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An Application of Interactive Fuzzy Satisficing Approach with Particle Swarm Optimization for Multiobjective Emergency Facility Location Problem with A-distance

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Abstract— **This paper extends optimal location problems for emergency facilities to multiobjective programming problems by considering the following two objectives: one is to minimize the maximal distance of paths from emergency facilities to hospitals via accidents, and the other is to maximize frequency of accidents that emergency facilities can respond quickly. In order to find a satisfying solution of the formulated problems, an interactive fuzzy satisfying method with particle swarm optimization is proposed. Computational results for applying the method to examples of multiobjective emergency facility location problems are shown.**

I. Introduction

In this paper, we consider emergency facility location problems (EFLPs), such that ambulance service stations [9], fire stations [12], etc. Matsutomi and Ishii [9] considered EFLPs with the situation that if an accident occurs, the nearest emergency facility sends ambulances to it and injured people are conveyed to the nearest hospital. In such EFLPs, there are the following two important factors.

One is distance (or norm); for details about relation between facility location and norm, see Martini [7]. There are two norms widely used in studies about EFLPs. One is the Euclidean norm [1]. About this norm, it is assumed that it can be traveled to any orientations at any points. However, this assumption does not usually holds in cases of facility location in urban areas. The other is block norm [11], [15], [16]. About this norm, it is assumed that it can be traveled to given several allowable orientations of movement with weights at any points. Rectilinear distance is regarded as one of block norms such that there are two allowable orientations which cross at right angles with the same weights. EFLP with rectilinear distance are often studied [1], [17]. A-distance defined by Widmayer et al. [18] is also regarded as one of block norms such that there are several allowable orientations of movement with the same weights. Matsutomi and Ishii [9] consider EFLP with the A-distance. In this paper, we propose a new EFLP based on the EFLP with the A-distance.

The other is criteria of optimality for facility location. In general EFLP [2], [6], [12], an objective for facility location is to minimize the maximal distance between emergency facilities and the scenes of accidents. In addition to the above objective, in this paper, we consider another new objective which is to maximize frequency of accidents that emergency facilities can respond quickly to. We formulate multiobjective EFLP with the above two objectives. Since multiobjective programming problems do not have complete optimal solution generally, in order to find a satisfying solution for the decision maker, we apply interactive fuzzy satisficing method proposed by Sakawa et al [13]. In this method, we need to efficiently find an optimal solution for each of the minimax problems with the corresponding reference membership values. We propose to apply particle swarm optimization (PSO) method proposed by Kennedy et al [3].

The organization of the paper is as follows. In Section II, we give the definition of A-distance and its properties. In Section III, we formulate multiobjective EFLP with the A-distance. For the formulated EFLP, first we propose the method to compute the objective values for each location in Section IV. In order to find a satisfying solution for the decision maker, we introduce the interactive fuzzy satisficing method proposed by Sakawa and Yano [13] in Section V. In order to solve the minimax problems in this method, we proposed a PSO method considering characteristics of EFLP in Section VI. In section VII, we show results for applying the method to examples of multiobjective EFLPs. Finally, we make mention of conclusions and future remarks in Section VIII.

II. A-distance

In this section, we describe the definition of A-distance and its properties. We consider the situation that there are a orientations which can only move in the plane $\boldsymbol{R}^2$. The orientations are represented as the angles between the corresponding straight lines to orientations and the Cartesian x-axis; for example, orientation 0 is the x-axis and orientation $\pi/2$ is the y-axis. Let $A = \{\alpha_1, \dots, \alpha_a\}$ be a set of orientations such that $0 \le \alpha_1 \le \cdots \le \alpha_a < \pi$. A line, a half line, or a line segment is called to be A-oriented if its orientation is one of those in A. Then, the A-distance between two points $\boldsymbol{p}_1$ and $\boldsymbol{p}_2 \in \boldsymbol{R}^2$

is represented as follows:

$$d_A(\boldsymbol{p}_1,\boldsymbol{p}_2) := \begin{cases} d_2(\boldsymbol{p}_1,\boldsymbol{p}_2), \text{if } \boldsymbol{p}_1 \text{ and } \boldsymbol{p}_2 \text{ are in} \\ \qquad\qquad \text{an } A\text{-oriented line,} \\ \min_{\boldsymbol{p}_3 \in \boldsymbol{R}^2} \{d_A(\boldsymbol{p}_1,\boldsymbol{p}_3) + d_A(\boldsymbol{p}_3,\boldsymbol{p}_2)\}, \\ \qquad\qquad \text{otherwise,} \end{cases} \quad (1)$$

where $d_2(\cdot,\cdot)$ means the Euclidean distance. The rectilinear distance is represented as the A-distance with $A = \{0, \pi/2\}$.

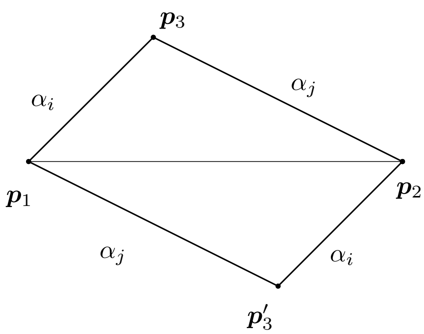

Fig. 1. A-distance

As shown in Figure 1, if $\boldsymbol{p}_1$ and $\boldsymbol{p}_2$ are not in any A-oriented lines, there exists at least one point $\boldsymbol{p}_3 \in \boldsymbol{R}^2$ such that

$$d_A(\boldsymbol{p}_1,\boldsymbol{p}_2) = d_2(\boldsymbol{p}_1,\boldsymbol{p}_3) + d_2(\boldsymbol{p}_3,\boldsymbol{p}_2). \quad (2)$$

For a point $\boldsymbol{p}$ and a distance d, the locus of all points $\boldsymbol{p}'$ with $d_A(\boldsymbol{p},\boldsymbol{p}') = d$ is called A-circle with center $\boldsymbol{p}$ and radius d.

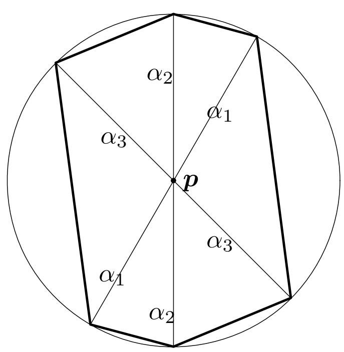

Fig. 2. A-circle in cases that $A = \{\pi/3, \pi/2, 3\pi/4\}$

As shown in Figure 2, A-circle has its boundary of $2a$-gon whose corner points are the intersections of the circle with center $\boldsymbol{p}$ and radius d and the A-oriented lines through $\boldsymbol{p}$.

For two points $\boldsymbol{p}_1$ and $\boldsymbol{p}_2$, bisector of $\boldsymbol{p}_1$ and $\boldsymbol{p}_2$ with the A-distance is defined as follows:

$$B_A(\boldsymbol{p}_1,\boldsymbol{p}_2) = \{\boldsymbol{p} \mid d_A(\boldsymbol{p}_1,\boldsymbol{p}) = d_A(\boldsymbol{p}_2,\boldsymbol{p})\}. \quad (3)$$

Let $Q = \{\boldsymbol{p}_1,\ldots,\boldsymbol{p}_n\}$ be a set of n points in $\boldsymbol{R}^2$. Then, Voronoi polygon $V_A(\boldsymbol{p}_i)$, $i = 1,\ldots,n$, with the A-distance is defined as follows:

$$V_A(\boldsymbol{p}_i) = \bigcap_{j \neq i} \left\{\boldsymbol{p} \mid d_A(p,\boldsymbol{p}_i) \leq d_A(\boldsymbol{p},\boldsymbol{p}_j), \boldsymbol{p} \in \boldsymbol{R}^2\right\}. \quad (4)$$

Sides and vertices of Voronoi polygons are called Voronoi edges and Voronoi points, respectively. The set of all Voronoi polygons, which can be regarded as a partition of $\boldsymbol{R}^2$, is called Voronoi diagram with the A-distance. The computational time to construct Voronoi diagram for Q, denoted by $VD_A(Q)$, is estimated at most $O(n \log n)$ shown by Widmayer et al. [18]. Figure 3 shows an example of Voronoi diagram with $A = \{0, \pi/2\}$.

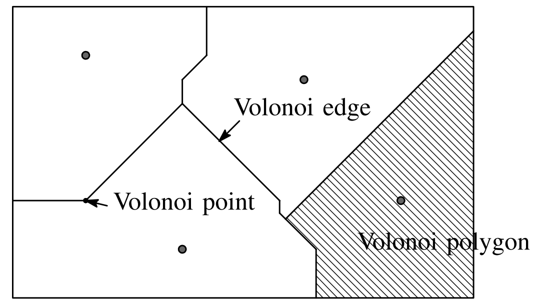

Fig. 3. Voronoi diagram with the A-distance

III. Formulation of multiobjective EFLP

In this section, we formulate multiobjective EFLP with A-distance. Let $S \subset \boldsymbol{R}^2$ be a closed convex polygon in which accidents occur and the decision maker needs to locate emergency facilities. We consider the situation that if an accident occurs at a point, the nearest emergency facility to the point sends ambulances to the point and then injured people in the accident are conveyed from the point to the nearest hospital.

First, we show the minimax criterion about path from the emergency facilities to the hospitals via the points of accidents. Let $\boldsymbol{h}_1,\cdots,\boldsymbol{h}_m \in S$ be sites of m hospitals, let $\boldsymbol{y}_1,\cdots,\boldsymbol{y}_n \in S$ be sites of n emergency facilities, and $Y = (\boldsymbol{y}_1,\cdots,\boldsymbol{y}_n)$. Then, if an accident occurs at a point $\boldsymbol{p} \in S$, A-distance for the above path is represented as follows:

$$u(Y,\boldsymbol{p}) := \min_{i=1,\ldots,n} d_A(\boldsymbol{y}_i,\boldsymbol{p}) + \min_{j=1,\ldots,m} d_A(\boldsymbol{p},\boldsymbol{h}_j). \quad (5)$$

Because the decision maker does not know where accidents occur in S beforehand, one of our objectives is regarded to cope with any accident points in S quickly. Then, the first objective function is represented as follows:

$$f_1(Y) := \max_{\boldsymbol{p} \in S} u(Y,\boldsymbol{p}) \quad (6)$$

Secondly, we show a new criterion about frequency of accidents. We assume that the decision maker knows points where accidents frequently occur in S, called accident points. There are k accident points whose sites are denoted by $\boldsymbol{a}_1,\ldots,\boldsymbol{a}_k \in S$, and each of their accident points has a weight about frequency of accidents, denoted by $w_1,\ldots,w_k > 0$, respectively. Let $\gamma > 0$ be an upper limit of the response time from the emergency facilities to the hospitals that a medical treatment for injured people can be in time. The other of our objectives is regarded as to maximize sum of the weights of frequency of accident points that the emergency facilities can cover for a given $\gamma > 0$. Then, the second objective function is represented as follows:

$$f_2(Y) := \sum_{i \in I_\gamma(Y)} w_i, \tag{7}$$

where

$$I_\gamma(Y) := \{\boldsymbol{a}_i | u(Y, \boldsymbol{a}_i) \leq \gamma\} \tag{8}$$

Therefore, multiobjective EFLP is formulated as follows:

$$P_M : \left.\begin{array}{ll} \text{minimize} & f_1(Y) \\ \text{maximize} & f_2(Y) \\ \text{subject to} & Y = (\boldsymbol{y}_1, \ldots, \boldsymbol{y}_n) \in S^n \end{array}\right\} \tag{9}$$

IV. Formulation of multiobjective EFLP

In order to find an optimal solution of problem P_M, we need to compute the two objective values for each solution. For the second objective function f_2, we can compute its objective value by measuring the A-distances from the emergency facilities to the hospitals via the k accidents points. In this section, we propose the method to compute the first objective value for each location.

Matsutomi and Ishii [9] proved the following lemma for the single objective EFLP with f_1 in the case of one emergency facility, that is, $n = 1$:

Lemma 1: For the single objective EFLP with f_1 that $n = 1$, $\boldsymbol{p} \in S$ which maximizes $u(Y, \boldsymbol{p})$ is one of the following points:

- vertices of the boundary of S,
- intersections of Voronoi edges of each $V(\boldsymbol{h}_1), \ldots, V(\boldsymbol{h}_m)$ and the boundary of S.

We extend Lemma 1 to the following theorem for our proposing multiobjective EFLPs that $n \geq 2$:

Theorem 1: For problem P_M, $\boldsymbol{p} \in S$ which maximizes $u(Y, \boldsymbol{p})$ is one of the following points:

- vertices of the boundary of S,
- intersections of Voronoi edges of each $V(\boldsymbol{h}_1), \ldots, V(\boldsymbol{h}_m)$ and the boundary of S,
- Voronoi points of each $V(\boldsymbol{h}_1), \ldots, V(\boldsymbol{h}_m)$,
- Voronoi points of each $V(\boldsymbol{x}_1), \ldots, V(\boldsymbol{x}_n)$,
- intersections of Voronoi edges of each $V(\boldsymbol{x}_1), \ldots, V(\boldsymbol{x}_n)$ and the boundary of S,
- intersections of Voronoi edges of each $V(\boldsymbol{h}_1), \ldots, V(\boldsymbol{h}_m)$ and Voronoi edges of each $V(\boldsymbol{x}_1), \ldots, V(\boldsymbol{x}_n)$.

Such points in Theorem 1 can be found by drawing Voronoi diagram for hospitals and Voronoi diagram for each location of emergency facilities. Then, we can find objective value of $f_1(Y)$ by computing the maximal distance for paths from emergency facilities to hospitals via these points.

V. Interactive fuzzy satisficing approach

In this section, we introduce the interactive fuzzy satisficing method proposed by Sakawa and Yano [13] in order to find a satisficing solution of problem P_M for the decision maker.

About decision making in real world, it is generally for the decision maker to want to make an objective value be better than a certain value, rather than to maximize/minimize its objective value. Such an objective, called a fuzzy objective, includes vagueness based upon judgment of the decision maker. In this study, we represent the two objectives of problem P_M as fuzzy objectives provided by membership functions, denoted by μ_1 and μ_2.

Now we introduce an example of membership functions for each objective function. Let d_e denote the distance such that the decision maker is quite satisfied if the first objective value is less than d_e, and d_ℓ denote the distance such that she/he is satisfied to a certain degree if its objective value is more than d_e but less than d_ℓ. Then, we use the following linear membership function for the former objective:

$$\mu_1(f_1(Y)) := \begin{cases} 1, & \text{if } f_1(Y) < d_e, \\ \dfrac{f_1(Y) - d_e}{d_\ell - d_e}, & \text{if } d_e \leq f_1(Y) < d_\ell, \\ 0, & \text{if } f_1(Y) \geq d_\ell. \end{cases} \tag{10}$$

Next, one of the simplest ways to provide membership function for the latter objective is as follows:

$$\mu_2(f_2(Y)) := \frac{f_2(Y)}{\sum_{i=1}^{k} w_i} \tag{11}$$

Then, problem P_M is transformed as the following multiobjective fuzzy programming problem:

$$P_Z : \left.\begin{array}{ll} \text{maximize} & \mu_1(f_1(Y)) \\ \text{maximize} & \mu_2(f_2(Y)) \\ \text{subject to} & Y \in S^n \end{array}\right\} \tag{12}$$

Since there is generally no complete optimal solution about multiobjective programming problem including P_Z, the concept of the M-Pareto optimal solution is usually used for multiobjective fuzzy programming problems.

Definition 1: Solution Y^* is an M-Pareto optimal solution to problem P_Z if and only if there does not exist any solutions $Y \in S^n$ such that $\mu_i(f_i(Y)) \geq \mu_i(f_i(Y^*))$ for all $i = 1, 2$ and $\mu_j(f_j(Y)) > \mu_j(f_j(Y^*))$ for at least one $j \in \{1, 2\}$.

The interactive fuzzy satisficing method [13] is to find a satisfying M-Pareto optimal solution through interaction to the decision maker. Let $(\bar{\mu}_1, \bar{\mu}_2)$ be a pair of initial reference membership levels of membership function μ_1 and μ_2, respectively. Then, the interactive fuzzy satisficing method for EFLP is described as follows:

Interactive fuzzy satisficing method

Step 1: Provide two membership functions μ_1 and μ_2; for example, equation (10) and (11).

Step 2: Set the initial reference membership levels $(\bar{\mu}_1, \bar{\mu}_2) = (1, 1)$.

Step 3: For the given pair of reference membership levels $(\bar{\mu}_1, \bar{\mu}_2)$, solve the following corresponding minimax problem:

$$\left.\begin{array}{ll} \text{minimize} & \max_{i=1,2}\{\bar{\mu}_i - \mu_i(f_i(Y)) \\ & +\rho\sum_{j=1}^{2}(\bar{\mu}_j - \mu_j(f_j(Y)))\} \\ \text{subject to} & Y \in S^n \end{array}\right\} \quad (13)$$

Here, ρ is a positive number sufficiently small.

Step 4: If the decision maker is satisfied with the current levels of the M-Pareto optimal solution, STOP. Then the current M-Pareto optimal solution is a satisfying solution for the decision maker.

Step 5: Update the pair of current reference membership levels $(\bar{\mu}_1, \bar{\mu}_2)$ based on information of preference of the decision maker, the current values of the membership functions, etc. Return to Step 3.

In the interactive fuzzy satisfying method, we need to solve the minimax problems in Step 3. In the next section, we propose an efficient solving method for the minimax problem.

VI. Particle swarm optimization

PSO proposed by Kennedy and Eberhart [3] is based on the social behavior that a population of individuals adapts to its environment by returning to promising regions that were previously discovered [4]. This adaptation to the environment is a stochastic process that depends upon both the memory of each individual, called particle, and the knowledge gained by the population, called swarm.

In the numerical implementation of this simplified social model, each particle has the following three attributes: the position vector in the search space, the velocity vector and the best position in its track, and the best position of the swarm. The process can be outlined as follows.

Particle swarm optimization

Step 1: Generate the initial swarm involving N particles at random.

Step 2: Calculate the new velocity vector for each particle, based on its attributes.

Step 3: Calculate the new position of each particle from the current position and its new velocity vector.

Step 4: If the termination condition is satisfied, stop. Otherwise, go to Step 2.

To be more specific, for the position of the i-th particle at time t, denoted by $\boldsymbol{x}_i^{t+1}$, the new velocity vector of the i-th particle at time t, denoted by $\boldsymbol{v}_i^{t+1}$, is calculated by the following scheme introduced by Shi and Eberhart [14].

$$\boldsymbol{v}_i^{t+1} := \omega^t \boldsymbol{v}_i^t + c_1 R_1^t(\boldsymbol{p}_i^t - \boldsymbol{x}_i^t) + c_2 R_2^t(\boldsymbol{p}_g^t - \boldsymbol{x}_i^t) \quad (14)$$

In (14), R_1^t and R_2^t are random numbers between 0 and 1, $\boldsymbol{p}_i^t$ is the best position of the i-th particle in its track and $\boldsymbol{p}_g^t$ is the best position of the swarm. There are three problem dependent parameters, the inertia of the particle ω^t, and two trust parameters c_1, c_2.

Then, the new position of the i-th particle at time t, denoted by $\boldsymbol{x}_i^{t+1}$, is calculated from (15).

$$\boldsymbol{x}_i^{t+1} := \boldsymbol{x}_i^t + \boldsymbol{v}_i^{t+1}, \quad (15)$$

where $\boldsymbol{x}_i^t$ is the current position of the i-th particle at time t. The i-th particle calculates the next search direction vector $\boldsymbol{v}_i^{t+1}$ by (14) in consideration of the current search direction vector $\boldsymbol{v}_i^t$, the direction vector going from the current search position $\boldsymbol{x}_i^t$ to the best position in its track $\boldsymbol{p}_i^t$ and the direction vector going from the current search position $\boldsymbol{x}_i^t$ to the best position of the swarm $\boldsymbol{p}_g^t$, moves from the current position $\boldsymbol{x}_i^t$ to the next search position $\boldsymbol{x}_i^{t+1}$ calculated by (15). The parameter ω^t controls the amount of the move to search globally in early stage and to search locally by decreasing ω^t gradually. It is defined by (16)

$$\omega^t := \omega^0 - \frac{t(\omega^0 - \omega^{T_{\max}})}{0.75(T_{\max})}, \quad (16)$$

where $T_{\max}$ is the number of maximum iteration times, ω^0 is an initial value at the time iteration, and $\omega^{T_{\max}}$ is the last value. The searching procedure of PSO is shown in Fig. 4. Comparing the evaluation value of a particle after movement, denoted by $f(\boldsymbol{x}_i^{t+1})$, with that of the best position in its track, denoted by $f(\boldsymbol{p}_i^t)$, if $f(\boldsymbol{x}_i^{t+1})$ is better than $f(\boldsymbol{p}_i^t)$, then the best position in its track is updated as $\boldsymbol{p}_i^t := \boldsymbol{x}_i^{t+1}$. Furthermore, if $f(\boldsymbol{p}_i^{t+1})$ is better than $f(\boldsymbol{p}_g^t)$, then the best position in the swarm is updated as $\boldsymbol{p}_g^{t+1} := \boldsymbol{p}_i^{t+1}$.

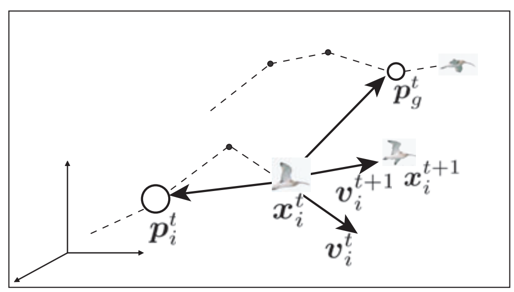

Fig. 4. Movement model for PSO

Such a PSO technique includes two problems. One is that particles concentrate on the best search position of the swarm and they cannot easily escape from the local optimal solution since the move direction vector $\boldsymbol{v}_i^{t+1}$ calculated by (14) always includes the direction vector to the best search position of the swarm. Another is that a particle after move is not always feasible for problems with constraints.

In order to settle the first problem, Matsui et al. [8] proposed the following leaving acts for particles which are on the best position of the swarm: (i) the particles move at random to a point in the feasible region, (ii) the particles move at random

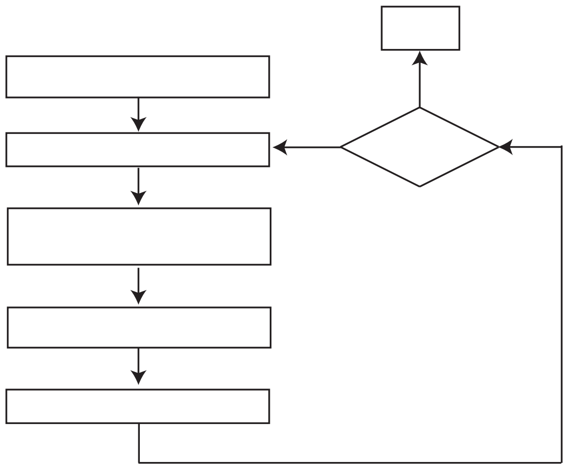

Fig. 5. PSO algorithm

to a point on the boundary of the feasible region, and (iii) the particles move at random to a point in a direction of some coordinate axis. Moreover, Matsui et al. [8] proposed the multiple stretching techniques, which is extended the stretching method proposed by Parsopoulos and Varahatis [10]. In the multiple stretching technique, the following functions for m local optimal solutions $\bar{\boldsymbol{x}}_k$, $k=1,\ldots,m$ are used:

$$G_k(\boldsymbol{x}) = f(\boldsymbol{x})+\gamma_1\|\boldsymbol{x}-\bar{\boldsymbol{x}}_k\|\Big(\mathrm{sign}\big(f(\boldsymbol{x})-f(\bar{\boldsymbol{x}}_{\min})\big)+1\Big) \quad (17)$$

$$H_k(\boldsymbol{x}) = G_k(\boldsymbol{x}) + \gamma_2\frac{\mathrm{sign}\big(f(\boldsymbol{x})-f(\bar{\boldsymbol{x}}_{\min})\big)+1}{\tanh\Big(\mu\big(G_k(\boldsymbol{x})-G_k(\bar{\boldsymbol{x}}_k)\big)\Big)} \quad (18)$$

$$S(\boldsymbol{x}) = \sum_{k=1}^{m} H_k(\boldsymbol{x})/m \quad (19)$$

Here, $\bar{\boldsymbol{x}}_{\min}$ is the best among m local optimal solutions. The value of $S(\boldsymbol{x})$ for a search position $\boldsymbol{x}$ is equal to the objective function value $f(\boldsymbol{x})$ of $\boldsymbol{x}$ if $f(\boldsymbol{x})$ is better than that of $\bar{\boldsymbol{x}}_{\min}$, while it takes a very large value if the distance between $\boldsymbol{x}$ and the nearest local optimal solution is less than a certain value. Otherwise, it takes a value depending on the distance.

In order to settle the second problem, Matsui et al. [8] proposed to generate initial particles in the feasible set by utilizing the homomorphism proposed by Koziel and Michalewicz [5]:

1) Find one feasible solution in PSO in consideration of the degree of violation of constraints, and set it a reference solution of homomorphism $\boldsymbol{r}$.
2) Generate a guaranteed initial search position with homomorphism: map N points generated randomly in a hypercube by homomorphism using base point solution $\boldsymbol{r}$ pursued in the above point in a feasible region of decision variable space, and set these points initial search position of a particle $\boldsymbol{x}_i^0$, $i=1,\ldots,N$.

Moreover, there are often cases that a particle after move is not always infeasible if we use the updating equation of search position mentioned above. To deal with such a situation, Matsui et al. [8] divided the swarm into two subswarms. In one subswarm, since the move of a particle to the infeasible region is not accepted, if a particle becomes infeasible after a move, it is repaired to be feasible. To be more specific, with respect to infeasible particles which violate constraints after move, we repair its search position to be feasible by the bisection method on the direction from the search position before move, $\boldsymbol{x}_i^t$, to that after move, $\boldsymbol{x}_i^{t+1}$. In the other subswarm, the move of a particle to the infeasible region is accepted.

In this study, based upon the above PSO method, we proposed to introduce a new fourth movement in (14). Let $\boldsymbol{q} \in S$ be maximizer of $u(\boldsymbol{x}_i^t, \boldsymbol{p})$, such a point can be found by using Theorem 1. In problem P_M, by approaching the nearest emergency facility to $\boldsymbol{q}$, objective value of $f_1(Y)$ may be improved. So we introduce such a movement to PSO method. Let $\boldsymbol{p}_a^t$ be position such that for $\boldsymbol{x}_i^t$, site of the nearest facility is changed to $\boldsymbol{q}$ and sites of the other facilities are fixed. Then, our proposing new velocity vector of the i-th particle at time t is represented as follows:

$$\begin{aligned}\boldsymbol{v}_i^{t+1} \;:=\; & \omega^t\boldsymbol{v}_i^t + c_1R_1^t(\boldsymbol{p}_i^t-\boldsymbol{x}_i^t)\\ & + c_2R_2^t(\boldsymbol{p}_g^t-\boldsymbol{x}_i^t) + c_3R_3^t(\boldsymbol{p}_a^t-\boldsymbol{x}_i^t),\end{aligned} \quad (20)$$

where c_3 is a trust parameter and R_3^t is a random number between 0 and 1.

VII. Numerical experiments

In this section, we apply interactive fuzzy satisficing approach with the PSO method to an example of our proposing multiobjective EFLPs. In this example, we consider an EFLP for two emergency facilities, that is $n=2$. We represent S as a convex hull including 20 points given in $[0,100]$ randomly. About A-distance, $A=\{0,\pi/4,\pi/2,3\pi/4\}$. About hospitals, we set $m=3$ and their sites are given in S randomly. About frequency of accidents, we set $\gamma=15$, and for each of 100 accident points, its site and weight are randomly given in S and $(0,1]$, respectively.

We illustrate the interactive fuzzy satisficing approach for the above example of multiobjective EFLP P_M. About parameters of PSO, we set that population size is 40, generation is 500, $c_1=c_2=2$, and $c_3=0.5$.

At Step 1, in order to represent fuzziness about two objectives, we use membership function (10) and (11) in Section V with setting $d_e=5$ and $d_\ell=120$.

At Step 3, we solve minimax problems with $(\bar{\mu}_1,\bar{\mu}_2)$ and $\rho=10^{-3}$ by applying PSO method. In order to verify efficiency of PSO method, we apply genetic algorithm for numerical optimization for constrained problem, GENOCOP [5], to minimax problems. About parameters of GENOCOP, we set that population size is 40 and generation is 500. Computational results for each $(\bar{\mu}_1,\bar{\mu}_2)$ at 20 times by PSO and GENOCOP are shown in Table I and II, respectively.

From Table I and II, PSO can find better solutions than GENOCOP by meanings of both mean and stability. This means that efficiency of PSO for such minimax problems.

At Step 4, the decision maker evaluates whether the M-Pareto optimal solution given by solving minimax problem at Step 3

TABLE I

COMPUTATIONAL RESULTS BY PSO

Minimax problem	1	2	3
$\bar{\mu}_1$	1.0	1.0	0.9
$\bar{\mu}_2$	1.0	0.8	0.8
Best	0.4104	0.3109	0.2608
Mean	0.4106	0.3113	0.2611
Worst	0.4113	0.3120	0.2614
Mean CPU Time (Sec)	9.5196	10.080	9.0718

TABLE II

COMPUTATIONAL RESULTS BY GENOCOP

Minimax problem	1	2	3
$\bar{\mu}_1$	1.0	1.0	0.9
$\bar{\mu}_2$	1.0	0.8	0.8
Best	0.4132	0.3111	0.2611
Mean	0.4302	0.3292	0.2764
Worst	0.4483	0.3440	0.2885
Mean CPU Time (Sec)	10.101	11.982	10.044

is satisfied or not. If its solution satisfies the decision maker, this algorithm is terminated. Otherwise, ask the decision maker to update the current reference membership levels $(\bar{\mu}_1, \bar{\mu}_2)$ by considering the current values of the membership functions, and resolve minimax problem to Step 3. In this example of EFLP, we assume that decision maker thinks that the first objective f_1 is more important than f_2. Then the decision maker hopes to improve the value of μ_1 even if the value of μ_2 is changed for the worse. However, the decision maker does not hope to go the value of μ_2 too bad. Then, an example of the interactive fuzzy satisficing methods is given in Table III.

TABLE III

RESULTS OF INTERACTIVE FUZZY SATISFICING APPROACH

Iteration	1	2	3
$\bar{\mu}_1$	1.0	1.0	0.9
$\bar{\mu}_2$	1.0	0.8	0.8
$\mu_1(f_1(Y^*))$	0.5901	0.6892	0.6395
$\mu_2(f_2(Y^*))$	0.5897	0.4892	0.5393
Mean CPU Time (Sec)	9.5196	10.080	9.0718

In Table III, the decision maker is not satisfied M-Pareto optimal solution at Iteration 1 because value of μ_1 is bad. Then, in order to improve value of f_1, at Iteration 2, she/he decreases $\bar{\mu}_2$, which is reference membership levels about f_2. M-Pareto optimal solution given at Iteration 2 is good about f_1, however, she/he is not satisfied because value of f_2 is too bad. Then, in order to improve value of f_2 a little, at Iteration 3, she/he decreases $\bar{\mu}_1$, which is reference membership levels about f_1. Then, she/he obtains a satisfying solution about both f_1 and f_2, so the algorithm is terminated.

VIII. CONCLUSIONS AND FUTURE RESEARCHES

In this paper, we proposed a new EFLP with A-distance by extending to multiobjective programming problem. In order to obtain a satisfying solution for the decision maker, we have proposed interactive fuzzy satisficing method with solving minimax problems by PSO. By applying an example of multi-objective EFLPs, we showed efficiency of PSO and illustrated the interactive fuzzy satisficing method.

In our proposing multiobjective EFLPs, we assume that S is convex polygon. However, in order to apply the EFLPs to more general cases, we need to consider various shapes of S which are non-convex, non-connected, etc. To construct solving methods for general shapes of S is a future research. Moreover, in cases that EFLP is large-scale, that is, S includes many hospitals and the decision maker locates many emergency facilities, it needs to efficiently find an optimal solution for the minimax problems in the interactive fuzzy satisficing methods. To consider large-scale EFLP is also a future research.

REFERENCES

[1] J. Elzinga, D.W. Hearn, *Geometrical solutions for some minimax location problems*. Trans. Sci., vol.6, pp.379-394, 1972.

[2] R.L. Francis, *A geometrical solution procedure for a rectilinear minimax location problem*. AIIE Trans., vol.4, pp.328-332, 1972.

[3] J. Kennedy, R.C. Eberhart, *Particle swarm optimization*, Proc. of IEEE Int. Conf. Neural Networks, Piscataway, NJ, pp.1942-1948, 1995.

[4] J. Kennedy, W.M. Spears, *Matching algorithms to problems: an experimental test of the particle swarm and some genetic algorithms on the multimodal problem generator*. Proc. of IEEE Int. Conf. Evolutionary Computation, Anchorage, Alaska, 1998.

[5] S. Koziel, Z. Michalewicz, *Evolutionary Algorithms, Homomorphous Mappings, and Constrained Parameter Optimization*. Evolutionary Computation, vol.7, No.1, pp.19-44, 1999.

[6] D.K. Kulshrestha, *A mini-max location problem with demand points arbitrarily distributed in a compact connected space*, J. Opl Res. Soc., vol.38, pp.447-452, 1987.

[7] H. Martini, A. Schŏbel *Median hyperplanes in normed spaces - a survey*. Discrete Applied Mathematics, vol.89, pp.181-195, 1998.

[8] T .Matsui, M .Sakawa, T. Uno, K. Kato, M. Higashimori, M. Kaneko, *Jumping pattern optimization for a serial link robot through soft computing technique*. Proceedings of Joint 3rd International Conference Soft Computing and Intelligent Systems and 7th International Symposium on advanced Intelligent Systems, FR-J4-3, 2006 (CD-ROM).

[9] T. Matsutomi, H. Ishii, *Minimax location problem with A-distance*. J. Opl Res. Soc., vol.41, pp.181-195, 1998.

[10] K.E. Parsopoulos, M.N. Varahatis, *Recent approaches to global optimization problems through Particle Swarm Optimization*. Natural Computing, vol.1, pp. 235-306, 2002.

[11] B. Pelegrin, F.R. Fernandez, *Determination of efficient points in multiple-objective location problems*. Navel Res. Log, vol.35, pp.697-705, 1988.

[12] D.R. Plane, T.E. Hendric, *Mathematical programming and the location of fire companies for the Denver fire department*. Opns. Res., vol.25, pp.563-578, 1977.

[13] M. Sakawa, H. Yano, *An interactive fuzzy satisfying method using augmented minimax problems and its application to environmental systems*. IEEE Transactions on Systems, Man, and Cybernetics, vol.SMC-15, pp.720-729, 1985.

[14] Y. Shi, R.C. Eberhart, *A modified particle swarm optimizer*. Proc. of IEEE Int. Conf. Evolutionary Computation, Anchorage, Alaska, 1998.

[15] J.F. Thisse, J.E. Hendric, R.E. Wendell *Some properties of location problems with block and round norm*. Opns. Res., vol.32, pp.1309-1327, 1984.

[16] J.E. Ward, R.E. Wendell, *Using block norm for location modeling*. Opns. Res., vol.33, pp.1074-1090, 1985.

[17] G.O. Wesolowsky, *Rectangular distance location under the minimax optimality criterion*. Trans. Sci., vol.6, pp.103-113, 1972.

[18] P. Widmayer, Y.F. Wu, and C.K. Wong, *On some distance problems in fixed orientations*. SIAM J. COMPUT., vol.16, pp.728-746, 1987.