

# Self-Tuning Robust Stability Fuzzy Digital Controller

Edson B.M. Costa

Federal University of Maranhão  
Av. dos Portugueses, s/n, Bacanga, CEP 65001-970,  
São Luís-MA, Brazil  
Email: edsonbmarques@hotmail.com

Ginalber L.O. Serra

Federal Institute of Education, Science and Technology  
Av. Getúlio Vargas, 04, Monte Castelo, CEP 65030-005,  
São Luís - MA, Brazil  
Email: ginalber@ifma.edu.br

**Abstract**—A self-tuning fuzzy control methodology via particle swarm optimization based a robust stability criterion, is proposed. The plant is modeled considering a Takagi-Sugeno (TS) fuzzy structure from input-output experimental data, by using the fuzzy C-Means clustering algorithm (antecedent parameters estimation) and weighted recursive least squares (WRLS) algorithm (consequent parameters estimation), respectively. An adaptation mechanism based on particle swarm optimization is used to tune recursively the parameters of a model based fuzzy PID controller, from the gain and phase margins specifications. Computational results for adaptive fuzzy control of a thermal plant with time varying delay is presented to illustrate the efficiency and applicability of the proposed methodology.

## I. INTRODUCTION

In general, the most practical control loops are characterized by changes in the plant due to uncertainty, nonlinearity, stochastic disturbances, change in the nature of the input, propagation of disturbances along the chain of unit processes, time-varying pure delay, etc. In all such situations, a conventional controller presents limitations to maintain the performance of the control loop at acceptable levels, and fuzzy adaptive control has been suggested as an alternative approach to these conventional control techniques in many applications [1], [2], [3], [4], [5], [6], [7].

The first adaptive fuzzy controller called the linguistic self-organizing controller was introduced in [8]. Since then, several other adaptive fuzzy control techniques have been proposed [9], [10], [11], [12], [13], [14], [15]. In which, the use of genetic algorithm, particle swarm optimization, simulated annealing and other bio-inspired intelligent optimization techniques for tuning adaptive fuzzy controllers can be emphasized as a recent topic of interest in the adaptive fuzzy control field [16], [17], [18], [19], [20], [21], [22], [23]. In [24], a genetic algorithm based adaptive fuzzy controller design has been proposed for the multi-variable control of temperature and relative humidity of a real world system by manipulating valve positions to adjust the water and steam flow rates for a air handling unit. In [25], an approach based on fuzzy rules for online continuous tuning of a low pass filter with adjustable gain in the feedback loop, is proposed. The fuzzy controller is optimally tuned using Particle Swarm Optimization (PSO) taking into accounts both the tracking error and the controller output signal range. In [26], a fuzzy adaptive control for pneumatic muscle actuator with simulated annealing tuning is

proposed. A hybrid adaptive approach was developed, where a conventional PD controller is placed into the feedforward branch and a fuzzy controller is placed into the adaptation branch. The fuzzy controller compensates for the actions of PD controller under conditions of inertia moment variation. The design of fuzzy controller is based on the results of optimization using simulated annealing algorithm.

In this context, a self-tuning robust stability adaptive fuzzy control methodology via particle swarm optimization, is proposed. The plant is identified by a TS fuzzy inference structure from input-output experimental data, by using the fuzzy C-Means clustering algorithm and WRLS algorithm for antecedent and consequent parameters estimation, respectively. An adaptation mechanism based on particle swarm optimization is used to tune the fuzzy PID controller parameters, via Parallel Distributed Compensation (PDC) strategy, based on gain and phase margins specifications, recursively, according to identified fuzzy model parameters of the plant. Computational results for adaptive fuzzy control of a thermal plant with time varying delay is presented to illustrate the the efficiency and applicability of the proposed methodology.

This paper is organized as follows: In Section II the proposed methodology, is presented. The structure of the TS fuzzy model and PID controller, the tuning formulas based on gain and phase margins specifications, the fuzzy model recursive estimation algorithm and particle swarm adaptation mechanism, are presented. In Section III computational results for adaptive fuzzy control of a thermal plant with time varying delay are shown. Finally, in Section IV the conclusions of this work are discussed.

## II. PROPOSED METHODOLOGY

The block diagram of the self-tuning robust stability fuzzy digital control methodology is depicted in Fig. 1. The TS fuzzy model is recursively identified using measurement input/output samples,  $u(k)$  e  $y(k)$ , taken from the plant. The normalized membership grade of each fuzzy rule,  $\mu^i$ , is determined from the membership functions obtained by a fuzzy clustering method, where the fuzzy C-Means clustering method is used in this paper. The consequent parameters of each fuzzy rule,  $\theta^i$ , is recursively estimated via WRLS method. An adaptation mechanism based on particle swarm optimization is used to online tune the fuzzy PID controller parameters,  $[\alpha^i, \beta^i, \gamma^i]$ , based on gain and phase margins specifications,  $MG_S$  and

$MP_S$ , to guarantee the robustness to variations in the plant behavior and tracking of the reference signal,  $r$ .

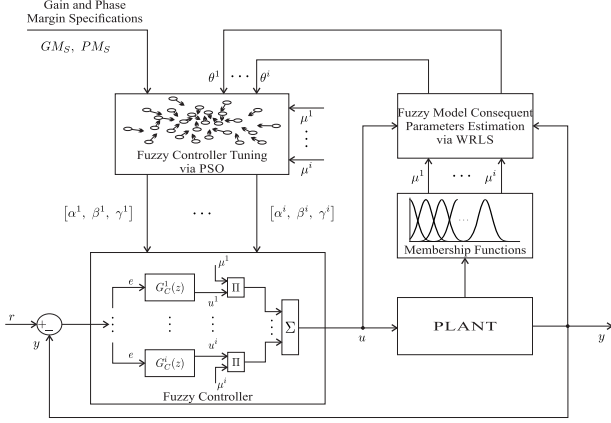


Fig. 1. Robust stability adaptive fuzzy control scheme.

In this paper, the plant is described by the following TS fuzzy model:

$$R^{i|i=1,2,\dots,l} : \text{IF } x_1 \text{ is } F_1^i \text{ AND } \dots \text{ AND } x_q \text{ is } F_p^i \text{ THEN}$$

$$G_p^i(z) = \frac{b_0^i + b_1^i z^{-1} + \dots + b_{n_u}^i z^{-n_u}}{1 + a_1^i z^{-1} + a_2^i z^{-2} + \dots + a_{n_y}^i z^{-n_y}} z^{-\tau_d^i/T} \quad (1)$$

where  $R^{(i)}$  denotes the  $i$ -th rule, and  $l$  is the number of rules in the rule base. In the antecedent part, the linguistic variables  $x_j$ ,  $j = 1, 2, \dots, q$ , belongs to a fuzzy set  $F_j^i$  with a truth value given by a membership function  $\mu_{F_j^i}(x_j) : \mathbb{R} \rightarrow [0, 1]$ . Each linguistic variable has its own discourse universe  $U_{x_1}, \dots, U_{x_n}$ , partitioned by fuzzy sets representing its linguistics terms, respectively. The consequent part of the  $i$ -th inference rule is composed of  $n_y$ -th order discrete-time transfer functions,  $G_p^i(z)$ , in which  $\tau_d^i$  is its time delay,  $a_{1,2,\dots,n_y}^i$  and  $b_{1,2,\dots,n_u}^i$  are its numerator and denominator parameters, respectively.

The TS fuzzy PID controller, to be designed based on gain and phase margins specifications according to PDC strategy, where the designed fuzzy controller shares the same fuzzy sets with the fuzzy model in the premise part with the controller and the plant [27], in the  $i^{i=1,2,\dots,l}$ -th rule, without loss of generality, is given by:

$$R^i : \text{IF } x_1 \text{ is } F_1^i \text{ AND } \dots \text{ AND } x_q \text{ is } F_p^i \text{ THEN}$$

$$G_c^i(z) = \frac{\alpha^i z^2 + \beta^i z + \gamma^i}{z^2 - z} \quad (2)$$

with:

$$\alpha^i = K_P^i + \frac{K_I^i T}{2} + \frac{K_D^i}{T} \quad (3)$$

$$\beta^i = \frac{K_I^i T}{2} - K_P^i - \frac{2K_D^i}{T} \quad (4)$$

$$\gamma^i = \frac{K_D^i}{T} \quad (5)$$

where  $K_P^i$ ,  $K_I^i$  and  $K_D^i$  are proportional, integral, and derivative fuzzy controller gains of the  $i$ -th inference rule, and  $T$  is the sample time, respectively [28].

From equations 1 and 2, the gain and phase margins of the fuzzy control system are given by:

$$\angle \left[ \sum_{i=1}^l \mu^i G_c^i(z, e^{j\omega_p}) G_P^i(z, e^{j\omega_p}) \right] = -\pi \quad (6)$$

$$GM = \frac{1}{\left| \sum_{i=1}^l \mu^i G_c^i(z, e^{j\omega_p}) G_P^i(z, e^{j\omega_p}) \right|} \quad (7)$$

$$\left| \sum_{i=1}^l \mu^i G_c^i(z, e^{j\omega_g}) G_P^i(z, e^{j\omega_g}) \right| = 1 \quad (8)$$

$$PM = \angle \left[ \sum_{i=1}^l \mu^i G_c^i(z, e^{j\omega_g}) G_P^i(z, e^{j\omega_g}) \right] + \pi \quad (9)$$

where the gain margin is given by equations (6) and (7), and the phase margin is given by equations (8) and (9), respectively. The  $\omega_p$  is called phase crossover frequency and  $\omega_g$  is called gain crossover frequency.

#### A. Fuzzy Model Recursive Estimation Algorithm

The consequent of the dynamic TS fuzzy model, given in the Eq. 1, can be represented by an ARX structure with input  $u(k)$  and  $i$ -th sub-model output  $y^i(k)$  in each iteration  $k$ . Considering  $D^i = -\tau_d^i/T$ , it is stated as

$$y^i(k) = \sum_{n=1}^{n_y} -a_n^i y^i(k-n) + \sum_{n=0}^{n_u} b_n^i u(k-(n+D^i)) \quad (10)$$

By applying a standard fuzzy inference method, that is, by using a singleton fuzzifier, product fuzzy inference, and center-average defuzzifier, the global TS fuzzy model output,  $\hat{y}$ , is obtained as following

$$\hat{y}(k) = \sum_{i=1}^l \mu^i(k) \left[ \sum_{n=1}^{n_y} -a_n^i y^i(k-n) + \sum_{n=0}^{n_u} b_n^i u(k-(n+D^i)) \right] \quad (11)$$

where  $\mu^i(x)$  is the normalized membership function satisfying

$$\mu^i(x) = \frac{\xi^i(x)}{\sum_{j=1}^l \xi^j(x)} \quad (12)$$

$$\xi^i(x) = \prod_{j=1}^q F_j^i(x_j), \quad (13)$$

$$\mu^i(x) \geq 0, \quad (14)$$

$$\sum_{i=1}^l \mu^i(x) = 1, \quad (15)$$

and  $F_j^i(x_j)$  is the membership grade of  $x_j$  in the fuzzy set  $F_j^i$ .

The time delay, given by  $D^i = -\tau_d^i/T$  samples can be obtained by any approach for time delay obtaining based on experimental data, such as cross-correlation method. And the sub-model parameters vector in  $i$ -th rule,  $(\hat{\theta}^i)^T =$

$[a_1^i \ a_2^i \ \cdots \ a_{n_y}^i \ b_0^i \ b_1^i \ \cdots \ b_{n_u}^i]$ , can be obtained in bath by least squares method, as following

$$\hat{\theta}^i = [\Phi^T M^i \Phi]^{-1} \Phi^T M^i Y \quad (16)$$

where  $Y = [y(1) \ y(2) \ \cdots \ y(N)]^T$  is the output vector of the dynamic system, considering  $N$  input-output pairs of observations  $\{u(k), y(k), k = 1, 2, 3, \dots, N\}$ .  $\Phi = [\varphi^T(1) \ \varphi^T(2) \ \cdots \ \varphi^T(N)]^T$  is the matrix of regression with regression vectors  $\varphi^T(k-1) = [-y(k-1) \ \cdots \ -y(k-n_y) \ u(k) \ \cdots \ u(k-n_u)]$ . And the matrix  $M^i$  is the diagonal weighting matrix of  $i$ -th rule, as follows

$$M^i = \begin{bmatrix} \mu^i(1) & 0 & \cdots & 0 \\ 0 & \mu^i(2) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mu^i(N) \end{bmatrix} \quad (17)$$

The recursive parameters estimation of the TS fuzzy model can be obtained by rewritten the Eq. 16 in the recursive form, as following

$$\hat{\theta}^i(k) = \left[ \sum_{n=1}^k \mu^i(n) \varphi(n) \varphi^T(n) \right]^{-1} \sum_{n=1}^k \mu^i(n) \varphi(n) y(n) \quad (18)$$

where  $P^i(k) = \left[ \sum_{n=1}^k \mu^i(n) \varphi(n) \varphi^T(n) \right]^{-1}$  is the fuzzy covariance matrix.

Then, it has

$$(P^{-1})^i(k) = (P^{-1})^i(k-1) + \mu^i(k) \varphi(k) \varphi^T(k) \quad (19)$$

The least-squares estimator  $\hat{\theta}^i(k)$ , equation (18), can be written as follows:

$$\hat{\theta}^i(k) = P^i(k) \left[ \sum_{n=1}^{k-1} \mu^i(n) \varphi(n) y(n) + \mu^i(k) \varphi(k) y(k) \right] \quad (20)$$

By writing (18) at time  $k-1$ , it has

$$\left[ \sum_{n=1}^{k-1} \mu^i(n) \varphi(n) \varphi^T(n) \right] \hat{\theta}^i(k-1) = \sum_{n=1}^{k-1} \mu^i(n) \varphi(n) y(n) \quad (21)$$

The left side of equation (21) can be represented in compact form as  $(P^{-1})^i(k-1) \hat{\theta}^i(k-1)$ . Substituting this result into the equation (20), it has

$$\hat{\theta}^i(k) = P^i(k) \left[ (P^{-1})^i(k-1) \hat{\theta}^i(k-1) + \mu^i(k) \varphi(k) y(k) \right] \quad (22)$$

From equations (19) and (22), it has

$$\hat{\theta}^i(k) = \hat{\theta}^i(k-1) + \mu^i(k) K^i(k) \xi^i(k) \quad (23)$$

where

$$K^i(k) = P^i(k) \varphi(k) \quad (24)$$

$$\xi^i(k) = y(k) - \varphi^T(k) \hat{\theta}^i(k-1) \quad (25)$$

The covariance matrix,  $P^i(k)$ , is computed by applying the matrix inversion lemma in the equation (19), and

$$P^i(k) = P^i(k-1) - P^i(k-1) \mu^i(k) \varphi(k) [\varphi^T(k) P^i(k-1) \mu^i(k) \varphi(k) + I]^{-1} \varphi^T(k) P^i(k-1) \quad (26)$$

This implies that

$$K^i(k) = P^i(k-1) \varphi(k) (\varphi^T(k) P^i(k-1) \mu^i(k) \varphi(k) + I)^{-1} \quad (27)$$

and

$$P^i(k) = P^i(k-1) - \mu^i(k) K^i(k) \varphi^T(k) P^i(k-1) \quad (28)$$

Therefore, the recursive weighted least squares is given by equations (23), (27) and (28). Considering the forgetting factor  $\lambda^i$  for  $i$ -th rule, the recursive estimation algorithm of the consequent parameters of the fuzzy model, is given by:

$$\xi^i(k) = [y(k) - \varphi^T(k) \hat{\theta}^i(k-1)] \quad (29)$$

$$K^i(k) = P^i(k-1) \varphi(k) [\varphi^T(k) P^i(k-1) \mu^i(k) \varphi(k) + \lambda^i I]^{-1} \quad (30)$$

$$\hat{\theta}^i(k) = \hat{\theta}^i(k-1) + \mu^i(k) K^i(k) \xi^i(k) \quad (31)$$

$$P^i(k) = \frac{1}{\lambda^i} (P^i(k-1) - \mu^i(k) K^i(k) \varphi^T(k) P^i(k-1)) \quad (32)$$

## B. Particle Swarm Adaptation Mechanism

In this paper, the adaptation mechanism is accomplished through a recursive multiobjective particle swarm optimization algorithm. The optimal parameters of the fuzzy PID controller are tuned, recursively, based on the fuzzy model parameters estimated by WRLS estimator and from the gain and phase margin specifications. The gain margin obtained in the  $k$ -th recursion,  $GM_O(k)$ , must be so near as possible of the specified gain margin,  $GM_S$ . In the same hand, the phase margin obtained in the  $k$ -th recursion,  $PM_O(k)$ , must be so near as possible of the specified gain margin,  $PM_S$ . The multi-objective adaptation mechanism is formulated by weighted-sum or scalarization method, in which the multiple objectives are combined into one single-objective scalar function,  $f(k)$ , by minimization of a positively weighted convex sum of the objectives

$$\text{minimize } f(k) = \delta_1 |GM_O(k) - GM_S| + \delta_2 |PM_O(k) - PM_S| \quad (33)$$

$$\text{subject to } GM_O(k) \geq GM_S \quad (34)$$

$$PM_O(k) \geq PM_S \quad (35)$$

$$GM_O(k) > 0 \quad (36)$$

$$PM_O(k) > 0 \quad (37)$$

where  $|GM_O(k) - GM_S|$  and  $|PM_O(k) - PM_S|$  are the objectives functions, the Eq. 34, 35, 36 and 37 are the constraint functions of the problem,  $\delta_1$  and  $\delta_2$  are weights, in which

$$\delta_1 + \delta_2 = 1, \delta_1 > 0, \text{ and } \delta_2 > 0 \quad (38)$$

Considering  $l$  rules and  $p$  particles, a  $3l$ -Dimensional swarm is initially generated in a search space defined around of the controller parameters encountered in the back iteration, as following:

$$\alpha_p^{i,0}(k) \sim U(\alpha^i(k-1) - \Delta_{\alpha^i}, \alpha^i(k-1) + \Delta_{\alpha^i}) \quad (39)$$

$$\beta_p^{i,0}(k) \sim U(\beta^i(k-1) - \Delta_{\beta^i}, \beta^i(k-1) + \Delta_{\beta^i}) \quad (40)$$

$$\gamma_p^{i,0}(k) \sim U(\gamma^i(k-1) - \Delta_{\gamma^i}, \gamma^i(k-1) + \Delta_{\gamma^i}) \quad (41)$$

where  $\Delta_{\alpha^i}$ ,  $\Delta_{\beta^i}$  and  $\Delta_{\gamma^i}$  are defined by expert;  $\alpha^i(k-1)$ ,  $\beta^i(k-1)$  and  $\gamma^i(k-1)$  are the controller parameters of the  $i | i=1,2,\dots,l$  - th rule in the back iteration. The recursive particle swarm adaptation mechanism is implemented as shown in Algorithm 1 and 2.

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**Algorithm 1:** Adaptation mechanism algorithm: part 1

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repeat
for each particle  $p \in 1, \dots, N$  do
  Compute the Gain and Phase Margin:

   $\angle[\sum_{i=1}^l \mu^i G_C^i(z, e^{j\omega p}) G_P^i(z, e^{j\omega p})] = -\pi$ 
   $GM_O(k) = \frac{1}{|\sum_{i=1}^l \mu^i G_C^i(z, e^{j\omega p}) G_P^i(z, e^{j\omega p})|}$ 
   $|\sum_{i=1}^l \mu^i G_C^i(z, e^{j\omega p}) G_P^i(z, e^{j\omega p})| = 1$ 
   $PM_O(k) = \angle[\sum_{i=1}^l \mu^i G_C^i(z, e^{j\omega p}) G_P^i(z, e^{j\omega p})] + \pi$ 

  if  $(GM_O(k) < GM_S$  or  $PM_O(k) < PM_S)$  then
    for each fuzzy rule  $i \in 1, \dots, l$  do
       $\alpha_p^i(k) = \alpha_p^i(k-1)$ 
       $\beta_p^i(k) = \beta_p^i(k-1)$ 
       $\gamma_p^i(k) = \gamma_p^i(k-1)$ 
    end
  end
else
  Evaluate the fitness of the  $p$ -th particle  $f_p$ 

   $f_p = \sqrt{\delta_1(GM_O(k) - GM_S)^2 + \delta_2(PM_O(k) - PM_S)^2}$ 

  end
  Update local best position
  if  $f_{(local)} < f_p$  then
    for each fuzzy rule  $i \in 1, \dots, l$  do
       $\alpha_{p(local)}^i(k) = \alpha_p^i(k)$ 
       $\beta_{p(local)}^i(k) = \beta_p^i(k)$ 
       $\gamma_{p(local)}^i(k) = \gamma_p^i(k)$ 
    end
  end
end

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### III. COMPUTATIONAL RESULTS

The computational results are based on experimental data from a thermal plant. The data acquisition platform is composed by virtual instrumentation environment, data acquisition

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**Algorithm 2:** Adaptation mechanism algorithm: part 2

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Update global best position
if  $f_{(global)} < f_{(local)}$  then
  for each fuzzy rule  $i \in 1, \dots, l$  do
     $\alpha_{p(global)}^i(k) = \alpha_{p(local)}^i(k)$ 
     $\beta_{p(global)}^i(k) = \beta_{p(local)}^i(k)$ 
     $\gamma_{p(global)}^i(k) = \gamma_{p(local)}^i(k)$ 
  end
end
Apply velocity update  $v_p(k+1)$ 
for each fuzzy rule  $i \in 1, \dots, l$  do
   $v_p^{[\alpha_p^i]}(k+1) = \omega v_p^{3i-2}(k) + c_1 r_1^{[\alpha_p^i]}(k) [\alpha_{p(local)}^i(k) - \alpha_p^i(k)] +$ 
     $+ c_2 r_2^{[\alpha_p^i]}(k) [\alpha_{p(global)}^i(k) - \alpha_p^i(k)]$ 
   $v_p^{[\beta_p^i]}(k+1) = \omega v_p^{3i-1}(k) + c_1 r_1^{[\beta_p^i]}(k) [\beta_{p(local)}^i(k) - \beta_p^i(k)] +$ 
     $+ c_2 r_2^{[\beta_p^i]}(k) [\beta_{p(global)}^i(k) - \beta_p^i(k)]$ 
   $v_p^{[\gamma_p^i]}(k+1) = \omega v_p^{3i}(k) + c_1 r_1^{[\gamma_p^i]}(k) [\gamma_{p(local)}^i(k) - \gamma_p^i(k)] +$ 
     $+ c_2 r_2^{[\gamma_p^i]}(k) [\gamma_{p(global)}^i(k) - \gamma_p^i(k)]$ 
end
Apply position update  $x_p(k+1)$ 
for each fuzzy rule  $i \in 1, \dots, l$  do
   $\alpha_p^i(k+1) = \alpha_p^i(k) + v_p^{[\alpha_p^i]}(k+1)$ 
   $\beta_p^i(k+1) = \beta_p^i(k) + v_p^{[\beta_p^i]}(k+1)$ 
   $\gamma_p^i(k+1) = \gamma_p^i(k) + v_p^{[\gamma_p^i]}(k+1)$ 
end
 $k = k+1$ 
until iterations number of PSO;

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hardware, sensor and actuator, as shown in Fig. 2. The thermal plant consists in a monophasic toaster AC 220 Volts, with functional temperature from 25 °C to 265 °C. The virtual instrumentation environment (Human Machine Interface) is based in LabVIEW software (LABoratory Virtual Instrument Engineering Workbench) which allows the designer to view, storage and process the acquired data. The data acquisition hardware performs the interface between sensors/actuators and the virtual instrumentation environment, and is composed by NI cRIO-9073 integrated system, the NI 9219 analog input module and the NI 9263 analog output module. The temperature sensor was the LM35, and the actuator is based on TCA 785 [29].

For recursive estimation of the consequent parameters of the TS fuzzy model, the input signal (RMS Voltage, in Volts) applied to monophasic toaster and the corresponding response (temperature, degree Celsius) were used as experimental data. A batch identification procedure was considered to obtain the initial conditions for implementation of the adaptive fuzzy control system: The antecedent membership functions of a TS fuzzy model (with two rules) were obtained using FCM

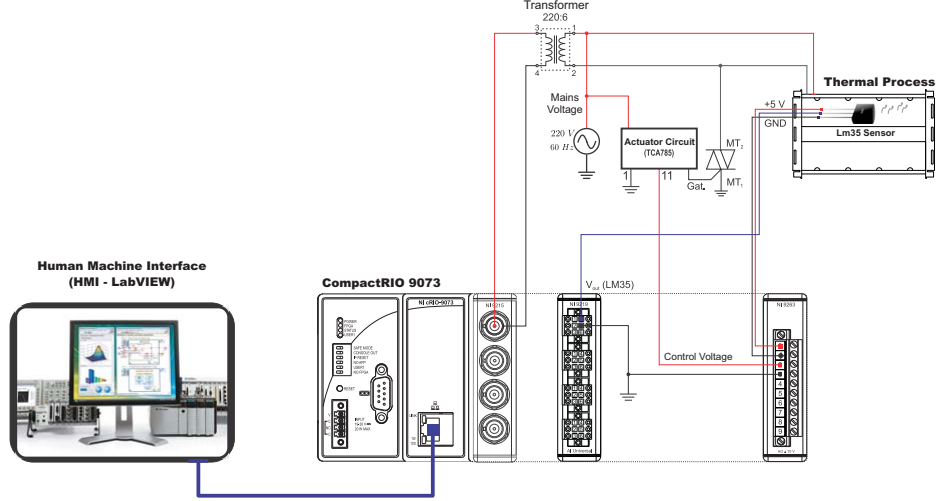


Fig. 2. Data acquisition platform based on virtual instrumentation.

(Fuzzy C-Means) algorithm; The time delay was estimated by computing the crosscorrelation function between input and output signals of the thermal plant, resulting in a time delay of 130 and 266 samples, corresponding to  $\tau_d^1 = 2.210$  seconds and  $\tau_d^2 = 4.522$  seconds with sample time of  $T = 17ms$ , for the second order transfer functions in first and second rules, respectively; The parameters of the transfer functions were  $b_0^1(0) = 0,0082$ ,  $b_1^1(0) = -0,0075$ ,  $a_1^1(0) = -0,5435$ ,  $a_2^1(0) = -0,456$ ,  $b_0^2(0) = 0,000352$ ,  $b_1^2(0) = 0,000248$ ,  $a_1^2(0) = -0,5648$ ,  $a_2^2(0) = -0,4348$  for the second order transfer functions in first and second rules, respectively; The forgetting factor adopted was of  $\lambda^1 = \lambda^2 = 0.997$  and covariance matrix was of  $P^i(0) = 10^{-5}I_4$ ; The gain and phase margins specified were  $5 - 70^\circ$ , and from a multiobjective particle swarm algorithm, the initial parameters of the fuzzy PID controller were:  $\alpha^1 = 2,8515$ ,  $\beta^1 = -2,8585$ ,  $\gamma^1 = 0,0081$ ,  $\alpha^2 = 3,0964$ ,  $\beta^2 = -3,1560$ ,  $\gamma^2 = 0,0606$ , and the gain and phase margins obtained were  $5 - 72,37^\circ$  and  $5,064 - 70,73^\circ$ , in the first and second rules, respectively. The results for TS fuzzy model parameters recursive estimation of the thermal plant, are shown in Fig. 3.

The recursive particle swarm adaptation of the fuzzy PID controller parameters is shown in Fig. 4. In this implementation of the particle swarm adaptation mechanism, the following conditions were adopted:  $\Delta_{\alpha^i} = \Delta_{\beta^i} = 1.0$ ,  $\Delta_{\gamma^i} = 0.01$ ,  $GM_s = 5.0$ ,  $PM_s = 70.0$ ,  $\delta_1 = 0.96$ ,  $\delta_2 = 0.04$ . It can be seen that according to variations in the parameters of the plant, as shown in Fig. 3, the corresponding parameters of the controller, as shown in Fig. 4, were satisfactorily estimated to guarantee the robust stability from the gain and phase margins instantaneously computed, as shown in Fig.5. It can be also observed that the gain margin is greater than or equal to 5 and the phase margin is greater than or equal to  $70^\circ$ , as required previously in the performance criterion.

The temporal response of the thermal plant and the control action are shown in Fig. 6 and Fig. 7, respectively. The initial set point for temperature was  $100^\circ C$ , and a changing to  $80^\circ C$  was applied at time of 5 minutes. A gain variation for the thermal plant of 1.2 and 0.8333 was considered at time of

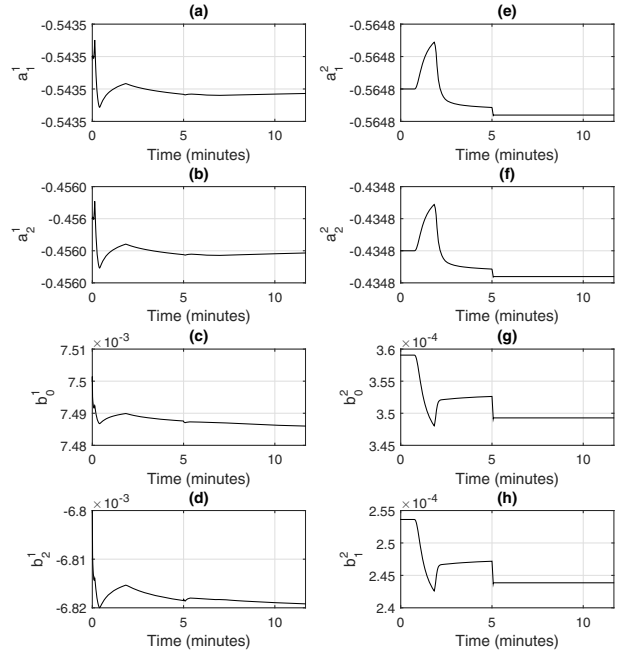


Fig. 3. Recursive parametric estimation: (a)-(d) consequent parameters of the first rule, (e)-(h) consequent parameters of the second rule.

1.8333 minutes and 6.8333 minutes, respectively. It can be seen the efficiency of the proposed methodology through the self-tune of the fuzzy PID controller based on gain and phase margins specifications to guarantee the robust stability in spite of variations in the plant behavior and tracking of the reference signal.

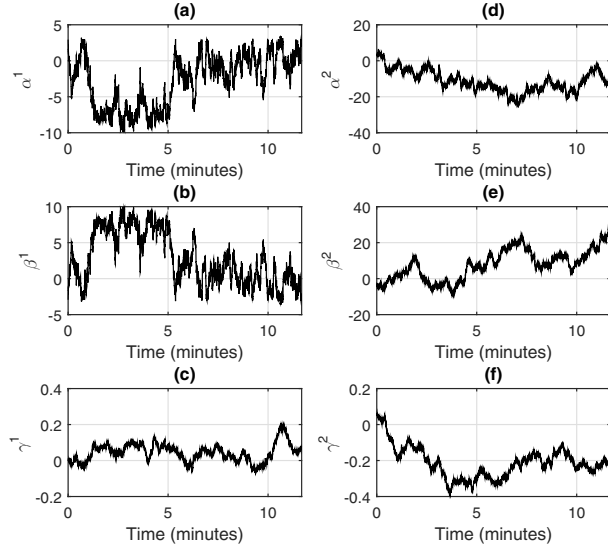


Fig. 4. Recursive estimation fuzzy PID controller parameters: (a)-(c) consequent parameters of the first rule, (d)-(f) consequent parameters of the second rule.

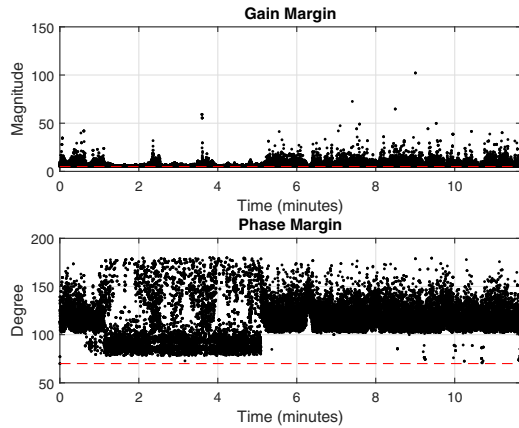


Fig. 5. Gain and Phase Margin Obtained for the Thermal Plant.

#### IV. CONCLUSIONS

In this paper, a self-tuning fuzzy digital control methodology based on gain and phase margins specifications, was proposed. The robust stability was satisfied via particle swarm adaptation mechanism used for self-tune of the fuzzy PID controller, in spite of variations in the plant behavior and track the reference trajectory as well. The development of an “evolving fuzzy system” scheme applied to proposed methodology is of particular interest.

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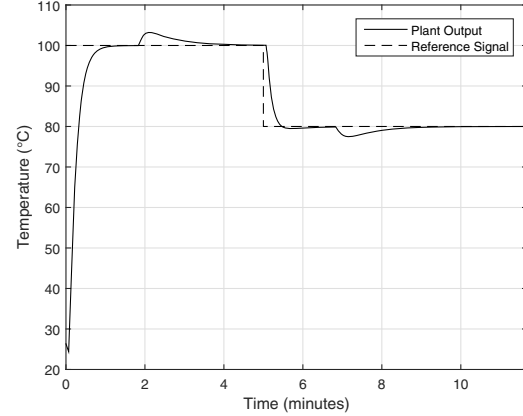


Fig. 6. Temporal response of the adaptive fuzzy PID control system.

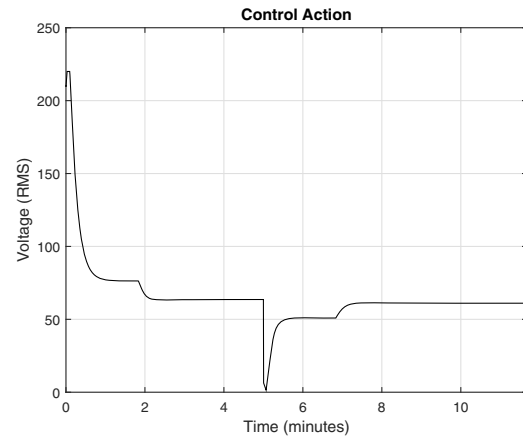


Fig. 7. Control action of the adaptive fuzzy PID controller.

encouraging and development of this research.

#### REFERENCES

- [1] G. Lakhekar and V. Saundarmal, “Novel adaptive fuzzy sliding mode controller for depth control of an underwater vehicles,” in *Fuzzy Systems (FUZZ), 2013 IEEE International Conference on*, July 2013, pp. 1–7.
- [2] R.-J. Wai, M.-W. Chen, and Y.-K. Liu, “Design of adaptive control and fuzzy neural network control for single-stage boost inverter,” *Industrial Electronics, IEEE Transactions on*, vol. 62, no. 9, pp. 5434–5445, Sept 2015.
- [3] Y. Yang, C. Hua, and X. Guan, “Adaptive fuzzy finite-time coordination control for networked nonlinear bilateral teleoperation system,” *Fuzzy Systems, IEEE Transactions on*, vol. 22, no. 3, pp. 631–641, June 2014.
- [4] M. Yue, C. An, Y. Du, and J. Sun, “Indirect adaptive fuzzy control for a nonholonomic/underactuated wheeled inverted pendulum vehicle based on a data-driven trajectory planner,” *Fuzzy Sets and Systems*, pp. –, 2015.
- [5] K. Das Sharma, A. Chatterjee, and A. Rakshit, “Lyapunov hybrid stable adaptive fuzzy tracking control approach for vision-based robot navigation,” *Instrumentation and Measurement, IEEE Transactions on*, vol. 61, no. 7, pp. 1908–1914, 2012.
- [6] C.-L. Hwang, C.-C. Chiang, and Y.-W. Yeh, “Adaptive fuzzy hierarchical sliding-mode control for the trajectory tracking of uncertain underactuated nonlinear dynamic systems,” *Fuzzy Systems, IEEE Transactions on*, vol. 22, no. 2, pp. 286–299, April 2014.

- [7] S. Tong, T. Wang, Y. Li, and B. Chen, "A combined backstepping and stochastic small-gain approach to robust adaptive fuzzy output feedback control," *Fuzzy Systems, IEEE Transactions on*, vol. 21, no. 2, pp. 314–327, April 2013.
- [8] T. Procyk and E. Mamdani, "A linguistic self-organizing process controller," *Automatica*, vol. 15, no. 1, pp. 15 – 30, 1979.
- [9] Y. Yang and J. Ren, "Adaptive fuzzy robust tracking controller design via small gain approach and its application," *Fuzzy Systems, IEEE Transactions on*, vol. 11, no. 6, pp. 783–795, Dec 2003.
- [10] Y. Yang and C. Zhou, "Adaptive fuzzy h<sub>∞</sub> stabilization for strict-feedback canonical nonlinear systems via backstepping and small-gain approach," *Fuzzy Systems, IEEE Transactions on*, vol. 13, no. 1, pp. 104–114, Feb 2005.
- [11] Y. Li, S. Tong, Y. Liu, and T. Li, "Adaptive fuzzy robust output feedback control of nonlinear systems with unknown dead zones based on a small-gain approach," *Fuzzy Systems, IEEE Transactions on*, vol. 22, no. 1, pp. 164–176, Feb 2014.
- [12] P. Huek and O. Cerman, "Fuzzy model reference control with adaptation of input fuzzy sets," *Knowledge-Based Systems*, vol. 49, pp. 116 – 122, 2013.
- [13] K. Saoudi and M. Harmas, "Enhanced design of an indirect adaptive fuzzy sliding mode power system stabilizer for multi-machine power systems," *International Journal of Electrical Power & Energy Systems*, vol. 54, pp. 425 – 431, 2014.
- [14] Q. Lu and M. Mahfouf, "Multivariable self-organizing fuzzy logic control using dynamic performance index and linguistic compensators," *Engineering Applications of Artificial Intelligence*, vol. 25, no. 8, pp. 1537 – 1547, 2012.
- [15] R.-J. Lian, "Design of an enhanced adaptive self-organizing fuzzy sliding-mode controller for robotic systems," *Expert Systems with Applications*, vol. 39, no. 1, pp. 1545 – 1554, 2012.
- [16] J.-S. Chiou, S.-H. Tsai, and M.-T. Liu, "A pso-based adaptive fuzzy pid-controllers," *Simulation Modelling Practice and Theory*, vol. 26, pp. 49 – 59, 2012.
- [17] R. Patel and V. Kumar, "Artificial neuro fuzzy logic pid controller based on bf-pso algorithm," *Procedia Computer Science*, vol. 54, pp. 463 – 471, 2015.
- [18] H. Z. Sabzi, D. Humberson, S. Abudu, and J. P. King, "Optimization of adaptive fuzzy logic controller using novel combined evolutionary algorithms, and its application in diez lagos flood controlling system, southern new mexico," *Expert Systems with Applications*, pp. –, 2015.
- [19] P. Hajebi and S. AlModarresi, "Online adaptive fuzzy logic controller using genetic algorithm and neural network for networked control systems," in *Advanced Communication Technology (ICACT), 2013 15th International Conference on*, Jan 2013, pp. 88–98.
- [20] R. L. Navale and R. M. Nelson, "Use of genetic algorithms and evolutionary strategies to develop an adaptive fuzzy logic controller for a cooling coil comparison of the aflc with a standard pid controller," *Energy and Buildings*, vol. 45, pp. 169 – 180, 2012.
- [21] —, "Use of evolutionary strategies to develop an adaptive fuzzy logic controller for a cooling coil," *Energy and Buildings*, vol. 42, no. 11, pp. 2213 – 2218, 2010.
- [22] H. Bevrani, F. Habibi, P. Babahajyani, M. Watanabe, and Y. Mitani, "Intelligent frequency control in an ac microgrid: Online pso-based fuzzy tuning approach," *Smart Grid, IEEE Transactions on*, vol. 3, no. 4, pp. 1935–1944, 2012.
- [23] O. Sahed, K. Kara, and M. Hadjili, "Pso-based fuzzy predictive control," in *IECON 2012 - 38th Annual Conference on IEEE Industrial Electronics Society*, 2012, pp. 2416–2421.
- [24] M. W. Khan, M. A. Choudhry, M. Zeeshan, and A. Ali, "Adaptive fuzzy multivariable controller design based on genetic algorithm for an air handling unit," *Energy*, vol. 81, pp. 477 – 488, 2015.
- [25] H. A. Hashim, S. El-Ferik, and M. A. Abido, "A fuzzy logic feedback filter design tuned with pso for adaptive controller," *Expert Systems with Applications*, vol. 42, no. 23, pp. 9077 – 9085, 2015.
- [26] A. Hosovsky, P. Michal, M. Tothova, and O. Biros, "Fuzzy adaptive control for pneumatic muscle actuator with simulated annealing tuning," in *Applied Machine Intelligence and Informatics (SAMi), 2014 IEEE 12th International Symposium on*, Jan 2014, pp. 205–209.
- [27] H. Wang, K. Tanaka, and M. Griffin, "Parallel distributed compensation of nonlinear systems by takagi-sugeno fuzzy model," in *Fuzzy Systems, 1995. International Joint Conference of the Fourth IEEE International Conference on Fuzzy Systems and The Second International Fuzzy Engineering Symposium., Proceedings of 1995 IEEE Int*, vol. 2, Mar 1995, pp. 531–538 vol.2.
- [28] R. Jacquot, *Modern Digital Control Systems 2e*, ser. Electrical and Computer Engineering Series. Marcel Dekker, 1995.
- [29] G. L. O. Serra(Ed.), *Highlighted Aspects From Black Box Fuzzy Modeling For Advanced Control Systems Design, Frontiers in Advanced Control Systems*. ISBN: 978-953-51-0677-7: InTech, DOI: 10.5772/45717, 2012.