Visualization Tools for Multicriteria Analysis of the Prototype Improvement Problem

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Abstract — One of the basic engineering optimization problems is the problem of improving a prototype. This problem is constantly encountered by industrial and academic organizations that produce and design various objects (e.g., motor vehicles, machine tools, ships, and aircraft). This paper presents an approach for improving a prototype by construction of the feasible and Pareto sets while performing multicriteria analysis. We introduce visualization methods that facilitate constructing the feasible and Pareto sets. Using these techniques, an expert can correctly state and solve the problem under consideration in a series of dialogs with the computer. Finally, we present a case study of applying these methods to a problem of improving a prototype of the ship.

I. INTRODUCTION

The problem of improving a prototype can be defined as follows: We want to improve a prototype of some object with respect to multiple criteria. The references [1-3,6-10,14] discuss similar multicriteria problems of improving a prototype of vehicles, machine tools and their units, parafoil-load delivery system, controllable descending system, etc. For example, after road, bench, full-scale and other tests, improving some important characteristics of a car is necessary. Furthermore, it is very important to make the process of operational development of a prototype as short as possible in terms of time.

To solve the above problem the prototype's operational development must take two stages. In the *first stage*, on the basis of different tests it is necessary to identify the mathematical model of the object and to determine its parameters. To this end, we can solve an identification problem, where we work with particular adequacy (proximity) criteria[1]. The questions of multicriteria identification and adequacy of mathematical models are discussed in detail in [1,2]. In the *second stage*, an expert formulates and solves the multicriteria optimization problem using performance criteria and the mathematical model whose adequacy was established in the first stage. Based on the results of optimization, improvements to the prototype are performed, and then the tests of the object are repeated. This cycle is iterated until an expert decides to terminate the operational development.

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[1] By adequacy (proximity) criteria we usually mean the discrepancies between the experimental and computed data, the latter being determined on the basis of a mathematical model [1,2].

In this paper we describe approaches for improving the basic performance criteria of the prototype by changing the design variable, functional, and criteria constraints while constructing the feasible and Pareto optimal solution sets. These approaches allow one to determine *the potential of the prototype for its improvement.*

In brief, the problem of improving a prototype considered in this paper possesses the following distinctive features:

- An expert often does not have sufficient information about the limits of variation of many design variables.

- The problem is essentially multicriteria. These criteria are usually contradictory.

- For these reasons, experts experience significant difficulties in correctly stating the problems of improving a prototype (definition of the feasible solution set) and, as a result, they end up solving *ill-posed problems*.

- The formulation and solution of the problem comprise a single process. The customary approach is that the expert first states the problem and then a computer is employed to solve it. This approach is untenable, since one can rarely formulate a problem completely and correctly before solving it. Thus, problems of this sort should be formulated and solved interactively.

- Construction of the feasible solution set is the most important step in formulating and solving the prototype improvement problems.

The above features suggest that one cannot solve the prototype improvement problems in a completely automatic fashion; we advocate solution of these problems via dialogs of an expert with computer.

We propose to solve the problem of improving the prototype by determining the feasible solution set via the *Parameter Space Investigation* (PSI) method that is widely used in various fields of industry, science, and technology [1, 2, 4, 7, 14]. Furthermore, we introduce tools for visualization that will guide experts in analyzing this problem. In some sense, the visualization procedures that we propose are diagnostic tools for an expert.

Finally, to illustrate the usefulness of techniques presented in this paper, we provide a case study that solves the problem of improving the preliminary ship design prototype by using the PSI method and various tools for visualization.

II. FORMULATION OF THE OPTIMIZATION PROBLEM

We assume that a prototype depends on r design variables $\alpha_1,...,\alpha_r$ representing a point $\alpha = (\alpha_1,...,\alpha_r)$ in the r-dimensional space. In the general case, one has to take into account *design variable, functional, and criteria constraints*.

The *design variable constraints* have the form $\alpha_j^* \leq \alpha_j \leq \alpha_j^{**}$, $j = 1,...r$. Constraints α_j^* and α_j^{**} define a parallelepiped Π in the r-dimensional design variable space. From the expert's perspective, the values of design variable constraints can be changed, if that leads to the improving the basic criteria.

The *functional constraints* can be written as follows: $C_l^* \leq f_l(\alpha) \leq C_l^{**}$, $l = 1,..., t$, where $f_l(\alpha)$ is a functional relation, C_l^* and C_l^{**} are some constants. Functional relations together with constraints are some requirements of the object that sometimes an expert can successively revise in order to improve the basic performance criteria. These can be norms, standards, and other requirements such as mass, overall dimensions, allowable stress in structural elements, and so on.

The operation of a prototype is described by the particular performance criteria $\Phi_\nu(\alpha)$, $\nu = 1,...,k$. All other things being equal, it is desired that these criteria are optimized. For simplicity, we assume that functions $\Phi_\nu(\alpha)$ are to be minimized. To avoid situations in which the expert regards the values of some criteria as unacceptable, we introduce *criteria constraints* in the form $\Phi_\nu(\alpha) \leq \Phi_\nu^{**}$, $\nu = 1,...,k$, where Φ_ν^{**} is the worst value of criterion $\Phi_\nu(\alpha)$ acceptable to an expert. These constraints are repeatedly revised during solution of the problem. The choice of Φ_ν^{**} is discussed in the following section.

The design variable, functional, and criteria constraints define the *feasible solution set $D \subset \Pi$*.

Let us formulate one of the basic problems of multicriteria optimization. It is necessary to define the feasible solution set D and find a set $P \subset D$ such that

$$\Phi(P) = \min_{\alpha \in D} \Phi(\alpha), \qquad (1)$$

where $\Phi(\alpha) = (\Phi_1(\alpha),...,\Phi_k(\alpha))$ is the criterion vector and P is the *Pareto optimal set*. We mean that $\Phi(\alpha) < \Phi(\beta)$ if for all $\nu = 1,..., k$, $\Phi_\nu(\alpha) \leq \Phi(\beta)$ and for at least one $\nu_0 \in \{1,...,k\}$, $\Phi_{\nu_0}(\alpha) < \Phi_{\nu_0}(\beta)$.

When solving this problem, one has to determine the vector of design variables $\alpha^O \in P$, which is the most preferred among the vectors belonging to set P.

III. CONSTRUCTION OF FEASIBLE AND PARETO OPTIMAL SETS WITH THE PARAMETER SPACE INVESTIGATION (PSI) METHOD

The PSI method is based on the search of the parallelepiped Π with points of uniformly distributed sequences (e.g., LPτ sequences), see [1, 2, 7, 14] for details. The method consists of three stages:

Stage 1. Compilation of Test Tables via Computer. First, one chooses N trial points $\alpha^1,...,\alpha^N$ that satisfy the functional constraints. Then all the particular criteria $\Phi_\nu(\alpha^i)$ are calculated at each of the points α^i; for each of the criteria a test table is compiled so that the values of $\Phi_\nu(\alpha^1),...\Phi_\nu(\alpha^N)$ are arranged in increasing order; i.e.,

$$\Phi_\nu(\alpha^{i_1}) \leq \Phi_\nu(\alpha^{i_2}) \leq ... \leq \Phi_\nu(\alpha^{i_N}), \quad \nu = 1,..., k, \quad (2)$$

where $i_1, i_2,...,i_N$ are the numbers of trials (a separate set for each ν). Taken together, the k tables form a complete test table.

Stage 2. Preliminary Selection of Criterion Constraints. This stage includes interaction with an expert. By analyzing inequalities (2), an expert specifies the criteria constraints Φ_ν^{**}. Actually, an expert has to consider one criterion at a time and specify the respective constraints. He analyzes one test table and imposes the criterion constraint. Then he proceeds to the next table, and so on. Note that the revision of the criteria constraints within the limits of the test tables does not lead to any difficulties for an expert.

Since we want to minimize all criteria, Φ_ν^{**} should be the maximum values of the criteria $\Phi_\nu(\alpha)$ which guarantee an acceptable level of the object's operation. If the selected values of Φ_ν^{**} are not a maximum, then many interesting solutions may be lost, since some of the criteria are contradictory. Moreover in some cases the feasible solution set may be empty.

In practice the expert imposes the criteria constraints in order to improve a prototype by all criteria simultaneously. If it is impossible, he improves a prototype by the most important criteria. In process of dialogs with computer the expert repeatedly revises criteria constraints and carries out the multicriteria analysis. The PSI method gives the expert valuable information on the advisability of revising various criteria constraints with the aim of improving the basic criteria. The expert sees what price he pays for making concessions in various criteria, i.e., what he loses and what he gains.

Stage 3. Solvability of Problem (1) via Computer. Let us fix a criterion, say $\Phi_{\nu_1}(\alpha)$, and consider the corresponding test table (2). Let S_1 be the number of the values in the table satisfying the selected criterion constraint:

$$\Phi_{\nu_1}(\alpha^{i_1}) \leq ... \leq \Phi_{\nu_1}(\alpha^{i_{S_1}}) \leq \Phi_{\nu_1}^{**} \quad (3)$$

Then criterion Φ_{ν_2} is selected by analogy with Φ_{ν_1} and the values of $\Phi_{\nu_2}(\alpha^{i_1}),...,\Phi_{\nu_2}(\alpha^{i_{S_1}})$ in the test table are considered. Let the table contain $S_2 \leq S_1$ values such that

$\Phi_{v_2}\left(\alpha^{i_j}\right) \leq \Phi_{v_2}^{**}$, where $1 \leq j \leq S_2$. Similar procedures are carried out for each criterion. Then if at least one point can be found for which all criteria constraints are valid simultaneously, then the set D is nonempty and problem (1) is solvable. Otherwise, the expert should return to Stage 2 and make certain concessions in the specification of Φ_v^{**}. However, if the concessions are highly undesirable, then one may return to Stage 1 and increase the number of points in order to repeat Stage 2 and Stage 3 using extended test tables [1].

The procedure is to be iterated until D is nonempty. Then the Pareto optimal set is constructed in accordance with the definition presented above. This is done by removing those feasible points that can be improved with respect to all criteria simultaneously.

Thus by using the PSI method, the criteria constraints are determined in the dialog of an expert with a computer. Then we should determine the Pareto set P, and after the analysis of P, find the most preferred solution $\Phi(\alpha^o)$, surpassing the prototype in all criteria, or at least the most important ones. As a rule, in engineering optimization problems, experts do not encounter serious difficulties in analyzing the Pareto optimal set and in choosing the most preferred solution. They have a sufficiently well-defined system of preferences. Moreover, the aforementioned set usually contains only a small number of solutions because of the strong constraints.

To investigate design variable space, we use *uniformly distributed sequences*. At present, the so-called LPτ sequences are among the best ones in terms of uniformity characteristics. We note that other generators, including the random number generators, are used as well in the PSI method [5].

The PSI method is implemented in the MOVI (*Multicriteria Optimization and Vector Identification*) software system [8].

Pseudo-criteria. In many situations an expert cannot determine functional constraints correctly. For example, in practical problems "good" solutions may lie beyond the limits imposed by the constraints. If an expert is informed of this, in some cases he will be ready to modify the constraints so that the "good" solutions will belong to the feasible solution set. The question is how to obtain such information. Instead of the function $f_l(\alpha)$, whose constraints C_l^* and C_l^{**}, $l = 1,...,t$ are soft, we introduce an additional criterion $\Phi_{k+l}(\alpha) = f_l(\alpha)$, which we will call *a pseudo-criterion*. However, to find the value of Φ_{k+l}^{**} one has to compile a test table containing $\Phi_{k+l}(\alpha)$. By using the aforementioned algorithm with the new test table, one can define Φ_{k+l}^{**} in a way that prevents the loss of interesting solutions.

In general, when solving a problem with soft functional constraints, one has to find the set D, taking all performance criteria into account, the functions $f_l(\alpha)$ being considered as

pseudo-criteria. In other words, one has to solve the problem with the constraints $\Phi_v(\alpha) \leq \Phi_v^{**}$, $v = 1,...,k, k+1,...,n$.

Notice, that the pseudo-criteria are not considered while constructing the Pareto optimal set.

IV. TOOLS FOR VISUALIZATION

The software package MOVI mentioned above provides various tools for visualization that facilitate multicriteria analysis process [6-8, 13]:

- **Graphs: "Criterion vs. Criterion".** After N tests, N1 design variable vectors have entered the test table. We consider projections of the *multidimensional points* $\Phi_v(\alpha^i)$, $v = 1,...,k$, $i = 1,...,N1$ onto the plane $\Phi_i \Phi_j$. These projections provide an expert with information about dependencies between criteria.

- **Graphs: "Criterion vs. Design Variable".** We consider projections of the *multidimensional points* $\Phi_v(\alpha^i)$, $v = 1,...,k$, $i = 1,...,N1$ onto the plane $\Phi_v \alpha_j$. These projections provide an expert with information about dependencies of criteria on design variables.

- **Histograms of Feasible Solutions.** The intervals $[\alpha_j^*; \alpha_j^{**}]$, $j = 1,...,r$ are divided into ten identical subintervals. Above each subinterval, the number of feasible designs entering this subinterval is indicated. The histograms show the role of the functional and criteria constraints in the design variable space and allow an expert to correct the initial design variable constraints accordingly.

- **Tables of Functional Failures**. These tables allow one to revise the rigid functional constraints in order to correct the feasible solution set, see [1, 8].

Analyzing the histograms and graphs reveals how the feasible and Pareto sets are distributed. In particular, the graphs and histograms allow an expert (1) to estimate the dependencies of the criteria on the design variables and the dependencies between the criteria, (2) to determine significant design variables, (3) to compare the values of criteria for a baseline design (prototype) with the results obtained by the PSI method. This information may prove to be helpful when correcting the ranges of the design variable, functional and criteria constraints in order to improve the results of the previous or initial optimization problems [1,2]. The examples of graphs and histograms are provided in the following sections.

V. CONSTRUCTION OF THE COMBINED FEASIBLE AND PARETO SETS

In many cases, analyzing the test tables leads us to conclude that correction of the boundaries of the initial parallelepiped (defining constraints on design variables) and construction of a new parallelepiped are advisable. Suppose that appropriate investigations have been performed in the new parallelepiped

and that the corresponding feasible set has been constructed. Now it is necessary to combine the feasible sets constructed in both these parallelepipeds and define the Pareto optimal solutions in the combined feasible set. The procedure of the repeated correction of parallelepipeds and construction of the combined feasible and Pareto optimal sets is essential for our problem. Constructing the combined Pareto optimal set allows one to estimate the contribution of each problem in this set and expediency of correcting the initial problem.

We also mention that construction of the combined Pareto optimal set is necessary for solving the optimization problems in a distributed mode [1, 8].

VI. EXAMPLE: THE PROBLEM OF IMPROVING A PRELIMINARY SHIP DESIGN PROTOTYPE

The mathematical model summarized below is based on the references [3, 11, 12]. Since we have a readily available mathematical model, we need only to conduct the second stage of the operational development as described in the Introduction.

A. Initial (or the first) statement of the optimization problem. The *design variable vector* of the prototype $\alpha^p=(\alpha_1^p,\ldots,\alpha_{14}^p)$ is given. We follow the general strategy to determine the initial design variable space, that is to construct the initial parallelepiped Π_1 such that the prototype is located in its center. The *vector of functional relations* $f = (f1,\ldots, f10)$ is specified according to our model. The *criteria* are Φ_1 (Seakeeping Rank [12]) and Φ_2 (Residuary Resistance Coefficient [11]). We want to maximize the criterion Φ_1 and to minimize the criterion Φ_2. The *vector of criteria of prototype* is known and is $\Phi^p=(8.608;1.968)$.

Since the functional constraints f5, …, f10 are nonrigid, i.e. they can change within some limits, it is difficult enough to formulate these constraints a priori. According to the PSI method, the nonrigid functional relations should be interpreted as the *pseudo-criteria*, i.e. $\Phi_3=f5$, $\Phi_4=f6$, $\Phi_5=f7$, $\Phi_6=f8$, $\Phi_7=f9$, and $\Phi_8=f10$. In this case, the constraints are defined during the solution of the problem (on the basis of analysis of the test table).

Therefore, we have the *criteria vector* $\Phi=(\Phi_1,\Phi_2, \Phi_3, \Phi_4, \Phi_5,\Phi_6\ \Phi_7, \Phi_8)$. We should construct the Pareto set taking into account only the performance criteria Φ_1 and Φ_2 because Φ_3, Φ_4, Φ_5, Φ_6, Φ_7, Φ_8 are pseudo-criteria.

Since we are solving the problem of improving the prototype, the criteria constraints equal to the values of the prototype criteria ($\Phi^p_1=\Phi^{**}_1$ and $\Phi^p_2=\Phi_2^{*}$) were accepted.

In summary, we want to investigate the problem with 14-dimensional design variables space and an 8-dimensional criteria space, keeping in mind complex constraints, which we need to correct in order to construct feasible solutions.

B. Solution of the initial statement of the optimization problem. We performed N=131,072 tests in Π_1 and only N1=1,487 vectors entered the test table. For investigation of

the design variable space, the LP_τ sequences were used in the MOVI software. The 129,585 solutions did not satisfy the functional constraints. Only the ND=240 vectors (including the prototype) entered to the feasible set, and NP=3 vectors are the Pareto optimal solutions. The remaining 1,247 vectors did not satisfy the criteria constraints. The Pareto optimal solutions in the criteria space are denoted as # 26087, # 81087, and # 75527. They significantly surpass the prototype by two criteria simultaneously. As a result of the multicriteria analysis of the Pareto optimal solutions, an expert has preferred a vector # 75527.

Graphs: "Criterion vs. Criterion". The graph of the first criterion versus the second criterion is shown in Figure 1. We see a complex relationship between the criteria. Three Pareto optimal solutions surpassed the prototype (denoted with numbers in the figure). An example of the dependency between a criterion and pseudo-criterion is shown in Figure 2. We see a linear relation on that figure.

Graph: "Criterion vs. Design Variable". Figure 3 shows a complex relationship between the criterion Φ_2 and the design variable α_{10}.

Histograms of Feasible Solutions. The distributions of feasible solutions for the range of the 2nd and 10th design variables in the initial parallelepiped Π_1 are shown in Figure 4a. We can see that there are large "gaps" that do not contain any feasible solutions. Similar distributions with "gaps" are observed for the 5th, 6th, and 9th design variables.

C. The second statement and solution of the optimization problem. Parallelepiped Π_2. The analysis of feasible solutions (including test tables, histograms, graphs of criterion vs. design variable, and graphs of criterion vs. criterion) has shown an expediency of the correction of the problem's initial statement. This can allow us to find Pareto optimal solutions that are better than ones obtained in the first statement of the problem.
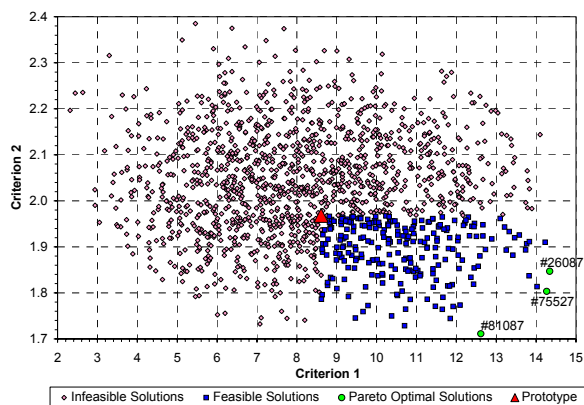

Figure 1. The dependency between criteria Φ_1 and Φ_2. Initial statement of the problem: ND=238; NP=3.
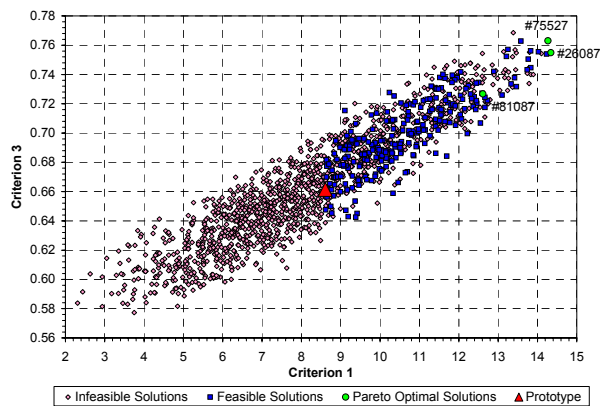
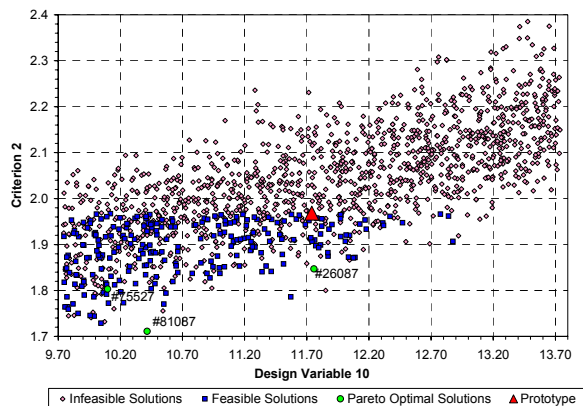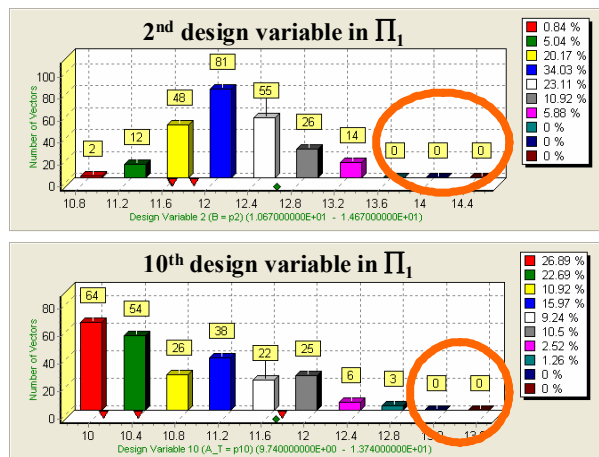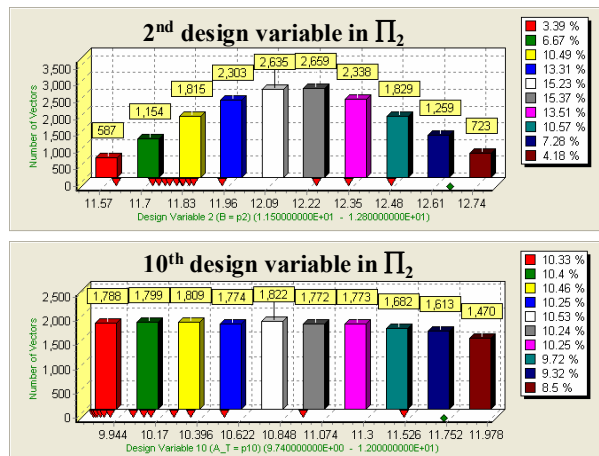**Figure 2.** The dependency between criterion Φ_1 and pseudo-criterion Φ_3.

Figure 3. The dependency between criterion Φ_2 and design variable α_{10}.

Figure 4. Histograms of the distribution of feasible and Pareto optimal solutions for the 2nd and 10th design variables: a) corresponds to initial parallelepiped Π_1 and b) corresponds to parallelepiped Π_2. The percentage of designs entering the corresponding interval is indicated on the right of each histogram. The prototype is marked with a green diamond. The Pareto optimal vectors are marked with red triangles. The "gaps" of the initial range for the 2nd and 10th design variables are circled.

Therefore, the design variable constraints in parallelepiped Π_1 were corrected and a new parallelepiped Π_2 was constructed. Again, the N=131,072 tests were conducted. The criteria constraints and functional constraints remained the same, as in the initial statement of the problem. The N1=18,270 vectors entered the test table. This time ND=17,302 vectors (including the prototype) entered the feasible set, and NP=14 are the Pareto optimal solutions, see Figure 6. Recall that in the initial statement there were only ND=240 and NP=3 feasible and Pareto optimal solutions, respectively. Histograms in Π_2 have much better distributions of feasible solutions (see Figure 4b).

After the combining the initial and new feasible sets, we have constructed a combined Pareto set. No solution belonging to the initial statement has entered into the combined Pareto set. In other words, all the results of optimization in the initial statement (parallelepiped Π_1) have been improved. The analysis of the Pareto solutions allowed us to give the preference to the vector # 74223. This vector surpassed all solutions in the initial statement by two criteria simultaneously, see Figure 6.

D. Final (or the third) statement and solution of the optimization problem. Parallelepiped Π_3. On the basis of analysis of the feasible solutions obtained in Π_2, parallelepiped Π_3 was constructed. More rigid criteria constraints Φ_1^{**}=14.342 and Φ_2^{**}=1.711 were formulated (previously they were Φ_1^{**}=8.608 and Φ_2^{**}=1.968). After the N=131,072 tests, the N1=34,986 vectors entered the test table. We have obtained feasible solutions ND=847 and Pareto solutions NP=7, see Figure 5. The smaller number of the feasible solutions compared to the previous problem can be explained by essentially stronger criteria constraints. The Pareto optimal solutions (# 113487, # 4145, # 68410, # 39801,

53988, # 72461, # 75110) are shown in Figure 5 and Figure 6. We observed the following:

- Solution #75110 surpasses 7 solutions from the second statement by two criteria simultaneously, see Figure 6.

- Solution # 113487 surpasses 5 solutions from the second statement by two criteria simultaneously, see Figure 6.

- Using the feasible solutions from the second and final statement, the combined Pareto set was constructed. The combined Pareto set includes all 7 Pareto optimal solutions from the final statement and only 2 solutions # 106467, # 49109 belonging to the second statement, see Figure 6. Further attempts to improve the obtained solutions have not given any new interesting results.

- The stability of the most interesting Pareto optimal solutions was investigated with respect to small variations of the parameters in the vicinity of these solutions. To this end, we constructed parallelepipeds centred at the Pareto optimal solutions and performed 1,024 tests in each parallelepiped. The corresponding variations in the criteria were small and insignificant, which indicated the stability of the solutions.

The overall dynamics of improving a prototype on the basis of two corrections of the problem statement are shown in Figure 6.

E. Main results. The problem of improving a preliminary ship design prototype has been solved. The prototype has been essentially improved. Improvement of the prototype has occurred, as a result of the definition of the feasible solution's area including: revising range of change of the design variables; weakening some constraints on pseudo-criteria (in comparison with the prototype); construction and analysis of histograms, graphs of criteria vs. criteria, criteria vs. design variables, and the combined Pareto optimal set.

To achieve the goal, we have corrected the problem statement twice. Each time the analysis of the obtained results allowed formulating the improved statement. In our case, three statements allowed us to construct the final combined Pareto set and to define the most preferable solution # 113487 from the final statement.

CONCLUSIONS

The process of constructing and analyzing the feasible solution set is of fundamental importance in multicriteria real-life problems, especially in the problem of improving a prototype. The main contribution of the present work is that we provide various visualization and analysis techniques that facilitate construction of the feasible solution set and thus help to solve the problem of improving a prototype. Finally, we presented a case study in which we aimed improving a preliminary ship design prototype. The prototype of the ship has been significantly improved by defining the feasible solution set.
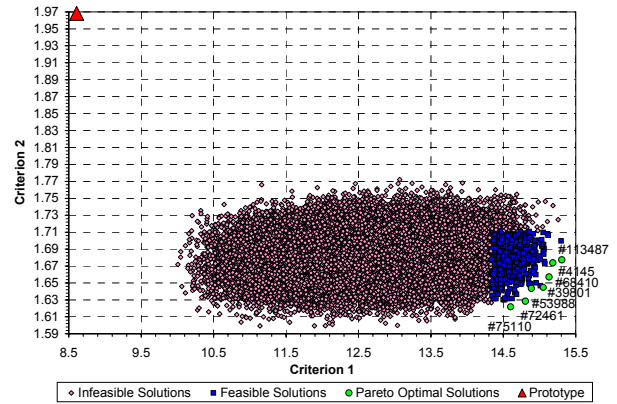

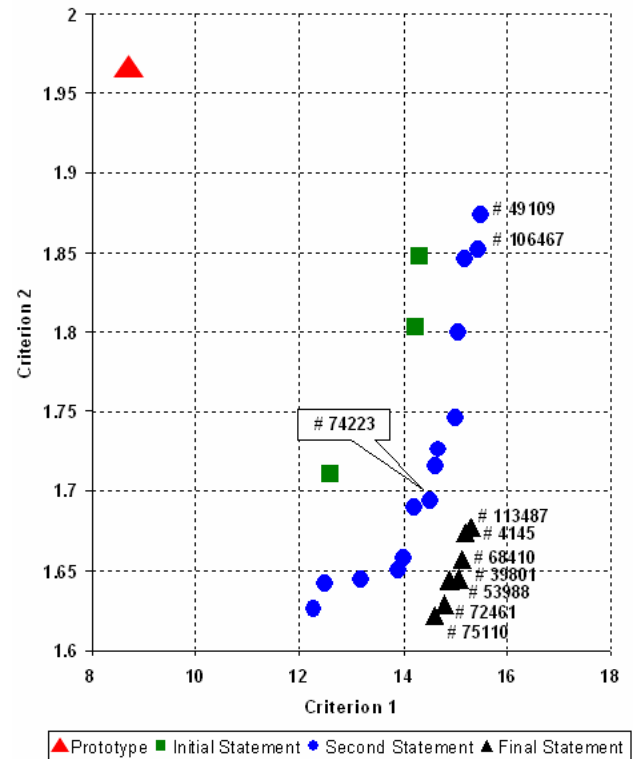**Figure 5.** The dependency between criteria Φ_1 and Φ_2. Final statement: ND=847; NP=7.

Figure 6. Pareto optimal solutions in the three statements and the combined Pareto set.

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