Information Dynamics at the Edge of Chaos: Measures, Examples, and Principles

Mikhail Prokopenko [1,2]
[1] CSIRO Information and Communications Technologies Centre
PO Box 76, Epping, NSW 1710, Australia
[2] School of Physics, University of Sydney, Australia
Email: mikhail.prokopenko@csiro.au

***Abstract*—We survey state-of-the-art methods of information dynamics and briefly discuss some of the popular measures, exemplifying their use in different contexts, including cellular automata, swarming behavior, modular robotics, and random Boolean networks. Several possible principles that generalize the patterns observed in the examples are also suggested. These principles are aimed at providing thermodynamic interpretations of critical phenomena exhibited by many complex Artificial Life systems.**

I. Introduction

In early 1990s, Chris Langton put forward a challenge: how can emergence of computation be explained in a dynamic setting, and how is it related to complexity of the system in point? [1]. In doing so, he suggested to dissect computation into its three primitive functions: the *transmission*, *storage* and *modification of information*, introducing a new term, "computation at the edge of chaos".

A system is said to exhibit the property of chaos if a slight change in the initial conditions results in large-scale but bounded differences in the result (see e.g. [2]). Importantly, there are transitions separating ordered and chaotic regimes, and by varying *control parameters* (e.g., the system composition and the strength of interactions within it) one may trigger these phase transitions. Following Ginzburg-Landau theory of phase transitions developed in physics, Haken introduced *order parameters* in explaining structures that spontaneously self-organize in nature [3], [4]. When energy or matter flows into a system typically describable by many variables, it may move away from equilibrium, approach a threshold (that can be defined in terms of some control parameters), and undergo a phase transition. At this stage, the behavior of the overall system can be described by only a few order parameters (degrees of freedom) that characterize newly formed patterns. In other words, the system becomes low-dimensional as some dominant variables "enslave" others, making the whole system to act in synchrony. A canonical physical example is laser: a beam of coherent light created out of the chaotic movement of particles.

Another example of coherent behavior is given by swarms of animals. Animal groups in nature often exhibit striking examples of spatial aggregation, e.g. schools of fish, swarms of locusts, herds of wildebeest, and flocks of birds [5], [6], [7]. Such aggregations may provide individuals with protection, mate choices, foraging, habitat assessment, migratory routes, etc. [8], [9]. Complex large-scale patterns and structures emerge within swarms through individual decisions based on perception of local conditions. For example, in response to a predator, many schools of fish display complex collective patterns, including compression, 'hourglass', 'vacuole', 'flash expansion', or form highly parallel translating groups [10]. It has been observed that small perturbations cascade through an entire swarm in a wave-like manner [11], with these cascades conjectured to embody information transfer [12].

Another well-studied model in the field of artificial life — random Boolean networks used to simulate Gene Regulatory Networks (GRNs) [13] — also exhibits well-known phase transition between ordered to chaotic dynamics: in terms of length of transients in phase space with respect to average connectivity or activity level.

We believe that many evolutionary and self-organization pressures can be characterized information-theoretically not only because this provides an approximation useful in designing biologically-inspired Artificial Life systems, but also because numerous optimal structures evolve/self-organize in nature when elements of information dynamics are maximized near specific phase transitions and error thresholds [14], [15], [16], [17], [18], [19], [20], [21], [22]. Guiding and maintaining self-organizing systems near critical points is studied within the field of Guided Self-Organization that attempts to meanigfully combine design and self-organization [23], [24], [25]. This survey, however, focuses specifically on *information dynamics of computation* within spatio-temporal systems, quantifying these elements on both global and local scale in space and time. The approach is exemplified in different contexts, including cellular automata, swarming behavior, modular robotics, and random Boolean networks.

In addition, we briefly discuss possible principles generalizing some of the patterns observed in the examples. This brings us closer to thermodynamic interpretations of critical phenomena present in many complex systems. In doing so, we point out some recent characterizations of order parameters via Fisher information, as well as an interpretation of transfer entropy as entropy production.

II. Information Dynamics

A. Measures

A recently developed framework of *information dynamics* studies the phenomenon of computation in a systematic way: it uncovers and analyses information-theoretic roots of each of the primitives, and synthesizes these parts back together in a

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multiplicity of ways, revealing different shapes that coherence and complexity may take [26], [27], [28], [29], [30].

The *active information storage* quantifies the information storage component that is directly in use in the computation of the next state of a process [26], [30]. More precisely, it is the average mutual information between the semi-infinite past of the process $x_n^{(k)} = \{x_{n-k+1}, \ldots, x_{n-1}, x_n\}$ (as $k \to \infty$) and its next state. The *local information storage* (or pointwise mutual information) is then a measure of the amount of information storage in use by the process at a particular time-step $n+1$:

$$a_X(n+1) = \lim_{k\to\infty} \log_2 \frac{p(x_n^{(k)}, x_{n+1})}{p(x_n^{(k)})p(x_{n+1})}. \quad (1)$$

For example, the local active information storage in Cellular Automata (CA) provides evidence that periodic nonmoving structures (e.g., blinkers in Conway's Game of Life) and background domains are dominant information storage processes in these systems, and reveals that local storage may sometimes misinform the observer when the computation is dominated by some other primitives.

In practice, one deals with finite-k estimates $a_X(n+1, k)$, as well as the finite-k estimates $A_X(k)$ of the average active information storage $A_X = \langle a_X(n+1)\rangle_n$. Interestingly, the sum of underestimates of the active information storage at each finite history length $k \geq 0$ is the *excess entropy* [30]:

$$E_X = \sum_{k=0}^{\infty}[A_X - A_X(k)]. \quad (2)$$

The *local information transfer*, based on *transfer entropy* [31], captures information transmission [27] from source Y to destination X, at a particular time-step $n+1$. Specifically, the local information transfer between a source and a destination agent is defined as the information provided by the source y_n about the destination's next state x_{n+1} that was not contained in the past of the destination $x_n^{(k)}$:

$$t_{Y\to X}(n+1) = \lim_{k\to\infty} \log_2 \frac{p(x_{n+1} \mid x_n^{(k)}, y_n)}{p(x_{n+1} \mid x_n^{(k)})}. \quad (3)$$

When applied to CA for filtering coherent structures it provides the quantitative evidence for the long-held conjecture that domain walls and traveling coherent structures (e.g., gliders in Conway's Game of Life) are the dominant information transfer agents in CA. This method is distinguished in using asymmetric, multivariate, information-theoretical analysis, which captures not only directional and non-linear relationships, but also collective interactions. Interestingly, the local information transfer can be negative, indicating that the source is misinformative about the next state of the destination.

The storage and transfer can be brought together in state-space diagrams $t(n,k)$ vs $a(n,k)$, distinguishing complex computation via a coherent structure in local information dynamics profiles, and identifying both clear and "hidden" coherent structures [29].

Sometimes it is useful to condition the local information transfer on another contributing process W, considering the *local conditional transfer entropy* [28]:

$$t_{Y\to X|W}(n+1) = \lim_{k\to\infty} \log_2 \frac{p(x_{n+1} \mid x_n^{(k)}, y_n, w_n)}{p(x_{n+1} \mid x_n^{(k)}, w_n)}. \quad (4)$$

Furthermore, *local information modification* can be quantified at each spatiotemporal point in a system — via separable information, a measure which locally identifies events where separate inspection of the sources to a computation is misleading about its outcome (as the points where "the whole is greater than the sum of the parts") [28]. Formally, one computes the *local separable information*:

$$s_X(n+1) = a_X(n+1) + \sum_i t_{Y_i\to X}(n+1), \quad (5)$$

adding up the local information storage and the local transfers from all available sources Y_i. Where $s(n+1)$ is positive, the separate or independent observations of the sources are informative overall about the next state of the information destination. Conversely, negative $s(n+1)$ is likely to indicate that an information modification event (e.g. a collision of gliders) takes place, with more significant modifications taking larger negative values, and some of the sources are misinformative.

The concept of synergy is a subject of vigorous research in complex systems [32], and artificial life in particular (e.g. [33]), promising to reveal origins of universal computation required to maintain systems near the edge of chaos.

B. Examples

Building up on these measures, one may investigate complex behavior in artificial life systems, including various phase transition between ordered and chaotic regimes.

For instance, recent studies by Wang et al. [34] quantitatively verified the hypothesis that the collective memory within a swarm can be captured by *active information storage*. Higher values of storage are associated with higher levels of dynamic coordination. This study revealed different ways in which a particle's spatial position is dynamically related to its information processing role. In this context, information cascades correspond to long range communications that either dynamically reorganize the swarm reducing the "fragility of mass behaviour" [35] or propagate incorrect decisions [36]. The study of Wang et al. [34] proposed and verified another hypothesis according to which information cascades are captured by *conditional transfer entropy* [27], [28], characterizing the communication aspect of collective but distributed computation within the swarm.

In random and small-world Boolean Networks it has been demonstrated that the information storage and information transfer are maximized on either side of the critical point — thus, explaining the phase transition of node's state dynamics in terms of the intrinsic distributed computation [37], [38]. Specifically, for fully-random topologies, the ordered phase of dynamics is dominated by information storage, while the chaotic phase is dominated by information transfer. In addition, the information dynamics behave similarly as the network topology itself undergoes an order – small-world – randomness transition. That is, more locally-connected, ordered networks with low activity tend to have their dynamics dominated by *information storage*, while more randomized networks with higher activity tend to be dominated by *information transfer*. In other words, small-world networks display something of a balance between these capabilities, with the capability for coherent information transfer being maximized near the small-world state [38].

Another example of complex computation in a distributed system that was successfully characterized with the information dynamics is a modular robotic system ("Snakebot") modelling a multi-segment snake-like organism, with actuators ("muscles") attached to individual segments ("vertebrae"), and without a global coordinating component such as central pattern generator. A particular side-winding locomotion is known to emerge as a result of individual control actions when the actuators are coupled within the system and follow specific evolved rules [39]. The side-winding locomotion has not only been measured but also evolved using information dynamics, utilizing *excess entropy* (storage) and *transfer entropy* (communications) [40], [41], [42]. In this instance, the adaptation of controllers occurs by evolution with the fitness function rewarding the regularity and richness of the actuators' multivariate series. The same adaptation can also be achieved during the agents lifetime — in other words, the time interval over which the fitness function is computed may be interpreted either as the full lifetime of the individual (leading to an evolutionary representation) or as a finite period within such lifetime (leading to an online learning).

Other interesting examples of online learning guided by information dynamics have been studied by Ay et al. [43], Der and Martius [44], and others. This approach has also been extended to form a basis for a guided self-organization of behavior [45], [46], [47].

C. Principles

The information-theoretic approach clearly separates different elements of distributed computation taking place in complex systems such as CA, swarms, complex networks, modular robots, etc. filtering and predicting relevant hot spots. For example, in swarms, it may predict a cascade's wavefront, collective memory's core, etc. In addition, this framework may reveal new biological/social principles that govern coherent behavior of living organisms. In particular, maximal information transfer in a swarm tends to quickly follow the stage with maximal collective memory. Similarly, in complex networks, information storage and information transfer are somewhat balanced near the small-world regime, providing quantitative evidence that small-world networks may indeed be capable of "combining" comparably large information storage and transfer. The robust side-winding locomotion in a Snakebot also required positive information transfer interleaved with significant storage (excess entropy).

Thus, one possible principle guiding and sustaining complex behavior is a *dynamic balance* between high information storage and information transfer, that supports cascades of information within the system while it operates near the edge of chaos. This principle may need to be complemented by an adequate capture of synergistic information, so that the system's components internally processing information create *novelty* by modifying stored and transferred information. We may conjecture that novelty is needed to make the balance between information storage and transfer dynamic rather than static or trivially oscillating. Otherwise, we may conceive a system that periodically synchronizes (via strong information cascades) its different components, and the components store some arbitrarily high information content, without generating anything new.

The concepts of synergy, and information modification in general, require a systematic consideration of the system from a thermodynamic perspective. Such a consideration not only enhances a representative power of formalisms, but also provides an explanation for generation or erasure of new information ("novelty dynamics"), via the exchange of energy between the system in point and its exterior. This is of highest importance for artificial life designers attempting to create local order against the relentless flow of entropy.

A recent study of thermodynamic phase transitions via statistical estimation theory explicitly related elements of the Fisher information matrix $F_{ij}(\theta)$ to the rate of change in the corresponding order parameters ϕ^i [48]:

$$F_{ij}(\theta) = \beta \frac{\partial \phi^i}{\partial \theta^j}, \tag{6}$$

where β is the inverse temperature (T) of the environment, $\beta = 1/k_B T$, and k_B is the Boltzmann constant. Specifically, second-order phase transitions were identified via divergences of individual elements of the *Fisher information matrix*. This result revealed the basic thermodynamic reasons behind similar empirical observations reported previously, e.g. in random Boolean networks. In other words, critical points are identified with the points where the observed variable is most sensitive to the control parameter / thermodynamic variable, diverging in infinite systems and maximized in finite-size systems.

Another recent study proposed a thermodynamic interpretation of transfer entropy near equilibrium, using a specialised Boltzmann's principle [49]. In particular, the local transfer entropy was shown, under some assumptions, to be proportional to the external entropy production ΔS_{ext}, attributed to the source of irreversibility Y:

$$t_{Y \to X}(n+1) = -\frac{\Delta S_{ext}}{k_B \ln 2}. \tag{7}$$

One possible interpretation of this result would relate an increase of predictability within the system (higher local transfer entropy) to negentropy, the entropy that the system exports (dissipates) to keep its own entropy low [50].

III. Conclusion

In this brief review of state-of-the-art methods of information dynamics, we have discussed some of the popular measures, exemplified their use, and sketched possible principles that generalize some of the patterns observed in the examples.

The most important lesson is that the information dynamics framework motivates us to switch from a singular view on complexity that suggested to characterize the latter with a single static measure, to a new perspective within a rich multi-faceted space where each primitive (storage, transfer and modification) becomes a separate dimension. Within such a space, complexity acquires shapes and trajectories defined by these basic axes, while coherence can be quantitatively studied via suitably defined state-diagrams. Crucially, such a view is supported by several recent thermodynamic interpretations of relevant primitives as well as the overall critical phenomena.

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