solutions to (25a) and (25b) are given as follows:

$$d^* = H^{-1}e \big/ e^T H^{-1}\, e , \tag{26a}$$

$$\lambda^* = -1 \big/ e^T H^{-1} e , \tag{26b}$$

where d^* is the derived ranking vector on the alternatives. The greater d_i^* is, the better the corresponding alternative S_i will be.

However, it should be noticed that the ranking vector d^* determined by (26a) has practical meanings only if $d^* \geq 0$. The problem left is to prove that d^* satisfies the nonnegative constraint (22c). There are two situations to be considered: $g_{ij}(d_i^* + d_j^*) = d_i^*$ holds for $\forall i, j$ and there is at least one $d_i^* \neq g_{ij}(d_i^* + d_j^*)$ holds for $\forall i, j$ [Xiao, 2001].

Theorem 1. Let $G = [g_{ij}]_{m\times m}$ be a possible degree matrix defined in (19). If $g_{ij}(d_i^* + d_j^*) = d_i^*$ holds for $\forall i, j$ and $i \neq j$, then $d^* \geq 0$.
The details on the proof of Theorem 1are omitted.

Theorem 2. Let $G = [g_{ij}]_{m\times m}$ be a possible degree matrix as defined in (19). If there is at least one $d_i^* \neq g_{ij}(d_i^* + d_j^*)$ for $\forall i, j$ and $i \neq j$, then matrix H in model (22) is positive definite and also invertible.
Proof. It is noticed that

$$z = d^T Hd = \sum_{i=1}^{m} \sum_{\substack{j=1 \\ j\neq i}}^{m} [d_i - (d_i + d_j) p_{ij}]^2 . \tag{27}$$

If there exists at least one $d_i^* \neq p_{ij}(d_i^* + d_j^*)$ for $\forall i, j$ and $i \neq j$, then $z > 0$. Furthermore, since H is symmetry, then it is clear that H is positive definite and is invertible.
Lemma 1. Let $H = [h_{ij}]_{m\times m}$ be a nth order square matrix satisfying $h_{ij} \leq 0$ for $i \neq j$. Then $H^{-1} \geq [0]$ if and only if all principal minor determinants of H are greater than zero.
Theorem 3. Let $G = [g_{ij}]_{m\times m}$ be a possible degree matrix as defined in (19). If there is at least one $d_i^* \neq p_{ij}(d_i^* + d_j^*)$ for $\forall i, j$ and $i{\neq}j$, then $H^{-1} \geq [0]$ holds.
Proof. Theorem 2 indicates that H is positive definite. Thus, all principal minor determinants of H are greater than zero. Furthermore, by (23b), it is clear that $h_{ij} \leq 0$ ($i \neq j$) holds if $0 \leq g_{ij} \leq 1$ for $i, j = 1,\cdots,m$ and $i \neq j$, i.e., all non-diagonal elements of H are less than zero. Therefore Theorem 3 holds.
Theorem 4. Let d^* be the ranking vector obtained by (26a). If there is at least one $d_i^* \neq g_{ij}(d_i^* + d_j^*)$ for $\forall i, j$ and $i \neq j$, then we have $d^* > 0$.
Proof. Let v_i denote the i th row vector of Q^{-1}, $i = 1,\cdots,m$. Since $Q^{-1} \geq [0]$, then $v_i \geq 0$, moreover $v_i e > 0$, i.e., there is at least one non-zero element in v_i. Let $w = e^T Q^{-1} e = \sum_{i=1}^{m} v_i e$. Since $v_i e > 0$, then $w > 0$. Therefore, $d^* = Q^{-1}e \big/ e^T Q^{-1}\, e > 0$ holds.

IV. AN ILLUSTRATIVE EXAMPLE

A robot user wants to select a robot and asks eight experts to help him make a decision. Four alternatives (i.e. S_1, S_2, S_3 and S_4) are provided for the user to choose. The attributes considered include: 1) R_1: costs (\$10,000), 2) R_2: velocity (m/s), 3) R_3: repeatability (mm), 4) R_4: load capacity (kg). Among the four attributes, R_2 and R_4 are of benefit type, and R_1 and R_3 are of cost type. The decision matrix with the four attributes (R_1, R_2, R_3 and R_4) and the four alternatives (S_1, S_2, S_3 and S_4) is presented as follows:

$$A = \begin{pmatrix} 3.0 & 1.0 & 1.0 & 70 \\ 2.5 & 0.8 & 0.8 & 50 \\ 2.2 & 0.7 & 2.0 & 90 \\ 1.8 & 0.5 & 1.2 & 110 \end{pmatrix} .$$

Suppose the experts $e_1, e_2, ..., e_8$ provide their opinions on the four alternatives to help the user. They express their opinions as followings: e_1 gives an ordered vector, O^1={3, 1, 2, 4}. e_2 gives an utility vector, U^2={0.7, 0.9, 0.6, 0.3}. e_3 expresses a vector of linguistic terms, L^3 =("fair", "good", "good", "very good"). e_4 presents a fuzzy preference relation matrix P^4 . e_5 uses the normal preference information, i.e., S_1 is preferred to S_3, and S_2 is preferred to S_4. e_6 provides a selected subset { S_3, S_4 }. e_7 gives a fuzzy selected subset {(S_2, "good"), (S_4 ,"very good")}. e_8 gives a pairwise comparison matrix on the four alternatives as follows:

$$H = \begin{pmatrix} 1 & 1/7 & 1/3 & 1/5 \\ 7 & 1 & 3 & 2 \\ 3 & 1/3 & 1 & 1/2 \\ 5 & 1/2 & 2 & 1 \end{pmatrix}.$$

Using the normalization functions above, the uniformed fuzzy preference relation matrices from these DMs are obtained respectively:

$$P^1 = \begin{pmatrix} - & 1/6 & 1/3 & 2/3 \\ 5/6 & - & 2/3 & 1 \\ 2/3 & 1/3 & - & 5/6 \\ 1/3 & 0 & 1/6 & - \end{pmatrix},$$

$$P^2 = \begin{pmatrix} - & 49/130 & 49/85 & 49/58 \\ 81/130 & - & 81/117 & 0.9 \\ 36/85 & 36/117 & - & 0.8 \\ 9/58 & 0.1 & 0.2 & - \end{pmatrix},$$

$$P^3 = \begin{pmatrix} - & 0.3077 & 0.3077 & 0.2 \\ 0.6923 & - & 0.5 & 0.36 \\ 0.6923 & 0.5 & - & 0.36 \\ 0.8 & 0.64 & 0.64 & - \end{pmatrix},$$

$$P^4 = \begin{pmatrix} - & 0.4 & 0.3 & 0.4 \\ 0.6 & - & 0.5 & 0.7 \\ 0.7 & 0.5 & - & 0.8 \\ 0.6 & 0.3 & 0.2 & - \end{pmatrix}, \quad P^5 = \begin{pmatrix} - & 0.5 & 1 & 0.5 \\ 0.5 & - & 0.5 & 1 \\ 0 & 0.5 & - & 0.5 \\ 0.5 & 0 & 0.5 & - \end{pmatrix},$$

$$P^6 = \begin{pmatrix} - & 0.5 & 0 & 0 \\ 0.5 & - & 0 & 0 \\ 1 & 1 & - & 0.5 \\ 1 & 1 & 0.5 & - \end{pmatrix}, \quad P^7 = \begin{pmatrix} - & 0.25 & 0.5 & 0 \\ 0.75 & - & 0.75 & 0.36 \\ 0.5 & 0.25 & - & 0 \\ 1 & 0.64 & 1 & - \end{pmatrix},$$

$$P^8 = \begin{pmatrix} - & 0.1093 & 0.2709 & 0.1782 \\ 0.8907 & - & 0.7517 & 0.6300 \\ 0.7291 & 0.2483 & - & 0.3599 \\ 0.8272 & 0.3700 & 0.6401 & - \end{pmatrix}.$$

The OWA operator with fuzzy linguistic quantifier "most" is used to aggregate the eight experts' opinions, with the corresponding weight vector being (0, 0, 0.15, 0.25, 0.25, 0.25, 0.1, 0)^T. The social fuzzy preference relation matrix is obtained as,

$$G = \begin{pmatrix} - & 0.3261 & 0.3553 & 0.2989 \\ 0.6739 & - & 0.5968 & 0.6729 \\ 0.6447 & 0.4032 & - & 0.5439 \\ 0.7011 & 0.3271 & 0.4561 & - \end{pmatrix}.$$

The ranking values of the four alternatives can be obtained by using (26), i.e., d_1 =0.1407, d_2 =0. 3824, d_3=0. 2611, d_4 =0.2159. Thus the ranking result of the alternatives is $S_2 \succ S_3 \succ S_4 \succ S_1$.

V. SUMMARY

This paper proposes a new approach to solve the MADM problem with eight types of preference information on alternatives. In the approach, the different preference information given by multiple DMs are transformed into fuzzy preference relation respectively and aggregated into a social fuzzy preference relation with fuzzy majority method. Based on the social fuzzy preference relation, an optimization model is employed to assess the ranking values of the alternatives. The proposed approach is an extension for the current alternative ranking methods.

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Appendix A. Preliminary knowledge on fuzzy set theory[15]

Definition 1. Let X be a classical set of objects, called the universe. The generic elements in X is denoted as x, i.e., $X=\{x\}$. A fuzzy set $\widetilde{A}$ in X is characterized by a membership function $\mu_{\widetilde{A}}(x)$ that associate each element in X with a real number in [0, 1].

A fuzzy set $\widetilde{A}$ is usually denoted by a set of pairs, $\widetilde{A}=\{(x,\mu_{\widetilde{A}}(x)), x\in X, \mu_{\widetilde{A}}(x)\in[0,1]\}$. When X is a definite set, i.e., $\{x_1, \ldots, x_n\}$, the fuzzy set $\widetilde{A}$ can be represented as

$$\widetilde{A}=\sum_{i=1}^{n} x_i \big/ \mu_{\widetilde{A}}(x_i) \quad \text{(A.1)}$$

When X is an infinite set, the fuzzy set $\widetilde{A}$ can be represented as,

$$\widetilde{A}=\int_x x \big/ \mu_{\widetilde{A}}(x)\,. \quad \text{(A.2)}$$

Definition 2. A triangular linguistic term $\widetilde{B}$ on set R^+, denoted by (u, α , β), is defined to be a fuzzy triangular number if its membership function $\mu_{\widetilde{B}}: R^+ \to [0, 1]$ is equal to

$$\mu_{\widetilde{B}}(x)=\begin{cases}\frac{1}{u-\alpha}x-\frac{\alpha}{u-\alpha}, & x\in[\alpha,u]\\ \frac{1}{u-\beta}x-\frac{\beta}{u-\beta}, & x\in[u,\beta]\\ 0, & otherwise\end{cases} \quad \text{(A.3)}$$

where $\alpha \le u \le \beta$, and u is the modal value. α and β stand for the lower and upper values of the support of the triangular linguistic term $\widetilde{B}$ respectively.

Definition 3. Let two triangular linguistic terms $\widetilde{A}$ = (u, α , β), and $\widetilde{B}$ =(v, γ , δ), then the operations on $\widetilde{A}$ and $\widetilde{B}$, i.e., addition, multiplication and division, are defined respectively as,

$$\widetilde{A}+\widetilde{B}=(u+v,\ \alpha+\gamma,\ \beta+\delta) \quad \text{(A.4)}$$

$$\widetilde{A}\times\widetilde{B}=(uv,\ \alpha\ \gamma,\ \beta\ \delta) \quad \text{(A.5)}$$

$$\widetilde{A}/\widetilde{B}=(u/v,\ \alpha/\gamma,\ \beta/\delta) \quad \text{(A.6)}$$

Definition 4. Let $\widetilde{A}$ be a fuzzy set on X and Y be [0, 1]. A Max-membership defizzification mapping function g from X to Y [6], is defined as

$$g(\widetilde{A})=z^*\in Y \quad \text{(A.7)}$$

where

$$z^*=\{x^* \mid x^*\in X, \mu_{\widetilde{A}}(x^*)\ge\mu_{\widetilde{A}}(x), \forall x\in X\} \quad \text{(A.8)}$$