

Novel Data Knowledge Representation with TSK-type Preprocessed Collaborative Fuzzy Rule based System

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Abstract—A novel data knowledge representation with the combination of structure learning ability of preprocessed collaborative fuzzy clustering and fuzzy expert knowledge of Takagi-Sugeno-Kang type model is presented in this paper. The proposed method divides a huge dataset into two or more subsets of dataset. The subsets of dataset interact with each other through a collaborative mechanism in order to find some similar properties within each-other. The proposed method is useful in dealing with big data issues since it divides a huge dataset into subsets of dataset and finds common features among the subsets. The salient feature of the proposed method is that it uses a small subset of dataset and some common features instead of using the entire dataset and all the features. Before interactions among subsets of the dataset, the proposed method applies a mapping technique for granules of data and centroid of clusters. The proposed method uses information of only half or less/more than the half of the data patterns for the training process, and it provides an accurate and robust model, whereas the other existing methods use the entire information of the data patterns. Simulation results show that proposed method performs better than existing methods on some benchmark problems.

I. INTRODUCTION

Fuzzy logic has been applied successfully in various applications such as vehicle control, agricultural engineering, astronomy, chemistry, geology, image analysis, medical diagnosis, shape analysis and target recognition, consumer products, and control of manufacturing systems etc. Fuzzy logic is one of the most widely use technologies for developing complex feedback control systems for inexpensive processors due to its simplicity and ease in adopting to any environment. Fuzzy logic has the capability of providing a good and robust decision from incomplete information or less available knowledge with perfectly addressing as it acts like human decision making, whereas other techniques such as, linear

control design and purely logic-based models, require exact equations to model real-world behaviors. There are basically two type of fuzzy inference systems: Mamdani type fuzzy inference system model and Takagi-Sugeno-Kang (TSK) type fuzzy inference system model. Both of them contain the expert knowledge of fuzzy logic with their own problem solving capability. Traditionally, Mamdani model [1] is widely accepted as a manner to build expert knowledge. It allows designers to describe the expertise in an intuitive way. However, Mamdani-type fuzzy inference system consumes large amount of effort for computation. On the other hand, Sugeno method [2], which is known as Takagi-Sugeno-Kang (TSK) fuzzy model, is computationally effective and works well with optimization and adaptive techniques. Therefore, TSK type model is very attractive to researchers to solve control problems, particularly for dynamics nonlinear systems. This paper's simulation results are based on TSK type model.

Fuzzy rule based modeling system or fuzzy control rules first extract expert knowledge from its knowledge bases, and then build expert systems. Fuzzy expert system is a combination of rules and membership function, which is generated by fuzzy c-means (FCM) clustering or some other clustering methodology [10-17]. The success of the Fuzzy expert system depends on the quality of acquired knowledge. Fuzzy control rule can be expressed in following way:

- a) IF temperature is very low and pressure is normal, THEN heat change would be slightly positive.
- b) IF it is raining and wind is fast and dark outside, THEN take an umbrella and a torch.

However, in many real situations it is not a good way to combine expert knowledge, and it is necessary to resort the learning system to acquire knowledge. There exist several methods for the learning of fuzzy rules. An example as follows:

suppose we have variety of information with the same patterns from different field applications. To get a comprehensive study of these varieties of information, knowledge based clustering [5] and collaborative clustering between datasets is recommended. Pedrycz [6, 7] introduced a collaborative clustering to solve the problem when some data cannot acquire directly from the dataset due to data confidentiality. In this kind of clustering algorithm, several subsets of patterns can be processed together with an objective of finding a common structure that is shared by all of them. In this study, preprocessed collaborative fuzzy clustering (PCFC) technique is applied to generate a number of rules to calculate the membership function. PCFC has the ability to extract good knowledge from the given unidentified information. We combined PCFC rule learning mechanism with TSK type model, which helps modeling system to design an accurate and robust model.

The rest of the paper is organized as follows: Section II gives introduction of FCM, PCFC, TSK type FIS, the proposed system, and flow diagram of the proposed system. Section III shows the simulation results on two nonlinear dynamic system, and compares the proposed method with the Matlab based Genfis2 method [9]. Finally, the conclusions are covered in Section IV.

II. BASIC PROCEDURE AND PROPOSED ALGORITHM

A. Fuzzy C-Means Clustering

Fuzzy c-mean (FCM) [3, 4] is one of the most common unsupervised clustering algorithms, which is originally described first by Bezdek in 1973. Variants of FCM [19-22] have been described with modified definitions for the norm and prototypes of the cluster centroids [24-26]. FCM clusters each data point to one or more clusters, and partitions a set of data $x_i \in R^d, i = 1, 2, \dots, N$ into a certain number c of fuzzy clusters by minimizing the following cost function.

$$J_m = \sum_{i=1}^N \sum_{j=1}^c u_{ij}^m \|x_i - v_j\|^2 \quad (1)$$

where m is any real number great than 1, u_{ij} is the degree of membership of x_i in the cluster j , x is the i -th data point of d -dimension data, v_j is the d -dimension of the cluster j , and $\|\cdot\|$ is any norm expressing the similarity between any measured data and the center.

B. Procedure of FCM

1. Set up a value of c (number of cluster);
2. Select initial cluster prototype V_1, V_2, \dots, V_c from $X_i, i = 1, 2, \dots, N$;
3. Compute the distance $\|X_i - V_j\|$ between objects and prototypes;

4. Compute the elements of the fuzzy partition matrix ($i = 1, 2, \dots, N; j = 1, 2, \dots, c$)

$$u_{ij} = \left[\sum_{l=1}^c \left(\frac{\|x_i - v_l\|}{\|x_i - v_j\|} \right)^{\frac{1}{m-1}} \right]^{-1} \quad (2)$$

5. Compute the cluster prototypes ($j = 1, 2, \dots, c$)

$$V_j = \frac{\sum_{i=1}^N u_{ij}^2 x_i}{\sum_{i=1}^N u_{ij}^2} \quad (3)$$

6. Stop if the convergence is attained or the number of iterations exceeds a given limit. Otherwise, go to step 3

C. Takagi-Sugeno-Kang Fuzzy Inference System

TSK fuzzy inference model proposed by Takagi, Sugeno, and Kang [2, 18], has been widely used in control and fuzzy modeling. The key idea of TSK is to divide input space into several fuzzy regions, and approximate each region by a simple model. All the systems can be regarded as a combination of a series of simple models. In general, TSK can be represented in mathematical form as

$$R_i: \text{IF } x_1(t) = A_{1,j}^{(i)} \text{ AND } \dots \text{ AND } x_n(t) = A_{n,j}^{(i)}, \\ \text{THEN } y_s^{(i)} = f_s^{(i)}(x(t)),$$

where R_i is i -th rule in TSK model, t denotes a sampling instant, and $X_s(t) = [x_1(t), \dots, x_s(t)]^T$ is input vector. Each $x_k \in R$, where $k = 1, \dots, s$, $A_{k,j}^{(i)}$ the j fuzzy set characterized by the i -th rule corresponding to the input x_k , y_s is the output of overall model, and $f_s(X(t))$ is a first-order polynomial, can be computed as

$$f_s(X(t), b^i) = \frac{\sum_{k=1}^{R_i} \alpha_s^{(i)} f^{(i)}(x_1, \dots, x_s)}{\sum_{i=1}^{R_i} \alpha_s^{(i)}} \quad (4)$$

R_i is the total number of rules, and $\alpha_s^{(i)}$ is the degree of matching between the i -th fuzzy and the s -th sample. $f^{(i)}(x_1, \dots, x_s)$ can be represented as

$$b_{0,s}^i + b_{1,s}^i x_1(t) + \dots + b_{s-1,s}^i x_s(t), \quad (5)$$

where $b_{k,s}^i \in R$. By above definition, $R_i = 2$ and

$$\alpha_s^{(i)} = \min \{ \mu_{A_{1,j}^{(i)}} x_1', \mu_{A_{2,j}^{(i)}} x_2' \}. \quad (6)$$

Fig. 1 shows the computation process of TSK inference model. Ting and Quek [19] demonstrated a more complicated TSK inference model. Typical cost function J_{TSK} measures how the TSK model approximates the real problem. The J_{TSK} can be expressed as

$$J_{TSK} = \sum_{t=1}^N D^2(t), \quad (7)$$

where $D(t)$ is the difference of output between the real system and the identified model, and N is the number of training samples.

A generic TSK inference model can be divided into four parts.

1. Partition the input space into r inference rules.
2. Identify the structure of each IF part.
3. Identify the constitution of each THEN part.
4. Calculate the predicted value.

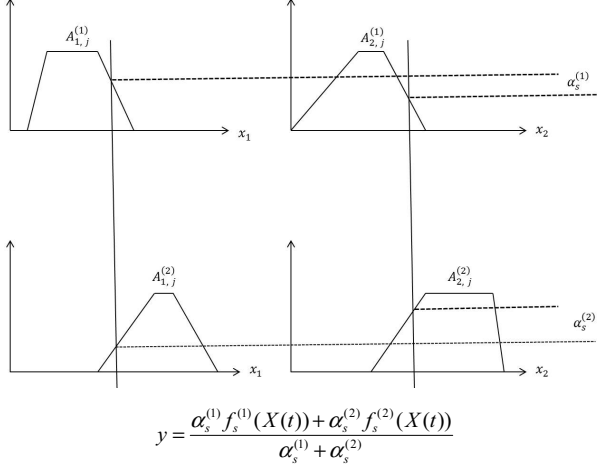


Fig.1 TSK inference model

D. Preprocessed Collaborative Fuzzy Clustering

Pedrycz [6] introduced a collaborative fuzzy clustering (CFC) to find the consistency between two or more datasets. Its variants: horizontal [8] and vertical collaborative fuzzy clustering [23], also exists. The minimization of objective function of CFC is defined as

$$Q[l] = \sum_{i=1}^N \sum_{j=1}^c u_{ij}^2[l] d_{ij}^2[l] + \sum_{\substack{m=1 \\ m \neq l}}^p \beta[l, m] \sum_{i=1}^N \sum_{j=1}^n \{u_{ij}[l] - u_{ij}[m]\}^2 d_{ij}^2[l] \quad (8)$$

where β is a user defined parameter based on datasets ($\beta > 0$), $\beta[l, m]$ denotes the collaborative coefficient with collaborative effect on dataset l through m , c is a number of cluster, $l=1, 2, \dots, P$ is a number of datasets, N is the number of patterns in the dataset, u represents the partition matrix, n is a

number of features, and d is an Euclidean distance between patterns and prototypes.

Preprocessed collaborative fuzzy clustering proposed by Prasad [27] is a mapping mechanism for prototype and partition matrix before collaboration phase. Since direct subtraction of $u_{ik}[l]$ and $u_{ik}[m]$ may lose the meaning of difference between two membership degrees $u_{ik}[l]$ and $u_{ik}[m]$ under different partition matrices of one pattern X_k to the same cluster, we have to find a constructive approach of the preprocessing in order to rearrange the rows order of $u_{ik}[l]$ corresponding to the rows order of $u_{ik}[m]$ in a rational way. The match rows pair is determined by

$$r = \arg \min_{j=1, 2, \dots, c} \sum_{i=1}^n (v_{ki}[l] - v_{ji}[m])^2 \quad (9)$$

where n is the number of features. The k -th row of $v[l]$ and the r -th row of $v[m]$ are considered to be matched row pair ($k=1, 2, \dots, c$). Similarly, this value is updated with $u_{ik}[l]$ and $u_{ik}[m]$.

The optimization task as shown in Eq. (8) is divided into two main parts those determine the partition matrix $U[l]$, and $v_1[l]$, $v_2[l]$, \dots , $v_c[l]$. These determination problems are calculated separately for each of the collaborating subsets of patterns. The Lagrange multipliers technique is used to determine the partition matrix in order to make constraint-free optimization. Preprocessed collaborative fuzzy partitioning is carried out through an iterative optimization of the objective function as shown in Eq. (8) with an update of partition matrix $U[l]$ and the prototype $v_i[l]$ as shown in Eq. (10) and Eq. (13) respectively. For optimization details please refer to [5, 6].

$$u_{st}[l] = \frac{\varphi_{st}[l]}{1 + \psi[l]} + \frac{1}{\sum_{j=1}^c d_{st}^2[l]} \left[1 - \sum_{j=1}^c \frac{\varphi_{jt}[l]}{1 + \psi[l]} \right], \quad (10)$$

where

$$\varphi_{st}[l] = \sum_{\substack{m=1 \\ m \neq l}}^p \beta[l, m] u_{st}[m] \quad (11)$$

$$\psi[l] = \sum_{\substack{m=1 \\ m \neq l}}^p \beta[l, m] \quad (12)$$

$$v_{st}[l] = \frac{\sum_{k=1}^N u_{sk}^2[l] x_{kt}[l] + \sum_{\substack{m=1 \\ m \neq l}}^p \beta[l, m] \sum_{k=1}^N (u_{sk}[l] - u_{sk}[m])^2 x_{kt}[l]}{\sum_{k=1}^N u_{sk}^2[l] + \sum_{\substack{m=1 \\ m \neq l}}^p \beta[l, m] \sum_{k=1}^N (u_{sk}[l] - u_{sk}[m])^2} \quad (13)$$

E. Procedure for PCFC

Based on the above discussions and the results, PCFC adds one more phase called phase II for mapping procedure before collaboration process and present the refined algorithm as follows:

1. Given: subsets of patterns X_1, X_2, \dots, X_p .
 2. Select: distance function, number of clusters (c), termination condition, and collaboration coefficient $\beta[l, m]$.
 3. Compute: initiate randomly all partition matrices $u[1], u[2], \dots, u[p]$
- Phase I
 For each data
 Repeat
 Compute prototype $\{v_j[l], j=1, 2, \dots, c$ and partition matrices $u[l]$ for all subsets of patterns}
 Until a termination condition has been satisfied
- Phase II
 Choose an approach for the preprocessing on cluster prototype and its corresponding partition matrices to adjust row order by using (9).
- Phase III
 Repeat
 For the matrix of collaborative links $\beta[l, m]$.
 Compute, prototype $v_j[l]$ and partition matrices $u[l]$ by using Eq. (10) and (13).
 Until a termination condition has been satisfied

Fig. 2 shows a general way of dividing a huge dataset into N numbers of equal data sites. Fig. 3 shows the representation of N number of classes for two datasets: *dataset1* and *dataset2*, after FCM. Here it can easily visualize how rows pair are mismatched and the mismatch could lead the system in a different direction for analyzing the data. Fig. 4 shows the correctness of N number of classes matching of *dataset1* and *dataset2* after applying the mapping mechanism for prototypes and partition matrices. In order to verify the mapping mechanism, the proposed method has used the paradigm of three classes, and then divided them equally into two subsets of dataset: *dataset1* and *dataset2*. Fig. 5(a) and 5(b) are clustered feature vectors of *dataset1* and *dataset2*, respectively. As it can be seen, in fig. 5(a) and 5(b), the first cluster (green color) of *dataset1* matches with the second cluster of *dataset2*, the second cluster (red color) of *dataset1* matches with the third cluster of *dataset2*, and the third cluster (blue color) of *dataset1* matches with the first cluster of *dataset2* - which are totally mismatched with each other. Fig. 5(c) and 5(d) show the plotting results after the mapping mechanism, where the effect of centroid mapping for prototype and row order mapping with the partition matrix can be seen. Now, we can easily take the difference(s) between rows of *dataset1* and *dataset2*, and easily do mapping between them.

F. Architecture of Proposed Model

The proposed method, shown in Fig. 6, combines the reasoning strengths of TSK type fuzzy inference system with the

knowledge representation ability of mapped collaborative fuzzy cluster, and gives a robust and reliable modeling system. Firstly, the given input data is divided into two or more equally sub groups of dataset, and FCM is applied on each sub-groups of dataset separately to calculate prototypes and partition matrix for each datasets. Secondly, PCFC updates all partition matrix and prototype by collaborating each of them and gets a common feature among them, and provides these features to the knowledge based sub system of fuzzy inference system. Thirdly, the inference engine uses the knowledge from the knowledge based sub system along with fuzzier information of given dataset. Instead of providing the entire data patterns, the proposed method just uses half of the data patterns for further modeling process, while other methods use the entire data patterns. By using just half of the data patterns, the proposed method is able to provide better or similar performance compare to methods, those use entire data patterns. The proposed method takes less computation time during training phase.

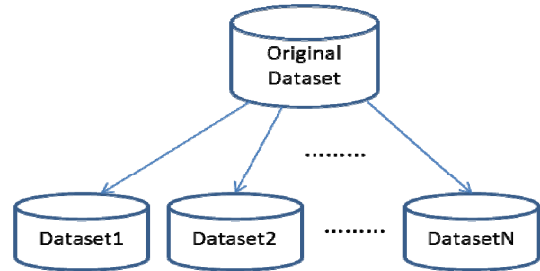


Fig. 2 Division of an original dataset into N different datasets

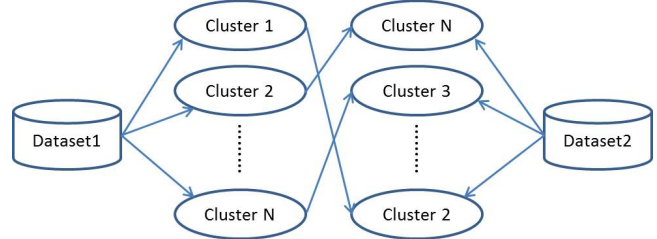


Fig. 3 Representation of N classes for *dataset1* and *dataset2* after FCM

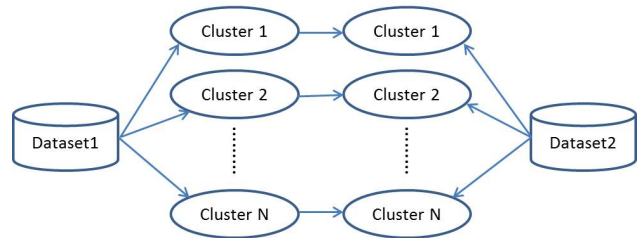
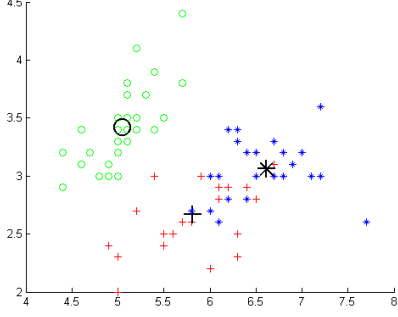
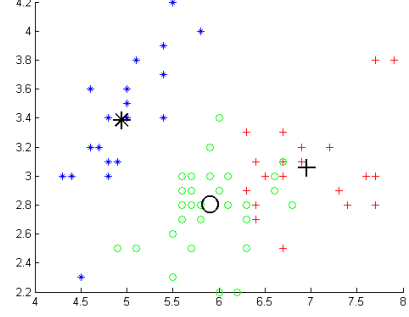


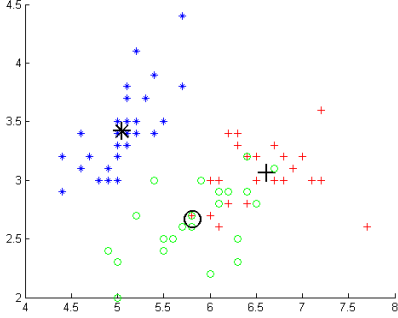
Fig. 4 Representation of N classes for *dataset1* and *dataset2* after preprocessing



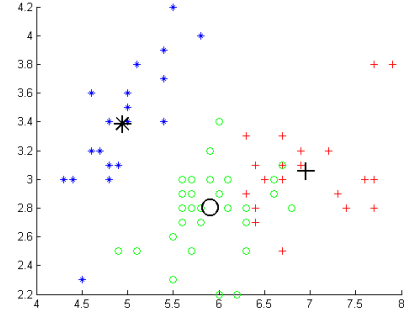
(a) Clustered feature vectors of *dataset1* based on FCM



(b) Clustered feature vectors of *dataset2* based on FCM



(c) Clustered feature vectors of *dataset1* based on FCM after mapping



(d) Clustered feature vectors of *dataset2* based on FCM after mapping

Fig. 5 Clustered feature vectors of *dataset1* and *dataset2*

III. SIMULATION RESULTS

A. Nonlinear Dynamics System Identification Problem-I

A nonlinear dynamics system identification problem is considered to illustrate the effect of the proposed method. The plant to be recognized is defined as

$$y(t+1) = \frac{y(t)y(t-1)[y(t)+2.5]}{1+y^2(t)+y^2(t-1)} + u(t) \quad (14)$$

If a series-parallel identification model is used for recognizing the plant, the model can be defined as

$$\hat{y}(t+1) = f\{y(t), y(t-1), u(t)\} \quad (15)$$

where $y(t+1)$ is the output and $u(t)=\sin(2\pi t/25)$ is the input; and this network includes three inputs and one output. The initial input values are considered as follows: $y(0)=0$ and $y(1)=0$. For each training and testing purpose, a set of 1000 data are extracted for this system identification problem.

Fig. 7, 8 and 9 show the output surface of given fuzzy inference system (FIS) using the first two inputs and the output of given dataset. Fig. 7 shows the output surface plot of FIS of Genfis3 by using the entire data patterns. Fig. 8 and 9 show the output surface plot of FIS of the proposed method by using the just half of the data patterns for each dataset. Fig. 8 (a) and (b)

show the surface plot when *dataset1* collaborates with *dataset2* without any mapping of cluster center and with mapping of cluster center, respectively. Similarly, Fig. 9 (a) and (b) show the surface plot when *dataset2* collaborates with *dataset1* without any mapping of cluster center and with mapping of cluster center, respectively.

B. Nonlinear Dynamics System Identification Problem-II

The nonlinear system to be recognized is defined as

$$y(t+1) = \frac{y(t)}{1+y^2(t)} + u^3(t) \quad (16)$$

where $y(t+1)$ is the output and $u(t)$ is the input signal that is generated by using the sinusoidal function given by $u(t)=\sin(2\pi t)/100$. The inputs $y(t)$ and $u(t)$ follow the uniform sample distribution in the interval $[-1.5, 1.5]$ and $[-1.0, 1.0]$, respectively.

For each training and testing purpose, a set of 400 for each data patterns are generated, respectively. Further, training dataset is divided into two datasets: *dataset1* and *dataset2*, those contain 200 patterns each. The proposed method uses the knowledge representation of 200 patterns of *dataset1/dataset2* for network training after applying the PCFC procedure. Table I shows the performance comparisons of the proposed method

with different values of collaborative coefficient (β) and Matlab based Genfis2 model. When *dataset1* collaborates with *dataset2*, the best average training and testing RMSE value is 0.0068 and 0.0070 for $\beta=0.08$, respectively. When *dataset2* collaborates with *dataset1*, the best average training and testing RMSE value is 0.0071 and 0.0070 for $\beta=2$, respectively. While for Genfis2

model, the training and testing RMSE value is 0.0134 and 0.0135, respectively. Fig. 7 shows desired and predicted output during training and testing at $\beta=1$ when *dataset1* collaborates with *dataset2*. Fig. 8 shows desired and predicted output during training and testing at $\beta=0.08$ when *dataset2* collaborates with *dataset1*.

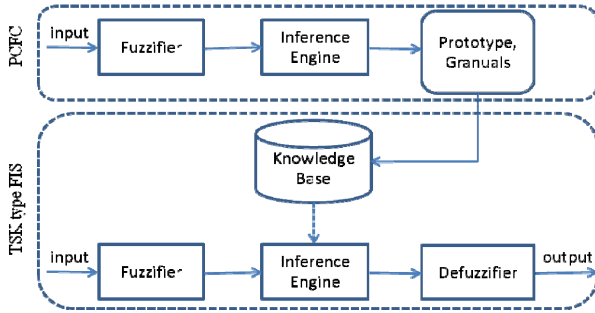


Fig. 6 Architecture of proposed method

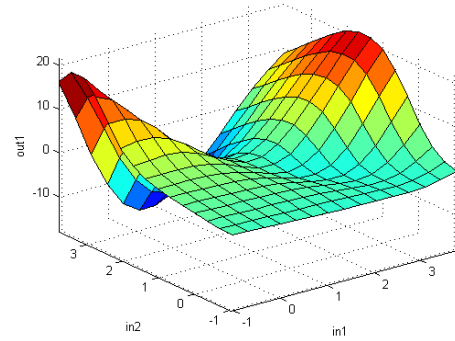
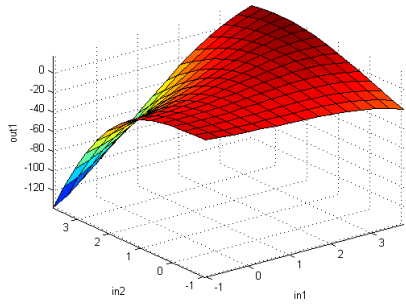
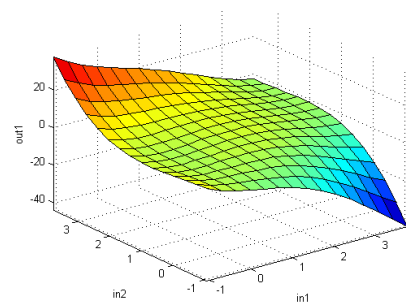


Fig. 7 Surface plot of FIS for FCM on the entire data pattern

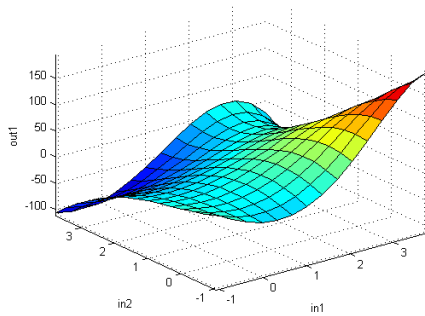


(a) *Dataset1* collaborates with *dataset2* without mapping

Fig. 8 Surface plot of FIS for CFC and PCFC for $\beta=0.5$

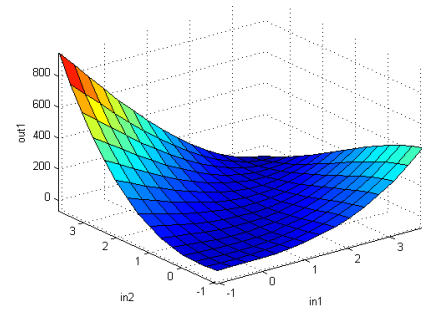


(b) *Dataset1* collaborates with *dataset2* with mapping



(a) *Dataset2* collaborates with *dataset1* without mapping

Fig. 9 Surface plot of FIS for CFC and PCFC for $\beta=0.5$



(b) *Dataset2* collaborates with *dataset1* with mapping

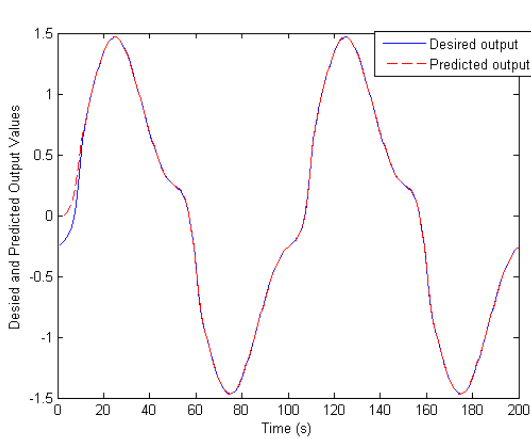
IV. CONCLUSIONS

In this paper, we proposed a new modeling strategy for TSK type fuzzy inference system based on collaborative fuzzy rule learning with cluster center mapping technique. The proposed method helps system modeling to find an accurate model based on given input information by using knowledge representation

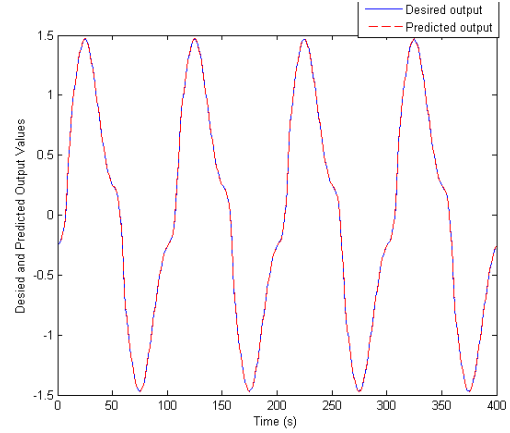
of PCFC. The proposed method is able to provide better or similar performance while using just halve of the given data patterns for training and keeping the lower computation time. For future work, we want to extend our work and compare with some other existing modeling systems with real world datasets and apply this model to deal with big data issue.

TABLE I. Training and testing RMSE of the proposed method and Genfis2

Model	Proposed Method (<i>dataset1</i> → <i>dataset2</i>)		Proposed Method (<i>dataset2</i> → <i>dataset1</i>)		Genfis2	
Process	Training RMSE	Testing RMSE	Training RMSE	Testing RMSE	Training RMSE	Testing RMSE
β	Mean	Mean	Mean	Mean	Mean	Mean
1	0.0082	0.0084	0.0079	0.0078	0.0134	0.0135
2	0.0101	0.0103	0.0071	0.0070	0.0134	0.0135
3	0.0084	0.0086	0.0072	0.0071	0.0134	0.0135
4	0.0088	0.0089	0.0091	0.0091	0.0134	0.0135
5	0.0097	0.0099	0.0096	0.0095	0.0134	0.0135
6	0.0095	0.0097	0.0065	0.0065	0.0134	0.0135
7	0.0106	0.0108	0.0090	0.0090	0.0134	0.0135
8	0.0098	0.0100	0.0091	0.0091	0.0134	0.0135
9	0.0093	0.0094	0.0079	0.0079	0.0134	0.0135
10	0.0099	0.0101	0.0073	0.0073	0.0134	0.0135
0.1	0.0084	0.0086	0.0088	0.0088	0.0134	0.0135
0.2	0.0092	0.0094	0.0077	0.0077	0.0134	0.0135
0.3	0.0085	0.0087	0.0092	0.0092	0.0134	0.0135
0.4	0.0091	0.0093	0.0079	0.0079	0.0134	0.0135
0.5	0.0103	0.0106	0.0087	0.0087	0.0134	0.0135
0.6	0.0090	0.0092	0.0097	0.0097	0.0134	0.0135
0.7	0.0082	0.0084	0.0087	0.0087	0.0134	0.0135
0.07	0.0088	0.0090	0.0112	0.0112	0.0134	0.0135
0.08	0.0068	0.0070	0.0095	0.0095	0.0134	0.0135
0.09	0.0081	0.0083	0.0085	0.0085	0.0134	0.0135

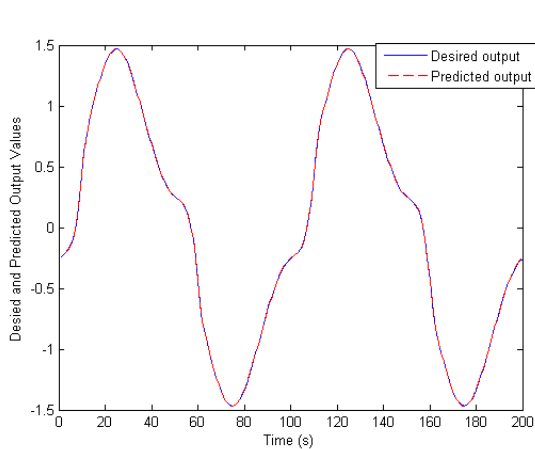


(a) Desired and predicted outputs during training

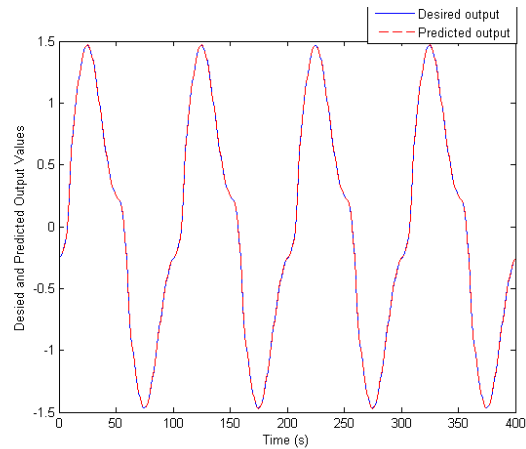


(b) Desired and predicted outputs during testing

Fig. 10 Desired and predicted outputs during training and testing at $\beta=1$ when *dataset1* collaborates with *dataset2*



(a) Desired and predicted outputs during training



(b) Desired and predicted outputs during testing

Fig. 11 Desired and predicted outputs during training and testing at $\beta=0.08$ when *dataset2* collaborates with *dataset1*

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