Abstract—In the framework of \( \Omega \)-sets, where \( \Omega \) is a complete lattice, we generalize the notion of a (universal) algebra, and to investigate its basic properties. Our techniques belong to the \( \Omega \)-algebras, equipped to is a complete lattice.

\( \Omega \)-algebras

I. INTRODUCTION

The topic of this research are \( \Omega \)-valued algebraic structures, where \( \Omega \) is a complete lattice.

Our research originates in both, in fuzzy structures and in \( \Omega \)-sets. Formally fuzzy set theory was found in 1965., by Zadeh’s known paper [21], and has become a developed theory since then. \( \Omega \)-sets appeared 1979., in the paper [10] by Fourman and Scott. Introducing \( \Omega \)-sets, they intended to model intuitionistic logic. An \( \Omega \)-set is a nonempty set \( A \) equipped with an \( \Omega \)-valued equality \( E \), with truth values in a complete Heyting algebra \( \Omega \). \( E \) is a symmetric and transitive function from \( \mathbb{E}^2 \) to \( \Omega \). \( \Omega \)-sets have been further applied to non-classical predicate logics, and also to theoretical foundations of Fuzzy Set Theory ([12], [14]).

We use \( \Omega \)-sets and in our approach \( \Omega \) is a complete lattice (not necessarily a Heyting algebra, or some other residuated lattice). The main reason for choosing this membership values structure is our usage of lattice operations meet and join (and not some additional operations existing in residuated lattices). Namely, these operations allow us to use cut-sets as a tool. In this setting, main algebraic and set-theoretic notions and their properties can be generalized from their classical origin (appearing on cut structures) to the lattice-valued framework ("cutworthy" properties, see [16]). This is not the case if other operations in residuated lattices are applied: if the operations are not meet and join, then many properties of cuts could not be transferred, generalized to fuzzy structures. So we deal also with lattice-valued sets. These were developed within the Fuzzy Set Theory in which the unit interval has been replaced by a complete lattice (firstly by Goguen [11]). This approach is widely used for dealing with algebraic topics (see e.g., [9], then also [18], [19]), and with lattice-valued topology (starting with [15] and many others). In the recent decades, along with the development of fuzzy logic, a complete lattice as a membership (truth values) structure is often replaced by a complete residuated lattice (see e.g., [1]).

A lattice-valued equality generalizing the classical one has been introduced in fuzzy mathematics by Hohle in [13], (see also [14]), and then it was used in investigations of fuzzy functions and fuzzy algebraic structures by many authors, in particular by Demirci ([8]), Bělohlávek and V. Vychodil ([2]) and others. Compatible fuzzy relations were also investigated from the early period (see, e.g., Murali ([17])).

Idsentities for lattice-valued structures with fuzzy equality were introduced in [20], and then developed in [3], [4], [5], [6]. In this framework, an identity holds if the corresponding lattice-theoretic formula is fulfilled. What is new in this approach is that an identity may hold on a lattice-valued algebra, while the underlying classical algebra does not satisfy the analogue classical identity.

II. PRELIMINARIES

A. Lattices, algebras

A partially ordered set \( \langle \Omega, \leq \rangle \), where every subset \( M \) has both a meet \( \bigwedge M \) and a join \( \bigvee M \) is a complete lattice. A complete lattice possesses the least and the greatest elements 0 and 1, respectively. A meet and a join of a two-element subset \( \{a, b\} \) of \( \Omega \) are binary operations, denoted by \( a \land b \) and \( a \lor b \), respectively.

A language (or a type) \( L \) is a set \( F \) of functional symbols, together with a set of natural numbers (arities) associated to these symbols. An algebra of type \( L \) is a pair \( (A, F) \), denoted by \( A \), where \( A \) is a nonempty set and \( F \) is a set of (fundamental) operations on \( A \). An \( n \)-ary operation in \( F \) corresponds to an \( n \)-ary symbol in the language. A subalgebra of \( A \) is an algebra of the same type, defined on a subset of \( A \), closed under the operations in \( F \). Terms in a language are regular expressions constructed by the variables and operational symbols (see [7]). If \( t(x_1, \ldots, x_n) \) is a term in the language of an algebra \( A \), then by \( t^A \) we denote the corresponding term-operation \( A^n \to A \) on \( A \) (as usual, \( t^A \) is obtained by replacing all functional symbols in \( t \) by the corresponding fundamental operations on \( A \), and variables by elements from \( A \)). An identity in a language is a formula \( t_1 \approx t_2 \), where \( t_1, t_2 \) are terms in the same language. An identity \( t_1(x_1, \ldots, x_n) \approx t_2(x_1, \ldots, x_n) \) is said to be valid on an algebra \( A = (A, F) \), or that \( A \) satisfies this identity, if for all \( a_1, \ldots, a_n \in A \), the equality \( t_1^A(a_1, \ldots, a_n) = t_2^A(a_1, \ldots, a_n) \) holds. An equivalence relation \( \rho \) on \( A \) which is compatible with respect to all fundamental operations, meaning that \( x_1 \rho y_1, \ldots, x_n \rho y_n \) imply \( f(x_1, \ldots, x_n) \rho f(y_1, \ldots, y_n) \), is a congruence relation on \( A \).
B. Ω-valued functions and relations

Throughout the paper, \((\Omega, \wedge, \vee, \leq)\) is a complete lattice with the top and the bottom elements 1 and 0 respectively.

An **Ω-valued function** \(\mu\) on a nonempty set \(A\) is a mapping \(\mu : A \to \Omega\).

For \(p \in \Omega\), a cut set or a \(p\)-cut of an \(\Omega\)-valued function \(\mu : A \to \Omega\) is a subset \(\mu_p\) of \(A\) which is the inverse image of the principal filter in \(\Omega\), generated by \(p\):

\[
\mu_p = \mu^{-1}(\uparrow(p)) = \{x \in A \mid \mu(x) \geq p\}.
\]

An **Ω-valued** (binary) relation \(R\) on \(A\) is an \(\Omega\)-valued function on \(A^2\), i.e., it is a mapping \(R : A^2 \to \Omega\).

\(R\) is symmetric if

\[
R(x, y) = R(y, x) \text{ for all } x, y \in A; \tag{1}
\]

\(R\) is transitive if

\[
R(x, y) \geq R(x, z) \land R(z, y) \text{ for all } x, y, z \in A. \tag{2}
\]

Let \(\mu : A \to \Omega\) and \(R : A^2 \to \Omega\) be an \(\Omega\)-valued function an \(\Omega\)-valued relation on \(A\), respectively. Then we say that \(R\) is an **Ω-valued relation on \(\mu\)** if for all \(x, y \in A\)

\[
R(x, y) \leq \mu(x) \land \mu(y). \tag{3}
\]

An \(\Omega\)-valued relation \(R\) on \(\mu : A \to \Omega\) is said to be reflexive on \(\mu\) if

\[
R(x, x) = \mu(x) \text{ for every } x \in A. \tag{4}
\]

A symmetric and transitive \(\Omega\)-valued relation \(R\) on \(A\), which is reflexive on \(\mu : A \to \Omega\) is an **Ω-valued equivalence** on \(\mu\).

Observe that an \(\Omega\)-valued equivalence \(R\) on \(A\) fulfills the **strictness property** (see [14]):

\[
R(x, y) \leq R(x, x) \land R(y, y). \tag{5}
\]

A **Ω-valued equivalence** \(R\) on \(A\) is an **Ω-valued equality**, if it satisfies the **strong separation property**:

\[
R(x, y) = R(x, x) \text{ implies } x = y. \tag{6}
\]

**Remark 1:** In [10] and then also in [14], the separation property is introduced by \(E(x, y) = 1\) implies \(x = y\). Obviously, the strong separation implies the separation.

A **lattice-valued subalgebra** of an algebra \(A = (A, F)\), here an **Ω-valued subalgebra** of \(A\) is a function \(\mu : A \to \Omega\) which is not constantly equal to 0, and which fulfills the following: For any operation \(f\) from \(F\) with arity greater than 0, \(f : A^n \to A, n \in \mathbb{N}\), and for all \(a_1, \ldots, a_n \in A\), we have that

\[
\bigwedge_{i=1}^{n} \mu(a_i) \leq \mu(f(a_1, \ldots, a_n)), \tag{7}
\]

and for a nullary operation \(c \in F\), \(\mu(c) = 1\). \tag{8}

Proposition 1: Let \(\mu : A \to \Omega\) be an \(\Omega\)-valued subalgebra of an algebra \(A\) and let \((a_1, \ldots, a_n)\) be a term in the language of \(A\). If \(a_1, \ldots, a_n \in A\), then the following holds:

\[
\bigwedge_{i=1}^{n} \mu(a_i) \leq \mu(t^A(a_1, \ldots, a_n)). \tag{9}
\]

\(\Box\)

An \(\Omega\)-valued relation \(R : A^2 \to \Omega\) on an algebra \(A = (A, F)\) is **compatible** with the operations in \(F\) if the following two conditions holds: for every \(n\)-ary operation \(f \in F\), for all \(a_1, \ldots, a_n, b_1, \ldots, b_n \in A\), and for every constant (nullary operation) \(c \in F\)

\[
\bigwedge_{i=1}^{n} R(a_i, b_i) \leq R(f(a_1, \ldots, a_n), f(b_1, \ldots, b_n)); \tag{10}
\]

\[
R(c, c) = 1. \tag{11}
\]

III. **Ω-algebras**

A. **Ω-set**

The following is defined in [10], and then adopted to a fuzzy framework in [6].

An **Ω-set** is a pair \((A, E)\), where \(A\) is a nonempty set, and \(E\) is a symmetric and transitive \(Ω\)-valued relation on \(A\), fulfilling the strong separation property (6).

For an **Ω-set** \((A, E)\), we denote by \(\mu\) the \(Ω\)-valued function on \(A\), defined by

\[
\mu(x) := E(x, x). \tag{12}
\]

We say that \(\mu\) is determined by \(E\). Clearly, by the strictness property, \(E\) is an \(Ω\)-valued relation on \(\mu\), namely, it is an \(Ω\)-valued equality on \(\mu\). That is why we say that in an **Ω-set** \((A, E)\), \(E\) is an **Ω-valued equality**.

Recall that by the definition of a cut, for \(p \in \Omega\), a \(p\)-cut of \(E : A^2 \to \Omega\) is a binary relation on \(A\) given by

\[(x, y) \in E_p \text{ if and only if } E(x, y) \geq p.\]

**Lemma 1:** If \((A, E)\) is an **Ω-set** and \(p \in \Omega\), then the cut \(E_p\) is an equivalence relation on the corresponding cut \(\mu_p\) of \(\mu\).

**Proof:** We prove reflexivity of \(E_p\) on \(\mu_p\): \((x, x) \in E_p\) if and only if \(E(x, x) = \mu(x) \geq p\), if and only if \(x \in \mu_p\). Similarly, one could prove symmetry and transitivity. \(\blacksquare\)

B. **Ω-algebra:** identities

Next we introduce a notion of a lattice-valued algebra with a lattice valued equality.

Let \(A = (A, F)\) be an algebra and \(E : A^2 \to \Omega\) an **Ω-valued equality** on \(A\), which is compatible with the operations in \(F\). Then we say that \((A, E)\) is an **Ω-algebra**. Algebra \(A\) is the underlying algebra of \((A, E)\).

Next we present some cut properties of **Ω-algebras**. These have been proved in [6], in the framework of groups.

**Proposition 2:** Let \((A, E)\) be an **Ω-algebra**. Then the following hold:

(i) The function \(\mu : A \to \Omega\) determined by \(E\) \((\mu(x) = E(x, x)\) for all \(x \in A\)) is an **Ω-valued subalgebra** of \(A\).
(ii) For every \( p \in \Omega \), the cut \( \mu_p \) of \( \mu \) is a subalgebra of \( A \), and

(iii) For every \( p \in \Omega \), the cut \( E_p \) of \( E \) is a congruence relation on \( \mu_p \).

Next we define how identities hold on \( \Omega \), according to the approach in [20].

Let and \( w(x_1, \ldots, x_n) \approx v(x_1, \ldots, x_n) \) (briefly \( u \approx v \)) be an identity in the type of an \( \Omega \)-algebra \((A, E)\). We assume, as usual, that variables appearing in terms \( u \) and \( v \) are from \( x_1, \ldots, x_n \). Then, \((A, E)\) satisfies identity \( u \approx v \) (i.e., this identity holds on \((A, E)\)) if the following condition is fulfilled:

\[
\bigwedge_{i=1}^{n} \mu(a_i) \leq E(u^A(a_1, \ldots, a_n), v^A(a_1, \ldots, a_n)),
\]

for all \( a_1, \ldots, a_n \in A \) and for the term-operations \( u^A \) and \( v^A \) on \( \mathcal{A} \) corresponding to terms \( u \) and \( v \) respectively.

Hence, identity \( u \approx v \) holds on \((A, E)\), if the inequality (13) is fulfilled in the lattice \( \Omega \) whenever variables are replaced by elements from \( A \), and the operational symbols are replaced by the corresponding operations.

If \( \Omega \)-algebra \((A, E)\) satisfies an identity, then this identity need not hold on \( \mathcal{A} \). On the other hand, if the supporting algebra fulfills an identity then also the corresponding \( \Omega \)-algebra does.

**Proposition 3:** [6] If an identity \( u \approx v \) holds on an algebra \( \mathcal{A} \), then it also holds on an \( \Omega \)-algebra \((A, E)\).

**Proof:** Suppose that \( x_1, x_2, \ldots, x_n \) are variables appearing in terms \( u, v \). If \( u \approx v \) holds on \( A \), then for any \( a_1, \ldots, a_n \in A \), \( u^A(a_1, \ldots, a_n) = v^A(a_1, \ldots, a_n) \), hence \( E(u^A(a_1, \ldots, a_n), v^A(a_1, \ldots, a_n)) = E(u^A(a_1, \ldots, a_n), u^A(a_1, \ldots, a_n)) = \mu(u(a_1, \ldots, a_n)) \geq \bigwedge_{i=1}^{n} \mu(a_i) \).

In the following we analyze basic properties of \( \Omega \)-algebras and how they are related to properties of the corresponding underlying algebras.

Let \((A, E)\) be an \( \Omega \)-algebra.

First we deal with nullary operations (constants), if they exist in \( A \). By formulas (8) and (11), if \( c \) is a constant in \( A \) (determined by a nullary operation in the language), then \( E(c, c) = \mu(c) = 1 \). The reason for this requirement included in the definition of compatibility is based on the following property.

**Lemma 2:** If \((A, E)\) is an \( \Omega \)-algebra and \( c \in F \) a constant nullary fundamental operation on \( A \), then \( E(c, c) \geq E(x, x) \), for every \( x \in A \).

**Proof:** Recall that we denote by \( \mu : A \rightarrow \Omega \) the \( \Omega \)-valued mapping defined by \( \mu(x) = E(x, x) \). By Proposition 2 (ii), for every \( p \in \Omega \), we have that \( \mu_p \) is a subalgebra of \( A \). Therefore, being of the same type as \( A \), \( \mu_p \) should contain \( c \). Hence, for every \( p \in \Omega \), \( x \in \mu_p \), if \( x \in \mu_p \) then also \( c \in \mu_p \). Therefore, for every \( x \in A \), \( E(x, x) = \mu(x) \leq \mu(c) = E(c, c) \).

**Proposition 4:** Let \((A, E)\) be an \( \Omega \)-algebra. For every term \( u(x_1, \ldots, x_n) \) in the language of \( A \),

\[
\mu(u^A(c_1, \ldots, c_n)) = 1,
\]

where \( u^A \) is the term operation on \( A \) corresponding to the term \( u \), and \( c_1, \ldots, c_n \) are constants (not necessarily different) from the set \( F \) of fundamental operations of \( A \).

**Proof:** Since \( \mu \) is an \( \Omega \)-valued subalgebra of \( A \), by (9) we have

\[
\mu(u^A(c_1, \ldots, c_n)) \geq \mu(c_1) \land \ldots \land \mu(c_n) = 1 \land \ldots \land 1 = 1.
\]

If \( u(x_1, \ldots, x_n) \) is a term in the language of an algebra \( A \), then a term operation \( u^A(x_1, \ldots, x_n) \) on \( A \) is idempotent, if \( A \) satisfies the identity \( u(x, \ldots, x) \approx x \).

**Theorem 1:** A term operation \( u^A(x_1, \ldots, x_n) \) is idempotent on an \( \Omega \)-algebra \((A, E)\) if and only if it is idempotent on \( A \).

**Proof:** If the identity \( u(x, \ldots, x) \approx x \) holds on \( A \), then for all \( x \in A \), \( u^A(x, \ldots, x) = x \), hence \( E(x, u^A(x, \ldots, x)) = \mu(x)\) and by (13), the same identity holds also on \((A, E)\).

Conversely, suppose that the identity \( u(x, \ldots, x) \approx x \) holds on \((A, E)\) i.e., let \( E(x, u^A(x, \ldots, x)) \geq \mu(x) \) for all \( x \in A \). Since \( E(x, u^A(x, \ldots, x)) \leq \mu(x) \) due to (3), we have \( E(x, u^A(x, \ldots, x)) = \mu(x) \), and by the strong separation property we conclude that for all \( x \in A \), we have \( u^A(x, \ldots, x) = x \).

Our main results are characterizations of \( \Omega \)-algebras in terms of subalgebras, congruence relations and classical quotient structures. Recall that for an \( \Omega \)-algebra \((A, E)\), the function \( \mu : A \rightarrow \Omega \), \( \mu(x) = E(x, x) \) is an \( \Omega \)-subalgebra of \( A \). Further, for every \( p \in \Omega \), the cut \( \mu_p \) is a subalgebra of \( A \), and the cut relation \( E_p \) is a congruence relation on \( \mu_p \).

**Theorem 2:** Let \((A, E)\) be an \( \Omega \)-algebra, and \( F \) a set of identities in the language of \( A \). Then, \((A, E)\) satisfies all identities in \( F \) if and only if for every \( p \in \Omega \) the quotient algebra \( \mu_p / E_p \) satisfies the same identities.

**Proof:** Let \((A, E)\) be an \( \Omega \)-algebra, fulfilling the set \( F \) of identities. For \( p \in \Omega \), consider the quotient algebra \( \mu_p / E_p \) of the subalgebra \( \mu_p \) of \( A \) over the congruence \( E_p \) on \( \mu_p \). We prove that this classical algebra satisfies all identities in \( F \).

Let \( u(x_1, \ldots, x_n) \approx v(x_1, \ldots, x_n) \) be an identity from \( F \). By assumption, for all \( x_1, \ldots, x_n \in A \), and for the corresponding term operations \( u^A, v^A \), we have

\[
\mu(x_1) \land \ldots \land \mu(x_n) \leq E(u^A(x_1, \ldots, x_n), v^A(x_1, \ldots, x_n)).
\]

In particular, for \( x_1, \ldots, x_n \in \mu_p \), we have

\[
\mu(x_1) \land \ldots \land \mu(x_n) \geq p,
\]

hence

\[
E(u^A(x_1, \ldots, x_n), v^A(x_1, \ldots, x_n)) \geq p,
\]
and
\[(u^A(x_1, \ldots, x_n), v^A(x_1, \ldots, x_n)) \in E_p,\]
since \(E_p\) is a congruence relation on the subalgebra \(\mu_p\). Therefore, these values belong to the same congruence class, in other words
\[\forall u^A(x_1, \ldots, x_n), \forall v^A(x_1, \ldots, x_n) \in E_p,\]
and by the compatibility of \(E_p\)
\[u^A((x_1)_E_p, \ldots, (x_n)_E_p) = v^A((x_1)_E_p, \ldots, (x_n)_E_p).\]
Therefore, the identity \(u \approx v\) holds on \(\mu_p/E_p\).

Conversely, assume that for every \(p \in \Omega\), the quotient algebra \(\mu_p/E_p\) satisfies every identity from \(\mathcal{F}\), i.e., if \(u \approx v\) is an identity from \(\mathcal{F}\), then for all \(x_1, \ldots, x_n\in \mu_p\), we have
\[u^A((x_1)_E_p, \ldots, (x_n)_E_p) = v^A((x_1)_E_p, \ldots, (x_n)_E_p),\]
then obviously
\[\forall u^A(x_1, \ldots, x_n), \forall v^A(x_1, \ldots, x_n) \in E_p,\]
For arbitrary \(x_1, \ldots, x_n \in A\), we take \(\mu(x_1) \wedge \ldots \mu(x_n) = p\), and by the above we get
\[E(u^A(x_1, \ldots, x_n), v^A(x_1, \ldots, x_n)) \geq p = \mu(x_1) \wedge \ldots \mu(x_n),\]
which proves that the identity \(u \approx v\) holds on \((A, E)\).

Next we prove that for special \(\Omega\)-equalities, the underlying algebra satisfies a set of identities if the corresponding \(\Omega\)-algebra does.

We call an \(\Omega\)-equality \(E : A^2 \rightarrow \Omega\) quasi-diagonal if for all \(x, y \in A\), such that \(x \neq y\),
\[E(x, y) \leq \bigwedge_{t \in A} E(t, t).\]

Theorem 3: Let \((A, E)\) be an \(\Omega\)-algebra, where \(E\) is a quasi-diagonal \(\Omega\)-equality. If \((A, E)\) satisfies a set of identities \(\mathcal{F}\), then also the underlying algebra \(A\) satisfies the same identities.

Proof: By the strong separation property (6), for every \(x \in A\), we have \(E(x, x) \neq 0\). Hence, since \(E\) is a quasi-diagonal \(\Omega\)-equality, it follows that for \(q = \bigwedge_{x \in A} \mu(x), \mu_q = A\), and \(E_q = \{(x, x) \mid x \in A\} = \Delta\) i.e., \(E_q\) is the equality relation \(\Delta\) on \(A\). By Theorem 2, for every \(p \in \Omega\), the quotient algebra \(\mu_p/E_p\) fulfills all identities in \(\mathcal{F}\). In particular, \(\mu_q/E_q\) fulfills these identities. Therefore, \(\mu_q/E_q = A/\Delta\). Obviously, \(A/\Delta\) is isomorphic with \(A\), hence \(A\) satisfies all identities in \(\mathcal{F}\).

IV. \(\Omega\)-SUBALGEBRA

Let \((A, E)\) be an \(\Omega\)-algebra, and \(E_1 : A \rightarrow \Omega\) a symmetric and transitive \(\Omega\)-relation on \(A\), so that the following holds: for all \(x, y \in A\)
\[E_1(x, y) = E(x, y) \wedge E_1(x, x) \wedge E_1(y, y)\]
(14)
Let also \(E_1\) be compatible with the operations in \(A\). Obviously, \((A, E_1)\) is an \(\Omega\)-algebra and we say that it is an \(\Omega\)-subalgebra of \((A, E)\).

The proof of the following proposition is straightforward, due to the definition \(\mu_1(x) = E_1(x, x)\), and since \(E_1\) is compatible with the operations in \(A\).

Proposition 5: If \((A, E)\) is an \(\Omega\)-subalgebra of an \(\Omega\)-algebra \((A, E), \) and \(\mu_1 : A \rightarrow \Omega\) is an \(\Omega\)-valued function on \(A\), defined by \(\mu_1(x) := E_1(x, x)\), then \(\mu_1\) is an \(\Omega\)-valued subalgebra of \(A\), i.e., it fulfills (7) and (8).

Next we prove that an \(\Omega\)-subalgebra \((A, E_1)\) of \((A, E)\) fulfills all the identities that the latter does.

Theorem 4: Let \((A, E_1)\) be an \(\Omega\)-subalgebra of an \(\Omega\)-algebra \((A, E)\). If \((A, E)\) satisfies the set \(\Sigma\) of identities, then also \((A, E_1)\) satisfies all identities in \(\Sigma\).

Proof: Let \(u \approx v\) be an identity from \(\Sigma\), with variables \(x_1, \ldots, x_n\). Then, since \(u \approx v\) holds in \((A, E)\), by the definition of \(E_1\) and the fact that it is compatible with operations on \(A\), by the definition of \(\mu_1\), and by Proposition 1, for all \(a_1, \ldots, a_n \in A\), we have
\[\bigwedge_{i=1}^{n} \mu_1(a_i) = \bigwedge_{i=1}^{n} E_1(a_i, a_i) = \bigwedge_{i=1}^{n} E(a_i, a_i) \wedge \bigwedge_{i=1}^{n} E_1(a_i, a_i) = \bigwedge_{i=1}^{n} \mu(a_i) \wedge \bigwedge_{i=1}^{n} E_1(a_i, a_i) \leq \bigwedge_{i=1}^{n} \mu_1(a_i) \leq \bigwedge_{i=1}^{n} E_1(a_i, a_i) \wedge \bigwedge_{i=1}^{n} \mu_1(a_i) \leq \bigwedge_{i=1}^{n} E_1(a_i, a_i) \wedge \bigwedge_{i=1}^{n} \mu_1(a_i) = \bigwedge_{i=1}^{n} E_1(a_i, a_i).\]

V. CONCLUSION

The paper introduces a new type of algebraic structures, in the framework of lattice-valued sets and a suitable fuzzy equality replacing the classical one. We deal with general properties of these algebras, using lattice-valued (fuzzy) identities. It turns out that this approach enables investigation of structures which do not satisfy identities (commutativity, associativity etc.), but their quotient cut substructures do.

The next task would be investigation of the corresponding congruences, homomorphisms and product structures.

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