Role of Boundary Dynamics in Improving Efficiency of Particle Swarm Optimization on Antenna Problems

(Invited Paper)

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***Abstract*— Attainment of global optimal solution and reduction of computational time and resources have always been a tradeoff issue in formulation of nature inspired algorithms; this tradeoff challenge has brought in a deluge of new algorithms proposing their efficacy over one another. Particle swarm optimization (PSO) algorithm and its variants are quite popular in optimizing antenna designs particularly due to their algorithmic simplicity and fast convergence rates. However, as a matter of the ever present tradeoff, fast convergence is often immature and leads to sub optimal solutions; another problem which may lead to inferior performances is the issue of stagnation. In some recent researches it has been shown that a class of boundary handling algorithms of PSO, known as position regulated boundary conditions (PRBCs), can minimize the tradeoff between 'immature fast convergence' and accomplishment of 'global optimal solution'; the problem of stagnation is also avoided in most of these cases. Here, the performance of these boundary algorithms are compared with that of other established ones over three different optimization targets of antenna designs namely, "Multi target optimization in linear antenna arrays", "Inset feed position optimization of rectangular patch antennas" and "Edge feed position optimization of rectangular patch antennas". Results show that the use of most of the PRBCs in PSO leads to impressive improvement in the optimization efficiency in terms of lesser computational time and attainment of global optimal solution.**

I. Introduction

Swarm algorithms being a subset of meta-heuristic optimization techniques find variety of applications in communication engineering [1-5,9-11,32-41] or science and technology in general [3-11]. The length and breadth of these applications are so vast that it is impossible to outline them in a single write-up. Among the myriad of these applications, we focus on the area of antenna optimization to fit into the aims and scopes of the special session on "*Nature Inspired Antenna Systems*". Moreover, antenna systems draw a considerable attention of researchers due to their complex and diverse design requirements apart from their indispensable role in the area of wireless communications. In antenna optimization problems the solution spaces are quite often multimodal in nature; therefore, nature inspired algorithms show better optimization results than their classical counterparts. Optimization techniques based on swarm intelligence are in use for over two decades now; particularly, techniques like PSO have drawn larger interest due to its easy implementation and faster computational time compared to other popular techniques like Genetic Algorithm (GA) or Differential Evolution (DE). The canonical form of PSO shown in Eqn.1 in general suffers from premature convergence and stagnation; several researches have analyzed these problems across various objective functions and benchmarks [12-18].This paper doesn't delve into detailed analysis of the aforesaid problems but it does provide a physical picture of those and suggests algorithmic changes to mitigate them.

Most of the practical optimization targets are seen as constrained problems where the entire solution space can be divided into feasible and infeasible regions; the interface between these regions may be termed as a boundary. Some of the techniques assume constrained optimization as part of unconstrained one and impose a penalty function [19] or a constriction factor [20]. However, most of the algorithms which deal with constrained optimizations try to control and confine the particles within the boundary or the feasible solution space [21-30]; different boundary conditions play a crucial role in PSO optimization of antenna array problems. Since its beginning, PSO with different boundary considerations has been applied to variety of problems [21-30] including those of electromagnetics and antennas [24-30]. Boundary behaviors of PSO for electromagnetics and antenna problems were first analyzed in [24]; ever since then, a particular class of boundary behavior which primarily regulates the velocities of the particles became the standard of boundary consideration. [24, 25] conclusively showed that condition which does not try to confine the particles in the solution space, i.e. invisible boundary, fetches better optimization results; nevertheless, an analysis of these boundary conditions shows that the particles which move out of the feasible solution space may be brought back for their increased participation in the solution finding process and better optimal solution. This leads to the proposition of a new class of boundary algorithms called position regulated boundary conditions (PRBCs) [34, 35]. A recent article [30] on analysis of different bound handling techniques also finds the standard conditions of [24, 25] as inferior. PRBCs do not control or regulate the velocity of the particles; instead, they reposition the particles, which go out of bounds, back into the solution space right in the next iterative cycle. One of the earlier cases of position regulated boundary was the hard boundary differentiated against soft boundary

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[10]; later an effective position regulated boundary called periodic boundary condition (PBC) was reported in [17] and applied to antenna array problems in [11]. Here, the various significant boundary conditions, including the one focused here, are compared with each other; the physics of PRBCs are analyzed and compared with those of velocity regulation algorithms. It is also shown that how significantly the PRBCs influence the inertia factor of PSO algorithm to obtain a quicker and matured convergence. Before beginning with the optimization, a brief review on optimization process and targets is presented where different ways of cost/fitness function formulation in context of antenna problems are described.

II. The Boundary Dynamics in PSO

A. Comparison of Boundary Conditions

Though velocity regulated boundary conditions have been the most established ones in context of PSO, they have some intrinsic problems as described ahead. For example, the

$$v_n = \omega * v_n + c_1 \text{rand}() * (p_{best,n} - x_n) + c_2 \text{rand}() * (g_{best,n} - x_n) \quad (1)$$

$$\text{when } x_n \rightarrow g_{best,n} \text{ and } p_{best,n} \text{ then } v_n \rightarrow v_{minimum,n}$$

maximum velocity (V_{max}) has to be heuristically set between 10-20% of the dynamic range of each dimension [2] or may be equal to that dynamic range [2].Other techniques suggests an alternate expression of velocity with a constriction factor[20] or new velocity updating mechanism [22]. Despite enforcement of such conditions, the particles with any finite velocity may go beyond the solution space. The more modern approaches, like absorbing or damping boundary [24, 25], may lead to unnecessary accrual of particles near the boundary and also adversely affect the particles intelligence of finding the global minimum (i.e. particularly, when the global minimum isn't near the boundary). Moreover, forced minimization of the velocity (as shown in Eqn.1) of a particle can let it conclude that it has reached its g_{best} and p_{best}. Such a situation is show in Fig.1 where either the velocity is reduced to zero at a fixed boundary or particles' positions are truncated to the fixed boundary; here the fixation is at 0.8 and 0.1 for a two variable optimization target; amplitudes of six element linear antenna arrays were optimized.

Few more conditions, described in [24, 25], assign a negative velocity to the particles which go out of solution space and hence bring them back; such conditions have to work outside the boundary and can't immediately reposition the particles inside but would rather take some unknown iterative steps to do so. Unprecedented oscillations in the particles may also be seen when negative velocities are assigned. Random velocities also fail to reposition the particles immediately into the solution space [23]. Assigning higher velocities can also take more iterative steps for the particles to converge. The better approach of boundary conditions shown in [24, 25] is that of invisible wall/boundary or floating boundary [21]; the problem is, a significant number of particles may escape the solution space and those which escape can't contribute to further exploration and hence reduce the chances of finding an optimal global minimum.

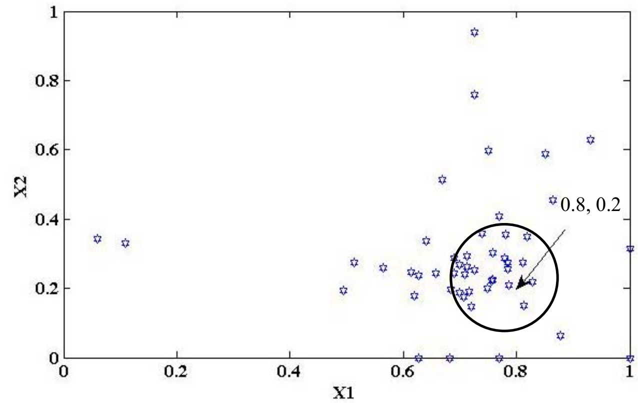

Fig.1. Accumulation of particles near a fixed boundary in a two dimensional solution space

It may now be readily concluded that velocity regulation in some situations can give rise to unwanted results in the PSO process. The invisible boundary described in [24-25] is almost an equivalent of floating/ no-boundary [21]. Therefore, more suitable boundary conditions need to be sought. Position regulation has the advantage of immediately relocating the particles within the solution space thus avoiding additional iterations and working beyond the boundaries. Control or regulation of the velocities of the particles need not be done either; however, both positions and velocities may be regulated to give a hybrid boundary condition [28].

B. Position Regulated Boundary Conditions

Before the classification of PRBCs some of these appeared in [27-28, 36]. Few possible PRBCs are described in this subsection with a comparison of their convergence characteristics in Fig.3 where six isotropic elements linear antenna array was used for side lobe reduction. The convergence results are averaged over fifty independent runs. Fig 2 pictorially represents the different PRBCs. *Fixed Position Relocation:* It is the simplest of all PRBCs where the particles are repositioned to particular fixed values within the solution space. These values are usually the boundary positions as in [27] or may be different. The intrinsic weakness of this method is that the particles here are easily trapped into a local minimum near the boundary as shown in Fig 1. An uncontrolled accumulation, or at times stagnation [12-18] of these particles, may be seen near the boundary limits; however, this method can outperform other boundary regulation methods when the global minimum is near the boundary. *Random Position Relocation:* To avoid the unintended accumulation of particles near the boundary in the preceding case, a random repositioning of particles is suggested here.

Random repositioning of particles helps in better exploration of the solution space and therefore has clear advantages over the fixed repositioning method to find the global minimum solution. Nevertheless, due to its random nature the method is rather inconsistent in its behavior when compared with other methods. *Symmetric Position Relocation*: First proposed in [34], this method relocates the particles escaping a boundary to an equidistant position '*l*' inside that boundary as shown in Fig.2(c).

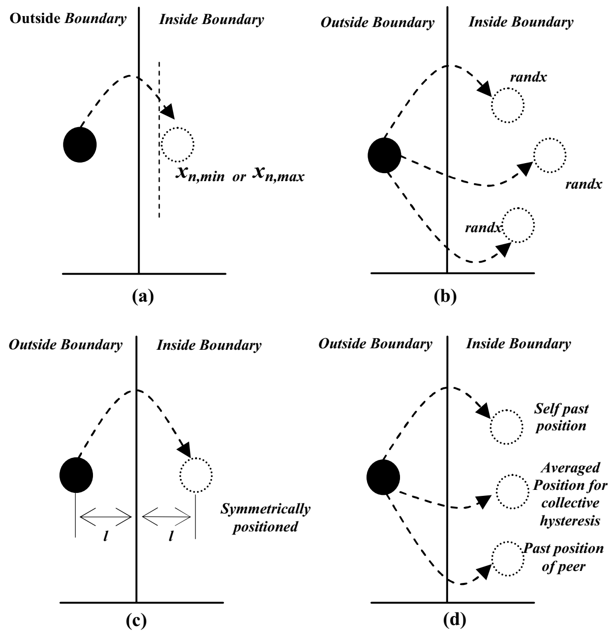

Fig.2. (a) Fixed/Hard Boundary (b) Random Boundary (c) Symmetric Boundary (d) Hysteretic Boundary

Here both the lower and higher boundary behave as mirrors where the distance of the escaped particle from the respective boundary outside the solution space is calculated; the particle is then repositioned inside the solution space with the same distance from its boundary.

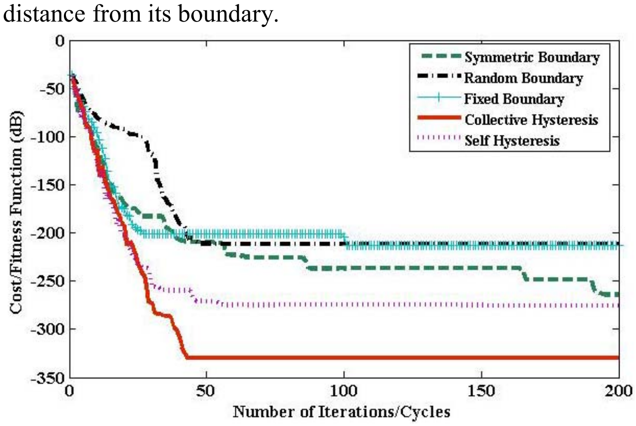

Fig.3. Comparison of convergence characteristics of different PRBCs

The method, though similar, is significantly different from the PBC (Periodic Boundary Condition) and is shown to perform better than it in an antenna optimization problem [34]. Lastly, another category of PRBCs were introduced in [35] known as *Hysteretic Boundary Conditions:* this category of algorithms reposition the errant particles back to their previous or hysteretic positions inside the boundary; two sub classifications within this category were defined as self and collective hysteretic conditions shown in Fig. 2(d); these were found to be most promising in the optimization targets described here.

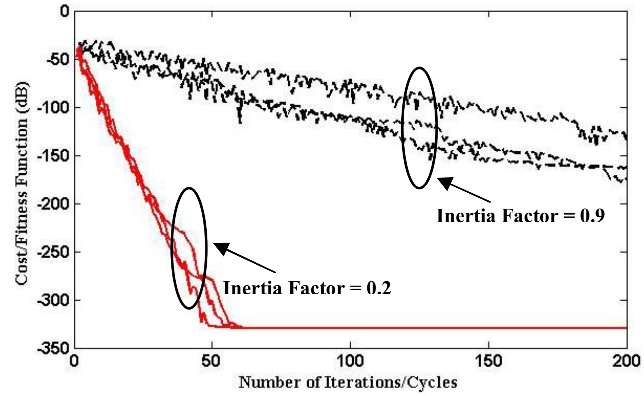

Fig.4. Convergence characteristics of collective hysteresis for three independent runs with inertia factors of 0.2 and 0.9.

Fig.4 shows the convergence results of collective hysteresis (with a fixed weight of 0.8) over three independent runs. It is very important to note here that a lower inertia factor of 0.2 gives quicker and matured convergence to the global optimum after around 50 iterations where as a factor 0f 0.9 is not able to converge even after 200 iterations; this is strikingly different from the established fact that 0.9 inertia factor is most suitable for good optimization results [24, 25]. The results are similar for other PRBCs. In usual circumstances it is quite obvious that a higher inertia factor shall take more number of iterations to converge; however, in such situations the convergence is matured and apparently reaches the global optimum unlike the present case where it doesn't seem to converge at all within the stipulated number of iterations. It may be interpreted from this result that due to the increased number of participation from the errant particles, which are brought back inside the boundary, chance of finding the global solution increases within lesser number of iterations. Therefore, our hypothesis of utilizing the errant particles to explore further solutions seems to be correct as opposed to the interpretations drawn in [24-25]. The weight factor of *collective hysteresis* is fixed to '0.8' in all the results.

III. Optimization Procees and Targets

Optimization problems within the broad purview of electromagnetics and antenna systems are fairly intricate. It has been shown in few research reports that some metaheuristic algorithms which show an excellent performance in several theoretical benchmark functions are not so impressive in antenna optimization targets [38-41]; this indicates that the algorithms should be benchmarked in their respective areas of application before claiming their superiority in the area; certainly, this aspect is in congruity with the famous no free lunch theorem [37]. Meta-heuristic optimization technique in general involves two major processing blocks namely the 'Optimization Algorithm' and the 'Cost/Fitness Function' as shown in Fig.5. Search spaces in most of the antenna optimization problems are either large, multimodal or complex and sometimes a combination of these. Therefore, to make the algorithms more effective, some optimization techniques initially reduces the search space by developing a coarse model before developing a finer model as in space-mapping [42] or at times a sub-optimal search space is obtained before metaheuristic algorithms work on them [43].

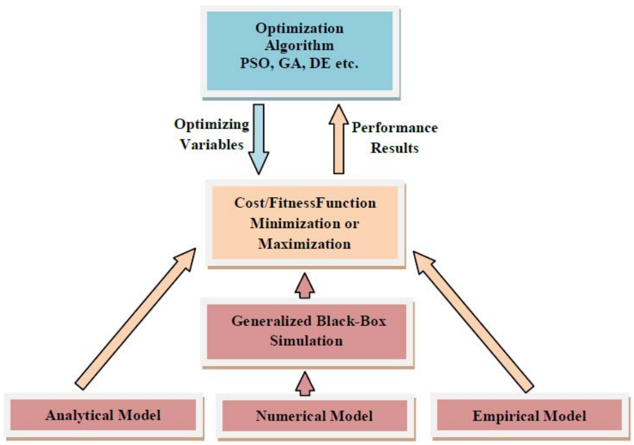

Fig.5. Optimization process with different ways of cost/fitness function formulation

To begin with an optimization problem, one has to formulate the cost/fitness function; this function is based on a mathematical model and targets minimization or maximization of a particular output/result. Mathematical models of antenna systems can be analytical, empirical or numerical depending on the complexity of the system (Fig.5), these models are mostly confined to a particular application or design situation, a small change in the physical structure of an antenna may require a completely new mathematical model. However, these models are able to provide physical insights into the design problems in hand. Modern systems demand versatility and higher design accuracies leading to the use of black-box simulation tools, like CST or HFSS, in the optimization process; these tools have a remarkable ability to model any electromagnetic structure from user provided inputs and numerical modelling. In an optimization process these black-box simulation tools replace the role of analytical or empirical models mentioned before. Such tools make an optimization process almost autonomous that seldom require any expert human intervention; nevertheless, the vital engineering requisite of drawing a physical insight and developing a fundamental understanding of a problem is almost lost in such cases. In the subsequent sections three optimization examples are taken which shall touch the aforesaid categories of mathematical modelling and also demonstrate the optimization efficiency of the boundary algorithms in consideration.

IV. Multi-target Optimization in Linear Antenna Arrays

Linear antenna arrays have been one of the easiest and most popular areas where optimization algorithms have been extensively used for radiation pattern synthesis. However, the scope of fixing multi target or multi criteria optimization is relatively less explored. Here, we take the example of antenna array factor, an analytical model, which shall be optimized for multiple targets or goals specified below.

- Symmetric windowed null placements.
- Overall symmetric side lobe reduction
- Minimization of FNBW between optimized and un-optimized patterns.

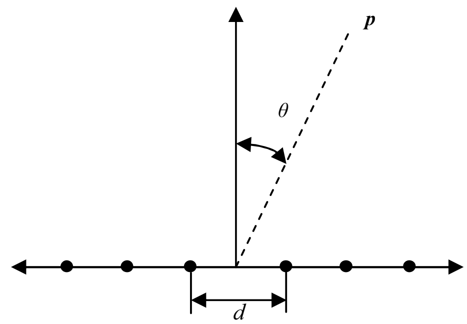

Fig.6. A linear antenna array

Here a linear antenna array of twelve elements is considered where the optimization variables are the individual amplitude of the elements and the uniform inter-element spacing. Fig.6 shows the topology of linear arrays. Eqn.2 shows the array factor [31] used to formulate the fitness function and Fig.7 compares the performance of boundary algorithms over one another. The optimized and the un-optimized array factor plots are compared in Fig.8; it can be seen that an overall side lobe reduction of more than -35dB is attained and a symmetric windowed nulls bellow -55dB are placed from 42^0 to 62^0 and from 118^0 to138^0. The FNBW also remains intact. The optimized uniform spacing is found to be 0.88λ.

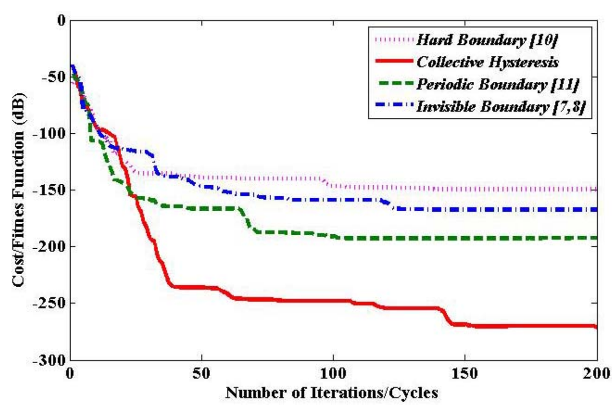

Fig.7. Convergence characteristics of collective hysteresis with other established boundary algorithms in linear antenna array example.

$$AF(u) = \frac{1}{\max_u \{|AF(u)|\}} \sum_{n=1}^{N} w_n e^{j2\pi ndu/\lambda} \quad (2)$$

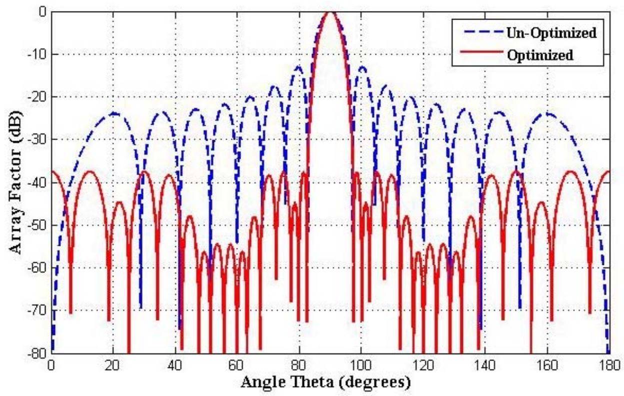

Fig.8. Array Factor plot of un-optimized and optimized linear antenna array with twelve isotropic elements.

V. Feed Position Optimization of Rectangular Patch Antennas

Optimization of feed position in microstrip patch antennas is important to achieve better impedance match resulting in minimization of return loss and maximization of the radiated power. However, analytical closed form models are not available in these cases and the designs are done using empirical models. In usual practice, further approximations need to be made in the empirical expressions so that those are reduced to simple equations; these approximations further compromise the accuracy of a design. Empirical results for an inset feeds are well established; therefore, these relations are taken here, without approximation, to device the cost function for minimization of return loss (S_{11}) indirectly from impedance matching; this example is taken as the first in this section. Secondly, the example of edge feed is illustrated where black-box simulation is used to minimize the S_{11} due to the unavailability of proper empirical formulation; though an edge feed can be modeled as a stepped impedance [46], good empirical relation to include the offset positions of the feed-line along the patch edge are not reported. In the second example an interface is established between MATLAB and CST microwave studio, using VBA based macro programming [44], to establish interoperability. The convergence characteristics shown in the examples are averaged over 50 independent runs.

A. Inset Position Optimization

Rectangular patch antenna with an inset feed is one of the most established feed line design technique but due to approximations in the traditional approach there is no room for multivariable optimization.Here, both y_0 and W_0 of Fig.9 act as the optimization parameters for the impedance matching cost /fitness function shown in Eqn.5

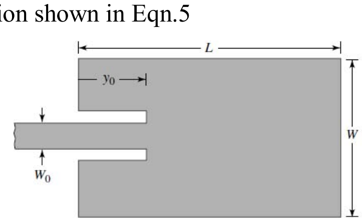

Fig.9. Rectangular patch antenna array with inset feed.

The idea is to minimize the difference between the impedance of the feed-line Z_c and at the inset point $R_{in}\,(y = y_0)$. Eqn.3 and Eqn. 4 are taken from [45] where the different parameters carry their usual meanings described in [45]. The impedance matching cost function is given below. It is worth mentioning here that the approximations done in Eqn. 3 in the traditional approach [22] is avoided here; moreover, feed line variations give another degree of freedom to the optimization process.

$$Z_c = \begin{cases} \dfrac{60}{\sqrt{\epsilon_{\text{reff}}}} \ln\left[\dfrac{8h}{W_0} + \dfrac{W_0}{4h}\right], & \dfrac{W_0}{h} \le 1 \\ \dfrac{120\pi}{\sqrt{\epsilon_{\text{reff}}}\left[\dfrac{W_0}{h} + 1.393 + 0.667 \ln\left(\dfrac{W_0}{h} + 1.444\right)\right]}, & \dfrac{W_0}{h} > 1 \end{cases} \quad (3)$$

$$R_{in}(y = y_0) = \frac{1}{2(G_1 \pm G_{12})}\left[\cos^2\left(\frac{\pi}{L}y_0\right) + \frac{G_1^2 + B_1^2}{Y_c^2}\sin^2\left(\frac{\pi}{L}y_0\right) - \frac{B_1}{Y_c}\sin\left(\frac{2\pi}{L}y_0\right)\right] \quad (4)$$

$$F_{\cos t/fitness} = \min\left\{\left|R_{in} - Z_c\right|\right\} \quad (5)$$

The performance of different boundary algorithms evaluated over the aforesaid cost function is given in Fig.10.

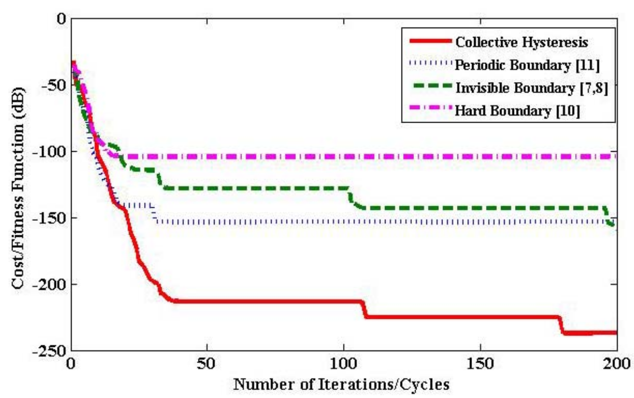

Fig.10.Convergence characteristics of collective hysteresis with other established boundary algorithms in inset fed rectangular patch example.

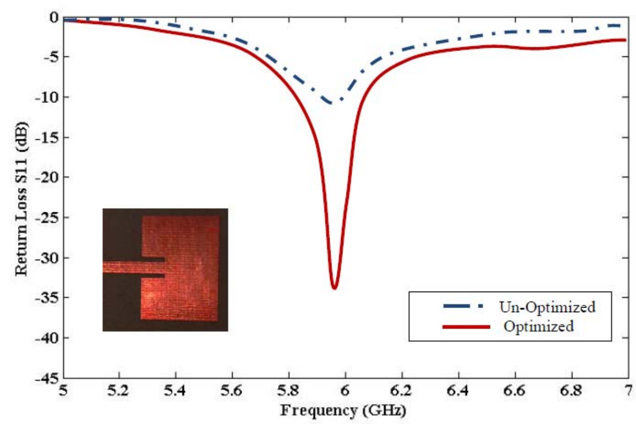

Fig.10.Convergence characteristics of collective hysteresis with other established boundary algorithms in inset fed rectangular patch example.

It is seen that collective hysteresis condition gives better result in this case. The return loss S_{11} is shown in Fig.11 along with the fabricated antenna. The antenna is designed over a microstrip PTFE dielectric of ε=2.2 and resonant frequency of 6 GHz.

B. Edge Position Optimization

As described before a black-box simulation approach is incorporated in this example where CST Microwave studio acts a black-box cost function and returns the performance of optimizing variables to MATLAB; this process continues till the total number of allocated iterations is completed. The cost function is directly fixed as the minimization of S_{11} with optimizing variables as the feed-line width W_0 and edge feed position l_0 as shown in Fig.12. The cost/fitness function is given in Eqn.6.

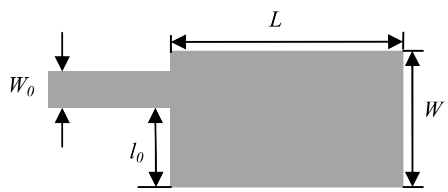

Fig.12. Rectangular patch antenna with edge feed

$$F_{cost/fitness} = \min\{S_{11}\} \quad (6)$$

The convergence curves of different boundary algorithms are shown in Fig.13 and the corresponding S_{11} plots of optimized and un-optimized results are shown in Fig.14.

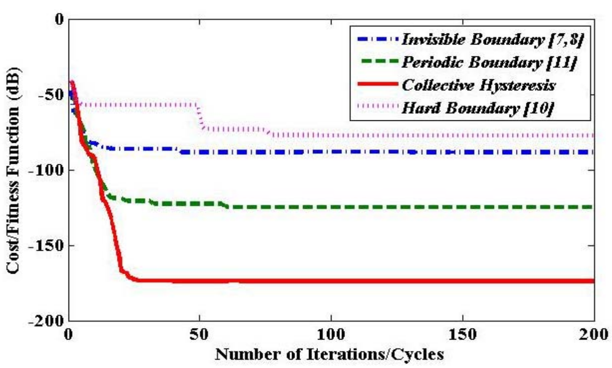

Fig.13. Rectangular patch antenna with edge feed

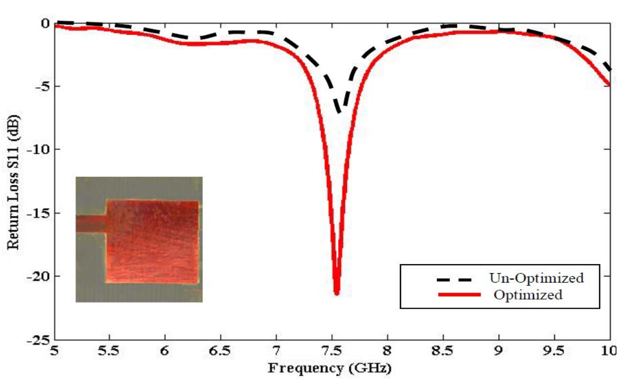

Fig.14. Rectangular patch antenna with edge feed

From the results it is apparent that collective hysteresis algorithm shows better performance in this case too. Fig.14 also shows the picture of the fabricated antenna. The antenna is designed at 7.5 GHz of frequency over a microstrip PTFE dielectric of ε=2.2.

VI. CONCLUSIONS

The various position regulated boundary conditions were evaluated. Collective hysteresis algorithm showed better performance in all the examples taken up here. An important fact is established that the inertia factor in PSO can be much less than 0.9 to get good and fast optimal results in case of PRBCs; this can be attributed to the fact that PRBCs increase the number of participants in search process. Three different examples on three different categories of cost/fitness function formulation were described. Scopes of the proposed methods are not limited to the antenna problems and are applicable in general to all possible optimization areas.

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