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DE vs. PSO: A Performance Assessment for Expensive Problems

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***Abstract*—This paper investigates the suitability of the Particle Swarm Optimization (PSO) and the Differential Evolution (DE) algorithms in solving expensive optimization problems. Eight PSO variants, and eight DE variants are experimentally compared among each other. The Comparing Continuous Optimizers (COCO) methodology was adopted in comparing these variants on the noiseless BBOB testbed. Based on the results, we provide useful insights regarding the algorithms' relative efficiency and effectiveness under an expensive budget of function evaluations; and draw suggestions about which algorithm should be used depending on what we know about our optimization problem in terms of evaluation budget, dimensionality, and function structure. Furthermore, we propose possible future research directions addressing the algorithms limitations. Overall, DE variants perform well in low dimensions, whereas in higher dimensions, several PSO variants surpass DE algorithms. Among the top performers, JADE and χPSO are the robust algorithms for solving expensive budget problems.**

I. Introduction

In recent years, the optimization community has been facing great challenges in designing computational methods/frameworks and algorithms that can successfully deal with the complexity of real-world optimization problems. Some of these problems are exceptionally difficult to solve exactly, due to the absence of objective function information [1], where the sole source of information about the objective function is available through point-wise evaluation (also known as black-box optimization). Typically, a single function evaluation requires some computational resources (e.g., time, money, power); an evaluation of one candidate solution might take a whole laboratory experiment [2]. Hence, it is desired for an optimization algorithm to be capable of achieving a good-quality solution with a limited (expensive) budget of function evaluation. However, an expensive budget setting is not the only challenge. Increased dimensionality of the solution space; differentiability, multi-modality and non-separability of the objective function are common in real-world problems. Subsequently, conventional methods fail to provide solutions on complex multimodal problems. Nevertheless, nature-inspired optimization algorithms, being simple and efficient, have emerged as an attractive alternative solver for such applications [3].

The two most promising optimization algorithms for solving complex optimization problems in the evolutionary computation (EC) community are Particle Swarm Optimization (PSO) [4] and Differential Evolution (DE) [5]. Recently [6], the performance of some DE variants algorithm on the BBOB testbed [7], [8] has been evaluated for cheap-budget settings. A similar study was conducted on the basic PSO algorithm [9] where it exhibited promising characteristics. Current state-of-the-art research in DE and PSO, on the other hand, has significantly enhanced their performance, and introduced a diverse collection of variants. The introduction of DE and PSO variants makes the picture unclear about which algorithm is suitable for an optimization problem, and it becomes even more puzzling under expensive-budget settings as studies of evolutionary algorithms are often based on a relatively cheap budget of function evaluations [10].

This paper aims to examine the relative effectiveness and efficiency of PSO and DE variants in solving expensive-budget optimization problems. Through experimental evaluation, it marks the differences in them; and investigates the suitability of the algorithms for various optimization problems. Furthermore, it draws several concluding points that may be fruitful for the EC community.

To achieve the paper's goals, the experimental evaluation must be able to discover and distinguish the good features of the variants over others, and show their differences over different stages of the search for various goals and situations. We have adopted the Comparing Continuous Optimizer (COCO) methodology [11] as it meets our requirements. It comes with a testbed of 24 scalable noiseless functions [8] addressing such real-world difficulties as ill-conditioning, multi-modality, and dimensionality.

The rest of the paper is organized as follows: Section II provides a brief description of the selected DE and PSO variants. In Section III, the numerical assessment of the algorithms is presented, starting with the experimental setup (Section III-A); afterwards, the procedure for evaluating the algorithms' performance is elaborated (Section III-B); followed by a discussion of the results (Section III-C). Section IV summarizes the main conclusions from this study, and suggests possible directions towards better PSO and DE algorithms for expensive-budget settings.

II. Selected DE & PSO Algorithms

The PSO and DE algorithms have been a hot topic of research in the past two decades for performance improve-

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TABLE I: DE and PSO variants considered in this paper. Shaded algorithms are the top 3 according to our experiments under DE and PSO category.

DE Algorithms	Description/
DE [5]	the classical algorithm; a population of candidate solutions undergoes mutation, crossover, and selection operations.
JADE [12]	tunes the mutation factor and the crossover probability over successive generations. Moreover, it employs a more explorative mutation strategy from the current generation, and an archive of parent solutions with better offspring.
DEAE [13]	derives an adaptive encoding technique for DE to make the search independent of the coordinate system.
DEctpb [14]	employs a mutation strategy similar to that of JADE, but with no adaptation.
JADEb[14]	employs DE's mutation strategy, but follows JADE's parameter adaptation technique.
MVDE [15]	applies several mutation operators; the next generation is produced via a tournament-selection process.
RL-SHADE [10]	improves on JADE by using a historical memory of successful parameter settings to guide the selection of future parameter values, its parameters are further tuned using the automated algorithm configuration tool, SMAC [16]. RL-SHADE has a restart strategy and reduces the population size linearly.
DE-F-AUC [17]	chooses among several mutation strategies based on their success profile using a multi-armed bandit technique.
PSO Algorithms	**Description**
PSO [4]	the classical algorithm; a population of candidtate solutions mimicks the collective behaviour of bird swarm
PSO-cf [18]	introduces a constriction factor to control the magnitude of particles flight
FIPS [19]	introduces a novel update mechanism through weighted sum average of all the particles
UPSO [20]	considers both local and global search in every step of the search process
CLPSO [21]	particles learn from any better performing particle
DMSPSO [22]	utilizes a novel dynamically changing neighbourhood topology for search process
SRPSO [23]	incorporates two unique learning strategies, namely, self-regulating inertia weight and self-perception on the global search direction
iSRPSO [24]	introduces guidance based directional update strategy for lesser performing particles

ment. In recent years, several promising variants of both the algorithms have been proposed and showed significant improvements in locating the optimum solutions. Here, we present the selected DE and PSO variants in this evaluation study. The selected algorithms are briefly described in Table I. The top 3 performing variants in each category are shaded in grey.

III. Numerical Assessment

A. Setup

For DE variants, the experimental setup can be found as follows. DE, JADE, and DEAE's settings are of [6]; DEctpb, JADEb's in [14]; MVDE's in [15], RL-SHADE's in [10], DE-F-AUC's in [17]. For PSO variants, we ran the selected PSO variants on 24 functions (15 instances per function) of the BBOB testbed with a budget of $10^3 \times D$ function evaluations. We used a MATLAB implementation of the algorithms retrieved from http://www.ntu.edu.sg/home/epnsugan/, with a modification on the terminating criteria of the variants to stop if the target function value f_t is achieved. The parameters of all the PSO variants are set to their standard values provided in the codes. The choice of the swarm size is not trivial; for DMSPSO, we used a swarm size of 60 as it was used by its authors [22]; for the rest of the variants, a swarm size of 50 is used as a popular choice for many numerical assessments of PSO algorithms. Investigating the effect of the swarm size is beyond the scope of this paper.

B. Performance Evaluation Procedure

The performance experiments are set up according to [11], where each algorithm is run on the functions given in [7], [8] with multiple trials per function. A set of target function values is specified per function. The algorithms are evaluated based on the number of function evalutions required to reach a target. The Expected Running Time (ERT) used in the figures and tables, depends on a given target function value, $f_\mathrm{t} = f_\mathrm{opt} + \Delta f$, and is computed over all relevant trials as the number of function evaluations executed during each trial while the best function value did not reach f_t, summed over all trials and divided by the number of trials that actually reached f_t [11], [25]. **Statistical significance** is tested with the rank-sum test for a given target Δf_t using, for each trial, either the number of needed function evaluations to reach Δf_t (inverted and multiplied by -1), or, if the target was not reached, the best Δf-value achieved, measured only up to the smallest number of overall function evaluations for any unsuccessful trial under consideration. As we are interested in expensive settings, the results are shown for function evaluations $\leq 1000 \times D$.

C. Performance Evaluation Discussion

Results for the six selected algorithms JADE, DE-F-AUC, JADEb, PSO-cf, DMSPSO, and UPSO[1] are reported in Figures 1, 2, 3, 4 and 5; and in Tables II and III. Although RL-SHADE is not among the top three DE variants, it exhibits an interesting performance. We show its behaviour in 2, 3, and 4. Overall, Fig. 1 shows that the ERT of all the algorithms grows roughly on an quadratic order with respect to the problem dimensionality for most of the functions; the dimensionality of the Katsuuras function (f_{23}) plays a vital role in its difficulty. One can notice the spread of the algorithms performance becomes more pronounced with higher dimensions. Based on the rest of the figures and tables, the following can be stated about the algorithms.

1) DE variants: In general, the performance of DE-F-AUC relatively degrades with increasing problem dimensions. JADE shows a robust topping performance on moderate and ill-conditioned functions with low- as well as high-dimension search space. Multi-modal and weakly structured multi-modal functions poise as hard problems for DE variants. RL-SHADE is presented here, although it is not among the top, as it showed a very promising performance. Note that it has only be given an evaluation budget of $100D$ (where the rest were given $> 1000D$). Within its allowed budget, RL-SHADE tops the rest of all algorithms. Running RL-SHADE for $1000D$ was not straightforward, because it required an involved set up and a training phase with the SMAC tool.

[1]The results of the other algorithms will be provided on the accompanying website of this paper, see Section IV

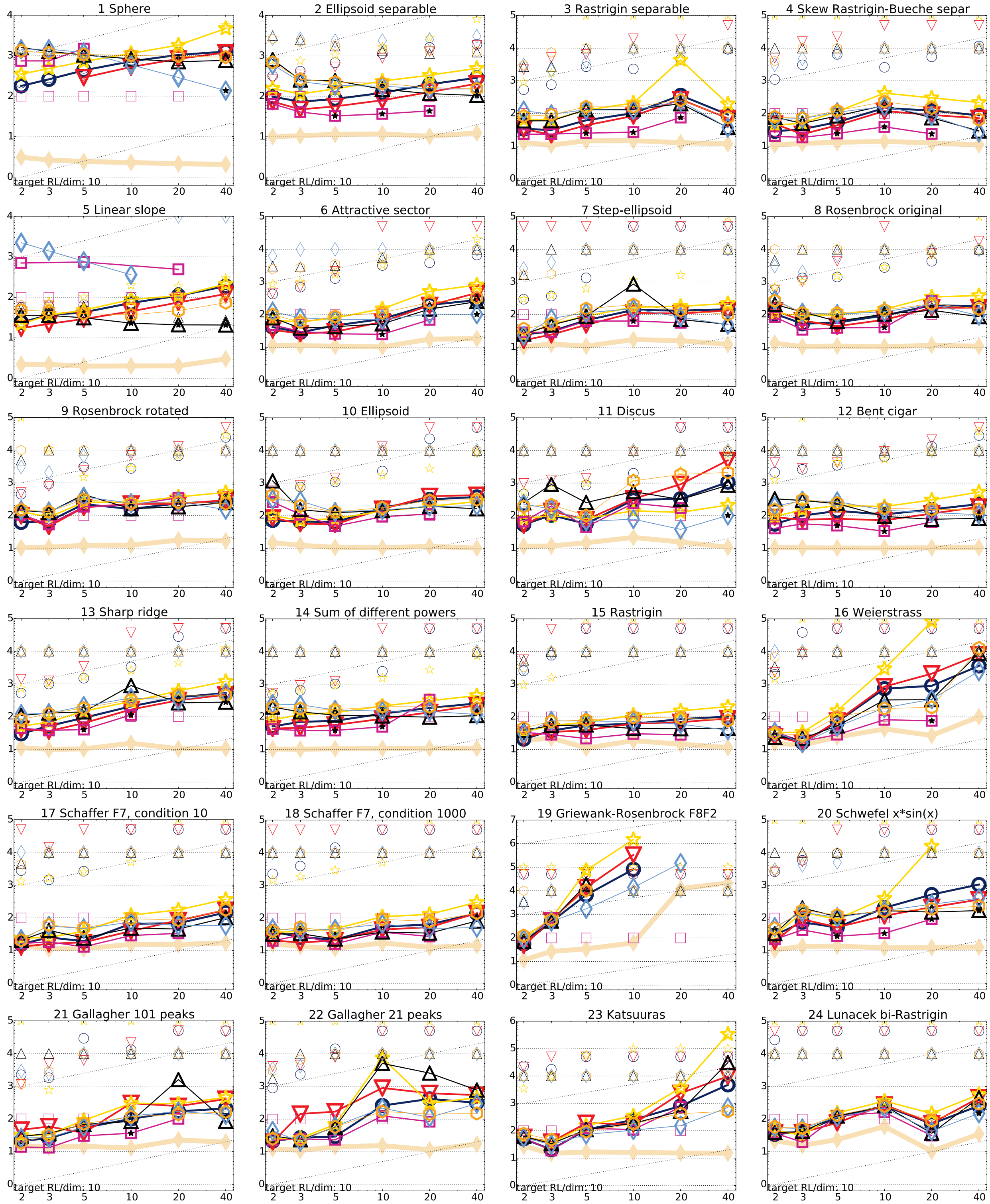

Fig. 1: Expected running time (ERT in number of f-evaluations as $\log_{10}$ value) divided by dimension versus dimension. The target function value is chosen such that the bestGECCO2009 artificial algorithm just failed to achieve an ERT of $10 \times \mathrm{DIM}$. Different symbols correspond to different algorithms given in the legend of f_1 and f_{24}. Light symbols give the maximum number of function evaluations from the longest trial divided by dimension. Black stars indicate a statistically better result compared to all other algorithms with $p < 0.01$ and Bonferroni correction number of dimensions (six). Legend: ○:JADE, ▽:JADEb, ⋆:DE-F-AUC, □:RL-SHADE, △:PSO-cf, ◇:DMSPSO, ⬡:UPSO

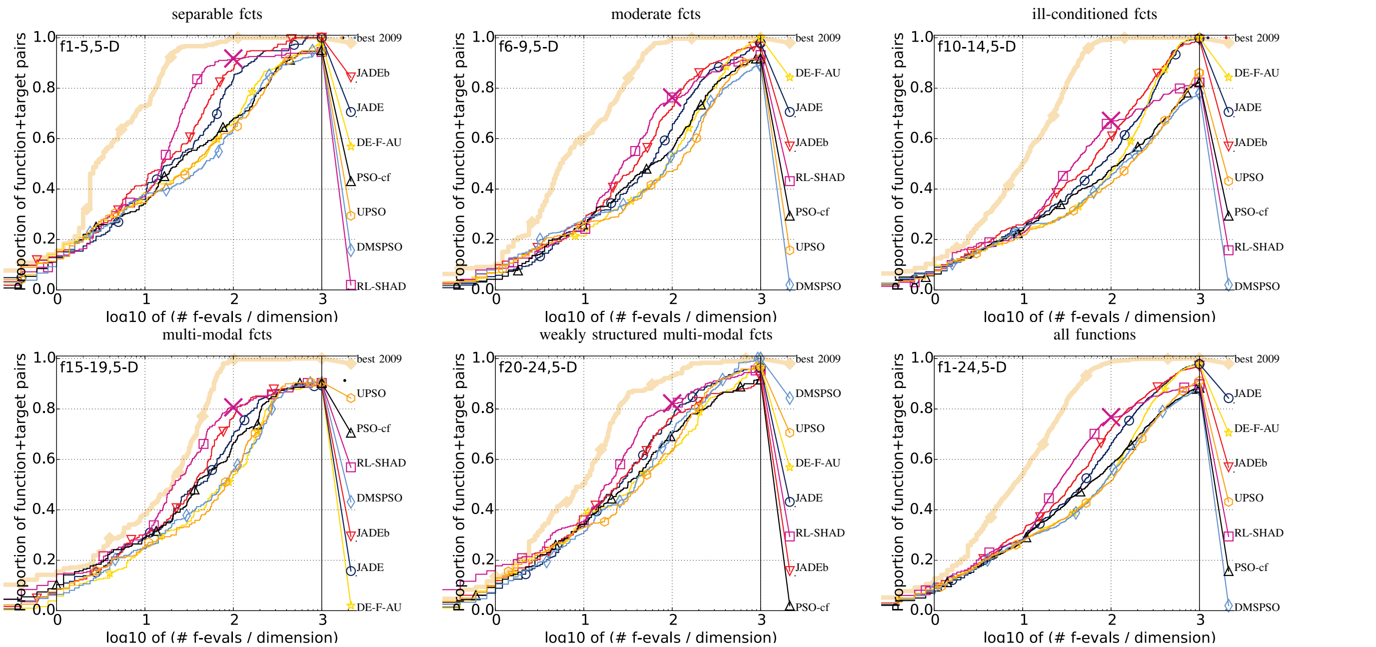

Fig. 2: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for all functions and subgroups in 5-D. The targets are chosen from $10^{[-8..2]}$ such that the bestGECCO2009 artificial algorithm just not reached them within a given budget of k × DIM, with $k \in \{0.5, 1.2, 3, 10, 50\}$. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each selected target.

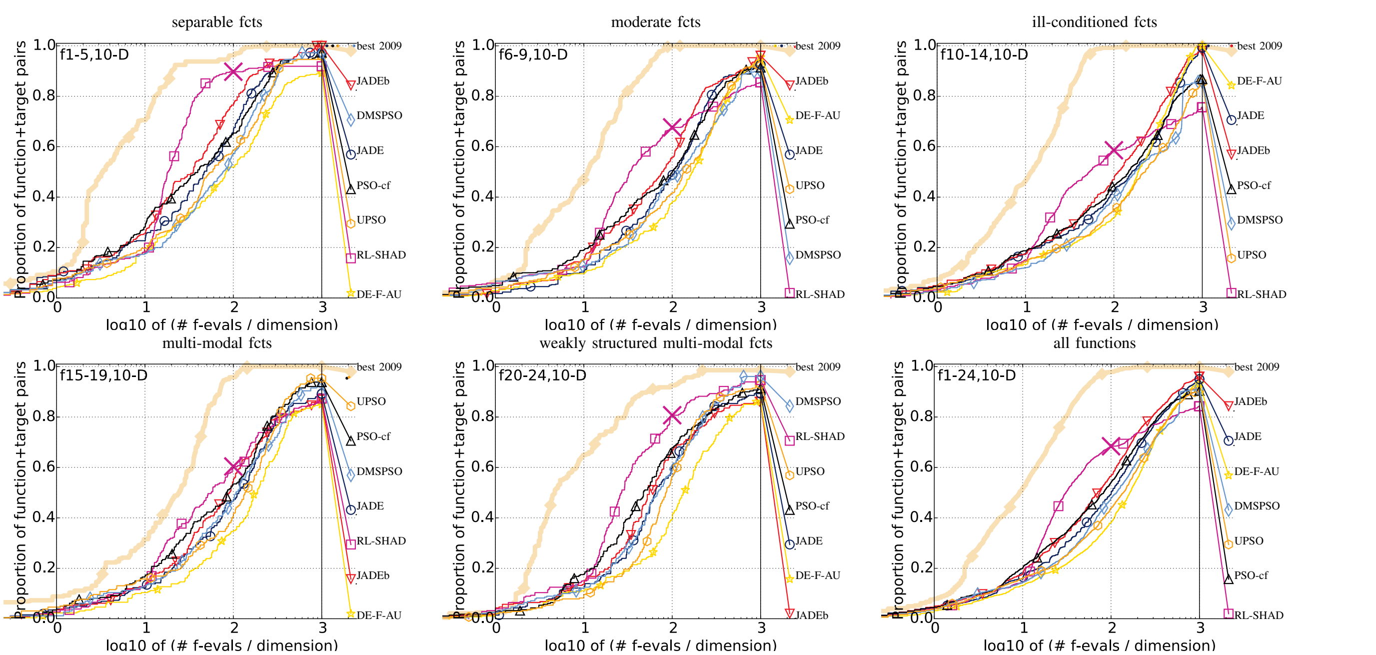

Fig. 3: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for all functions and subgroups in 10-D. The targets are chosen from $10^{[-8..2]}$ such that the bestGECCO2009 artificial algorithm just not reached them within a given budget of k × DIM, with $k \in \{0.5, 1.2, 3, 10, 50\}$. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each selected target.

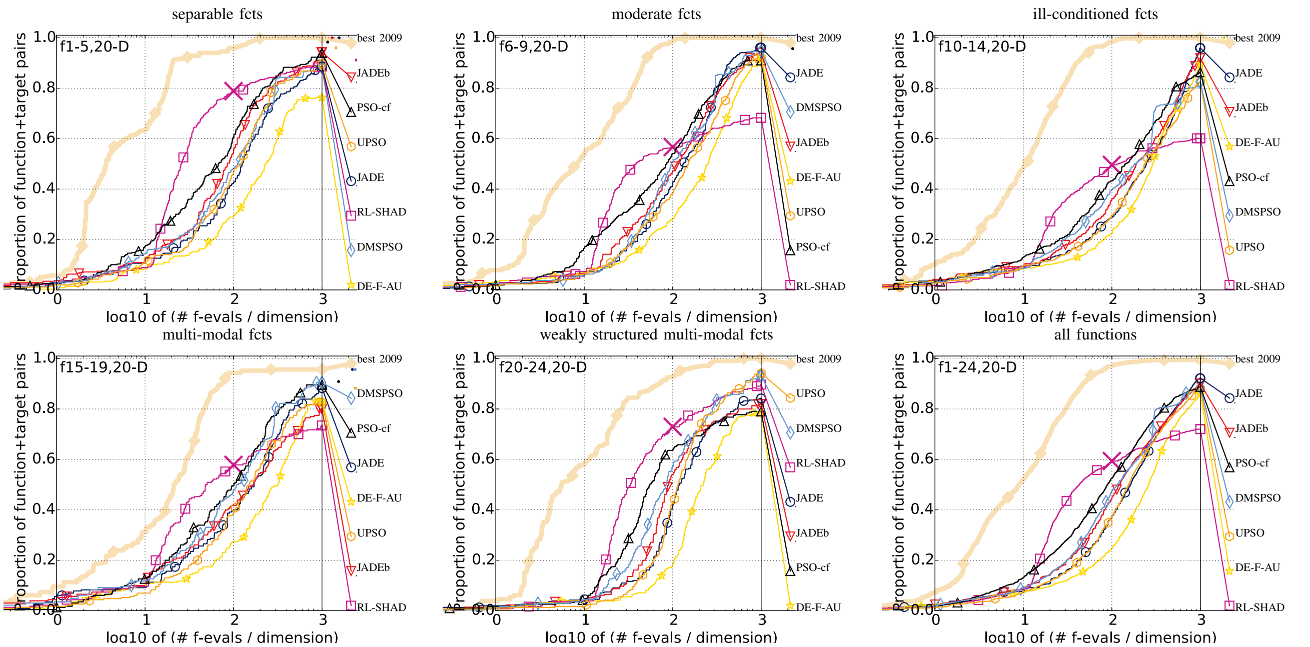

Fig. 4: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for all functions and subgroups in 20-D. The targets are chosen from $10^{[-8..2]}$ such that the bestGECCO2009 artificial algorithm just not reached them within a given budget of $k \times \mathrm{DIM}$, with $k \in \{0.5, 1.2, 3, 10, 50\}$. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each selected target.

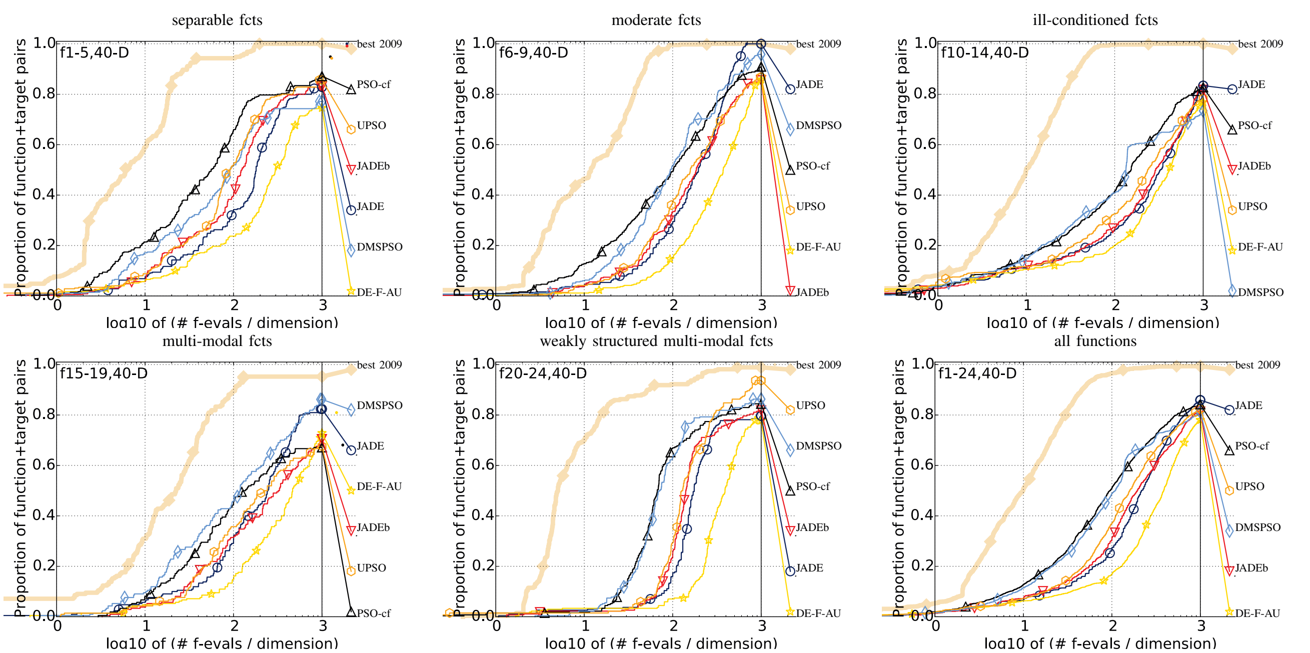

Fig. 5: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for all functions and subgroups in 40-D. The targets are chosen from $10^{[-8..2]}$ such that the bestGECCO2009 artificial algorithm just not reached them within a given budget of $k \times \mathrm{DIM}$, with $k \in \{0.5, 1.2, 3, 10, 50\}$. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each selected target.

#FEs/D	0.5	1.2	3	10	50	#succ
f1	*2.5e+1*:4.8	*1.6e+1*:7.6	*1.0e-8*:12	*1.0e-8*:12	*1.0e-8*:12	15/15
JADE	3.1(2)	3.6(3)	178(12)	178(19)	178(16)	15/15
JADEb	**1.5**(1)	**1.5**(2)	**118**(17)	**118**(20)	**118**(12)	15/15
DE-F-AU	**1.5**(0.4)	3.4(5)	294(9)	294(22)	294(14)	15/15
RL-SHAD	1.7(0.3)	2.7(4)	614(697)	614(420)	614(1096)	1/15
PSO-cf	2.6(3)	4.4(4)	426(30)	426(8)	426(25)	15/15
DMSPSO	4.4(9)	10(20)	493(3)	493(2)	493(3)	15/15
UPSO	2.4(2)	4.8(8)	441(11)	441(23)	441(21)	15/15
#FEs/D	0.5	1.2	3	10	50	#succ
f2	*1.6e+6*:2.9	*4.0e+5*:11	*4.0e+4*:15	*6.3e+2*:58	*1.0e-8*:95	15/15
JADE	1.3(2)	0.91(0.7)	5.3(2)	7.5(1.0)	33(2)	15/15
JADEb	**1.0**(0.2)	**0.86**(1)	**4.7**(2)	4.9(1)	**24**(3)	15/15
DE-F-AU	1.4(2)	0.98(0.8)	8.6(12)	13(1)	53(3)	15/15
RL-SHAD	2.5(4)	1.4(0.8)	5.2(2)	**2.9**(0.5)$^{\star 3}$	∞ *500*	0/15
PSO-cf	1.5(1)	1.8(0.8)	5.9(5)	21(7)	87(5)	15/15
DMSPSO	1.3(1.0)	0.87(0.7)	6.9(5)	16(7)	119(4)	15/15
UPSO	1.8(2)	1.4(2)	8.4(8)	20(4)	85(5)	15/15
#FEs/D	0.5	1.2	3	10	50	#succ
f3	*1.6e+2*:4.1	*1.0e+2*:15	*6.3e+1*:23	*2.5e+1*:73	*1.0e+1*:716	15/15
JADE	3.0(2)	**1.8**(2)	4.1(1)	4.3(1)	1.1(0.4)	15/15
JADEb	2.2(1)	2.0(3)	2.8(2)	3.0(2)	0.80(0.3)	15/15
DE-F-AU	1.7(2)	2.7(2)	6.3(5)	8.7(1)	3.4(2)	15/15
RL-SHAD	2.9(4)	2.5(2)	**2.5**(1)	**1.7**(0.4)*	**0.33**(0.4)$^{\star 3}$	14/15
PSO-cf	3.5(3)	2.7(1)	3.1(3)	9.0(5)	2.7(1)	15/15
DMSPSO	2.0(0.8)	2.6(2)	5.9(6)	8.9(3)	2.0(0.8)	15/15
UPSO	**1.5**(2)	2.2(3)	5.2(4)	11(3)	2.4(0.7)	15/15
#FEs/D	0.5	1.2	3	10	50	#succ
f4	*2.5e+2*:2.6	*1.6e+2*:10	*1.0e+2*:19	*4.0e+1*:65	*1.6e+1*:434	15/15
JADE	3.2(2)	2.1(0.6)	3.4(3)	4.1(2)	1.5(0.2)	15/15
JADEb	3.8(5)	2.2(2)	**2.4**(0.7)	3.3(1)	1.0(0.5)	15/15
DE-F-AU	**2.4**(4)	2.1(1)	5.8(6)	8.8(1)	5.0(2)	15/15
RL-SHAD	2.7(2)	2.0(2)	2.5(2)	**1.9**(0.6)$^{\star 2}$	**0.49**(0.2)$^{\star 3}$	15/15
PSO-cf	3.1(5)	**1.3**(1)	2.7(3)	6.4(3)	3.3(1)	15/15
DMSPSO	4.5(5)	3.3(3)	5.4(4)	8.3(2)	2.6(1)	15/15
UPSO	4.3(6)	3.2(7)	4.4(3)	9.3(3)	3.7(0.9)	15/15
#FEs/D	0.5	1.2	3	10	50	#succ
f5	*6.3e+1*:4.0	*4.0e+1*:10	*1.0e-8*:10	*1.0e-8*:10	*1.0e-8*:10	15/15
JADE	2.5(4)	3.2(2)	21(8)	21(6)	21(10)	15/15
JADEb	2.1(2)	**2.0**(1)	**15**(5)	**15**(5)	**15**(6)	15/15
DE-F-AU	2.7(2)	4.0(2)	24(20)	24(9)	24(15)	15/15
RL-SHAD	2.0(3)	3.1(3)	372(150)	372(300)	372(513)	2/15
PSO-cf	2.7(2)	3.3(3)	16(8)	16(9)	16(5)	15/15
DMSPSO	2.0(1)	8.3(6)	390(10)	390(5)	390(9)	15/15
UPSO	**1.8**(1)	4.3(3)	20(3)	20(3)	20(5)	15/15
#FEs/D	0.5	1.2	3	10	50	#succ
f6	*1.0e+5*:3.0	*2.5e+4*:8.4	*1.0e+2*:16	*2.5e+1*:54	*2.5e-1*:254	15/15
JADE	2.2(3)	2.9(4)	5.6(5)	4.3(3)	4.8(1)	15/15
JADEb	**1.6**(1)	**1.5**(2)	**2.8**(3)	2.8(2)	**4.1**(0.9)	15/15
DE-F-AU	2.1(2)	2.4(0.7)	7.1(7)	7.4(5)	8.3(0.4)	15/15
RL-SHAD	3.0(10)	1.9(3)	3.8(2)	**2.4**(2)	14(14)	2/15
PSO-cf	3.0(3)	1.7(1)	4.1(2)	3.9(1)	8.3(2)	15/15
DMSPSO	2.0(2)	5.6(9)	12(15)	6.6(6)	16(3)	15/15
UPSO	2.4(4)	2.1(2)	7.5(4)	8.1(8)	12(2)	15/15
#FEs/D	0.5	1.2	3	10	50	#succ
f7	*1.6e+2*:4.2	*1.0e+2*:6.2	*2.5e+1*:20	*4.0e+0*:54	*1.0e+0*:324	15/15
JADE	3.4(4)	3.8(4)	5.7(2)	6.2(4)	**2.0**(0.7)	15/15
JADEb	2.3(3)	2.4(3)	**3.3**(5)	**4.5**(2)	57(0.4)	14/15
DE-F-AU	1.6(0.5)	1.8(3)	8.4(5)	8.3(2)	2.4(0.7)	15/15
RL-SHAD	1.5(2)	2.3(2)	3.5(2)	5.9(3)	2.8(2)	7/15
PSO-cf	1.8(2)	1.7(3)	3.6(4)	8.2(4)	14(0.7)	14/15
DMSPSO	1.9(2)	2.1(4)	5.5(7)	11(3)	3.6(1)	15/15
UPSO	**1.3**(1)	**1.5**(3)	6.4(6)	14(6)	6.0(2)	15/15
#FEs/D	0.5	1.2	3	10	50	#succ
f8	*1.0e+4*:4.6	*6.3e+3*:6.8	*1.0e+3*:18	*6.3e+1*:54	*1.6e+0*:258	15/15
JADE	3.2(6)	2.4(2)	4.7(4)	5.3(1)	**6.0**(2)	15/15
JADEb	**1.9**(1)	2.6(2)	4.6(4)	4.1(1)	10(12)	15/15
DE-F-AU	2.8(2)	2.3(1)	6.5(8)	8.9(3)	8.4(2)	15/15
RL-SHAD	3.3(5)	3.8(4)	**3.9**(1)	**3.6**(2)	14(26)	2/15
PSO-cf	3.0(3)	2.4(1)	5.5(4)	5.7(1)	25(48)	14/15
DMSPSO	2.2(2)	**2.3**(3)	4.8(5)	9.4(3)	16(8)	15/15
UPSO	2.5(1)	2.5(3)	6.2(3)	10(6)	11(2)	15/15
#FEs/D	0.5	1.2	3	10	50	#succ
f9	*2.5e+1*:20	*1.6e+1*:26	*1.0e+1*:35	*4.0e+0*:62	*1.6e-2*:256	15/15
JADE	18(5)	16(4)	14(2)	18(38)	21(3)	15/15
JADEb	16(7)	13(4)	12(3)	**14**(17)	**15**(11)	15/15
DE-F-AU	32(11)	29(4)	24(4)	21(4)	16(2)	15/15
RL-SHAD	**13**(4)	**11**(13)	**10**(7)	15(12)	∞ *500*	0/15
PSO-cf	31(20)	30(9)	26(9)	36(69)	669(895)	4/15
DMSPSO	38(14)	35(9)	32(7)	30(15)	48(19)	15/15
UPSO	39(10)	34(4)	30(5)	25(8)	881(952)	3/15
#FEs/D	0.5	1.2	3	10	50	#succ
f10	*2.5e+6*:2.9	*6.3e+5*:7.0	*2.5e+5*:17	*6.3e+3*:54	*2.5e+1*:297	15/15
JADE	1.6(1)	2.1(3)	1.6(1)	5.9(4)	6.6(1)	15/15
JADEb	2.2(3)	1.9(2)	1.4(1)	5.1(3)	6.5(3)	15/15
DE-F-AU	**0.98**(0.8)	1.7(2)	**1.4**(0.9)	7.6(4)	**4.6**(0.5)	15/15
RL-SHAD	1.6(0.9)	1.6(1)	1.5(2)	**4.5**(2)	12(9)	2/15
PSO-cf	1.6(1)	2.2(2)	2.0(2)	12(4)	141(70)	10/15
DMSPSO	1.2(2)	**1.4**(2)	1.6(4)	12(5)	40(33)	15/15
UPSO	1.8(2)	1.7(2)	2.0(2)	12(7)	206(331)	8/15
#FEs/D	0.5	1.2	3	10	50	#succ
f11	*1.0e+6*:3.0	*6.3e+4*:6.2	*6.3e+2*:16	*6.3e+1*:74	*6.3e-1*:298	15/15
JADE	1.7(1)	3.3(4)	6.3(3)	3.8(3)	6.3(1.0)	15/15
JADEb	1.6(1)	**1.5**(2)	5.1(4)	5.1(5)	8.3(3)	15/15
DE-F-AU	1.9(2)	2.8(2)	11(9)	5.6(3)	**4.6**(0.2)	15/15
RL-SHAD	**1.5**(2)	2.2(4)	**3.7**(3)	**3.1**(2)	∞ *500*	0/15
PSO-cf	2.1(2)	3.4(5)	7.4(8)	17(12)	281(223)	7/15
DMSPSO	2.1(2)	2.6(1)	11(5)	4.5(2)	46(28)	15/15
UPSO	2.8(3)	5.7(5)	12(11)	6.8(5)	2479(2309)	1/15
#FEs/D	0.5	1.2	3	10	50	#succ
f12	*4.0e+7*:3.6	*1.6e+7*:7.6	*4.0e+6*:19	*1.6e+4*:52	*1.0e+0*:268	15/15
JADE	**1.2**(0.9)	2.1(2)	3.6(2)	14(2)	13(4)	15/15
JADEb	1.6(0.7)	**2.0**(2)	**3.0**(1)	7.9(1)	15(6)	15/15
DE-F-AU	2.0(1.0)	3.3(2)	8.7(8)	20(3)	**13**(2)	15/15
RL-SHAD	1.4(1)	2.5(2)	3.3(1)	**4.8**(3)$^{\star 2}$	∞ *500*	0/15
PSO-cf	2.1(2)	2.8(2)	3.6(2)	23(3)	388(467)	5/15
DMSPSO	1.9(1.0)	8.0(12)	11(9)	26(4)	39(78)	15/15
UPSO	1.8(2)	2.7(3)	7.1(6)	24(4)	31(52)	14/15
#FEs/D	0.5	1.2	3	10	50	#succ
f13	*1.0e+3*:2.8	*6.3e+2*:8.4	*4.0e+2*:17	*6.3e+1*:52	*6.3e-2*:264	15/15
JADE	1.9(0.5)	2.8(4)	5.3(3)	8.9(3)	12(2)	15/15
JADEb	1.8(1)	**2.1**(2)	3.4(2)	6.1(0.5)	11(3)	15/15
DE-F-AU	2.9(0.8)	3.6(4)	9.3(8)	14(3)	**11**(0.9)	15/15
RL-SHAD	1.7(2)	2.6(2)	**3.4**(2)	**3.9**(1)$^{\star 3}$	∞ *500*	0/15
PSO-cf	1.2(0.5)	2.4(3)	3.9(4)	13(2)	1250(1519)	2/15
DMSPSO	**1.0**(1)	3.6(5)	10(8)	20(4)	100(179)	12/15
UPSO	1.7(2)	2.4(1)	4.8(3)	18(5)	189(148)	8/15
#FEs/D	0.5	1.2	3	10	50	#succ
f14	*1.6e+1*:3.0	*1.0e+1*:10	*6.3e+0*:15	*2.5e-1*:53	*1.0e-5*:251	15/15
JADE	**1.8**(2)	**0.95**(1)	2.2(1)	7.2(1)	12(0.9)	15/15
JADEb	2.2(2)	1.3(1)	1.8(0.5)	5.4(0.9)	**10**(2)	15/15
DE-F-AU	2.4(2)	2.2(1)	2.3(2)	13(2)	11(0.9)	15/15
RL-SHAD	2.3(2)	2.2(4)	2.6(3)	**3.6**(1)$^{\star 2}$	∞ *500*	0/15
PSO-cf	2.9(3)	1.4(1.0)	**1.6**(0.8)	11(3)	647(550)	4/15
DMSPSO	2.2(2)	1.1(1.0)	2.4(3)	16(4)	166(149)	11/15
UPSO	2.7(5)	2.3(6)	3.4(3)	16(3)	918(860)	3/15
#FEs/D	0.5	1.2	3	10	50	#succ
f15	*1.6e+2*:3.0	*1.0e+2*:13	*6.3e+1*:24	*4.0e+1*:55	*1.6e+1*:289	5/5
JADE	3.2(3)	3.5(4)	4.2(2)	5.0(2)	3.1(1)	15/15
JADEb	3.7(3)	1.8(1)	**2.7**(2)	3.5(2)	**2.6**(1)	15/15
DE-F-AU	4.3(7)	3.8(4)	4.5(5)	6.1(4)	3.9(1)	15/15
RL-SHAD	2.5(3)	2.8(2)	3.1(0.7)	**2.0**(0.7)	3.0(3)	7/15
PSO-cf	**2.3**(1)	2.5(2)	4.3(3)	5.1(3)	7.4(3)	15/15
DMSPSO	2.5(2)	3.8(5)	8.3(8)	6.7(4)	5.1(2)	15/15
UPSO	2.4(5)	**1.7**(2)	7.0(4)	7.2(2)	4.9(2)	15/15
#FEs/D	0.5	1.2	3	10	50	#succ
f16	*4.0e+1*:4.8	*2.5e+1*:16	*1.6e+1*:46	*1.0e+1*:120	*4.0e+0*:334	15/15
JADE	1.7(2)	1.5(1)	2.2(3)	2.9(0.9)	7.5(5)	15/15
JADEb	**1.2**(0.6)	1.4(1)	2.2(2)	3.1(2)	5.3(6)	15/15
DE-F-AU	1.9(0.8)	2.4(5)	2.7(2)	6.4(7)	19(15)	15/15
RL-SHAD	1.8(2)	1.4(2)	**1.3**(1)	**1.2**(0.7)	**1.7**(1)	10/15
PSO-cf	1.5(3)	**1.4**(1)	2.7(3)	2.2(3)	4.7(2)	15/15
DMSPSO	1.8(4)	2.0(2)	1.8(1)	2.1(2)	2.9(2)	15/15
UPSO	1.5(2)	2.3(0.9)	4.1(5)	4.1(4)	3.3(3)	15/15
#FEs/D	0.5	1.2	3	10	50	#succ
f17	*1.0e+1*:5.2	*6.3e+0*:26	*4.0e+0*:57	*2.5e+0*:110	*6.3e-1*:412	15/15
JADE	3.3(4)	1.9(1)	1.7(1)	1.6(1)	1.6(0.5)	15/15
JADEb	3.1(5)	1.9(2)	1.6(0.4)	1.4(0.4)	**0.99**(0.4)	15/15
DE-F-AU	5.7(5)	2.2(2)	3.4(2)	3.2(2)	2.6(0.8)	15/15
RL-SHAD	**2.4**(2)	**1.0**(0.8)	**1.1**(0.9)	**1.3**(1)	1.8(2)	8/15
PSO-cf	2.5(2)	2.0(2)	2.1(2)	1.8(0.8)	2.4(0.5)	15/15
DMSPSO	5.5(7)	3.5(1)	4.1(4)	4.3(2)	4.2(2)	15/15
UPSO	4.3(5)	4.6(4)	4.3(2)	4.4(3)	3.1(2)	15/15
#FEs/D	0.5	1.2	3	10	50	#succ
f18	*6.3e+1*:3.4	*4.0e+1*:7.2	*2.5e+1*:20	*1.6e+1*:58	*1.6e+0*:318	15/15
JADE	2.7(8)	2.5(2)	2.1(2)	1.9(2)	**3.2**(1)	15/15
JADEb	1.4(1)	**1.6**(1)	**1.9**(2)	1.8(1)	4.3(1.0)	15/15
DE-F-AU	1.9(1)	3.5(2)	5.5(4)	4.0(3)	3.9(0.5)	15/15
RL-SHAD	1.5(0.6)	1.7(3)	2.2(1)	**1.4**(1)	4.4(3)	5/15
PSO-cf	1.5(1)	1.9(3)	3.8(4)	1.9(1)	4.8(2)	15/15
DMSPSO	**1.3**(1)	2.4(4)	4.9(3)	3.9(3)	7.3(5)	15/15
UPSO	1.8(1)	2.3(3)	2.4(4)	3.2(2)	4.9(1)	15/15
#FEs/D	0.5	1.2	3	10	50	#succ
f19	*1.6e-1*:172	*1.0e-1*:242	*6.3e-2*:675	*4.0e-2*:3078	*2.5e-2*:4946	15/15
JADE	192(166)	276(614)	183(342)	95(133)	81(69)	7/15
JADEb	401(185)	586(1121)	422(958)	154(152)	124(146)	5/15
DE-F-AU	2124(2720)	1726(2990)	967(1392)	322(219)	207(235)	6/15
RL-SHAD	∞	∞	∞	∞	∞ *500*	0/15
PSO-cf	545(796)	1478(1174)	1103(2055)	∞	∞ *5e4*	0/15
DMSPSO	**50**(18)	**78**(78)	**44**(31)	**34**(47)	**28**(18)	5/15
UPSO	303(467)	391(485)	1060(1351)	∞	∞ *5e4*	0/15
#FEs/D	0.5	1.2	3	10	50	#succ
f20	*6.3e+3*:5.1	*4.0e+3*:8.4	*4.0e+1*:15	*2.5e+0*:69	*1.0e+0*:851	15/15
JADE	2.2(2)	1.8(2)	5.1(1)	3.7(1)	2.3(1)	15/15
JADEb	2.3(2)	1.9(1)	5.4(3)	3.8(1.0)	**0.91**(0.3)	15/15
DE-F-AU	1.5(2)	1.7(2)	5.5(4)	6.7(2)	7.6(5)	15/15
RL-SHAD	**1.4**(1)	**1.3**(1)	**4.3**(3)	**2.0**(0.6)$^{\star 2}$	1.3(0.7)	6/15
PSO-cf	2.6(2)	1.9(1)	5.7(3)	8.0(4)	9.1(30)	14/15
DMSPSO	2.2(1)	2.3(2)	8.8(5)	7.6(4)	3.9(2)	15/15
UPSO	2.0(3)	1.8(2)	10(6)	8.9(2)	2.6(0.5)	15/15
#FEs/D	0.5	1.2	3	10	50	#succ
f21	*4.0e+1*:3.9	*2.5e+1*:11	*1.6e+1*:31	*6.3e+0*:73	*1.6e+0*:347	5/5
JADE	1.8(1)	1.6(2)	1.4(2)	3.7(2)	3.6(5)	15/15
JADEb	1.7(0.4)	1.7(0.7)	1.6(1)	2.6(2)	13(17)	15/15
DE-F-AU	1.2(2)	1.7(2)	2.7(4)	6.2(2)	4.3(2)	15/15
RL-SHAD	**1**(0.7)	1.4(1)	1.9(2)	**2.1**(0.9)	**2.3**(3)	7/15
PSO-cf	1.4(0.7)	**1.3**(2)	1.4(1)	4.7(3)	177(186)	7/15
DMSPSO	2.4(2)	1.6(1)	**1.2**(0.8)	3.9(3)	12(2)	15/15
UPSO	2.0(2)	2.7(2)	4.0(5)	5.0(5)	14(39)	14/15
#FEs/D	0.5	1.2	3	10	50	#succ
f22	*6.3e+1*:3.6	*4.0e+1*:15	*2.5e+1*:32	*1.0e+1*:71	*1.6e+0*:341	5/5
JADE	3.1(4)	2.4(4)	2.1(2)	2.1(1)	3.0(6)	15/15
JADEb	1.8(2)	1.4(2)	1.9(2)	12(36)	18(19)	15/15
DE-F-AU	**1.4**(1)	1.8(1)	3.0(4)	4.8(3)	229(733)	13/15
RL-SHAD	**1.4**(0.8)	1.9(2)	**1.3**(1)	**1.6**(2)	**2.9**(1)	6/15
PSO-cf	1.5(2)	1.6(2)	2.2(2)	4.0(3)	32(46)	13/15
DMSPSO	1.6(2)	2.5(4)	3.4(2)	4.3(1.0)	3.6(4)	15/15
UPSO	2.6(2)	**1.2**(0.5)	1.5(2)	4.1(3)	3.6(2)	15/15
#FEs/D	0.5	1.2	3	10	50	#succ
f23	*1.0e+1*:3.0	*6.3e+0*:9.0	*4.0e+0*:33	*2.5e+0*:84	*1.0e+0*:518	15/15
JADE	2.4(3)	2.6(4)	3.4(3)	6.5(10)	37(18)	15/15
JADEb	2.3(3)	2.2(2)	**2.6**(1)	11(14)	35(22)	15/15
DE-F-AU	2.4(3)	**1.8**(1)	4.3(8)	9.3(7)	9.3(4)	15/15
RL-SHAD	2.8(3)	2.8(3)	3.1(3)	7.3(5)	∞ *500*	0/15
PSO-cf	2.8(3)	3.0(2)	3.1(3)	6.6(4)	24(28)	15/15
DMSPSO	4.5(6)	3.1(2)	3.1(3)	**4.3**(3)	**5.2**(3)	15/15
UPSO	**1.4**(1)	3.7(2)	3.6(2)	5.5(3)	20(6)	15/15
#FEs/D	0.5	1.2	3	10	50	#succ
f24	*6.3e+1*:15	*4.0e+1*:37	*4.0e+1*:37	*2.5e+1*:118	*1.6e+1*:692	15/15
JADE	2.1(1)	3.7(3)	3.7(3)	4.7(3)	4.3(1)	15/15
JADEb	**2.0**(2)	**2.8**(3)	**2.8**(3)	**3.3**(2)	3.3(4)	15/15
DE-F-AU	2.7(4)	5.2(6)	5.2(3)	6.5(2)	3.4(0.9)	15/15
RL-SHAD	2.4(2)	3.7(2)	3.7(2)	5.3(10)	11(7)	1/15
PSO-cf	2.6(2)	4.2(2)	4.2(3)	5.3(4)	3.1(2)	15/15
DMSPSO	3.9(6)	6.0(3)	6.0(3)	4.9(1)	2.7(1)	15/15
UPSO	3.8(2)	6.7(3)	6.7(3)	5.6(2)	**2.5**(1)	15/15

TABLE II: Expected running time (ERT in number of function evaluations) divided by the respective best ERT measured during BBOB-2009 in dimension 5. The ERT and in braces, as dispersion measure, the half difference between 90 and 10%-tile of bootstrapped run lengths appear for each algorithm and run-length based target, the corresponding best ERT (preceded by the target Δf-value in *italics*) in the first row. #succ is the number of trials that reached the target value of the last column. The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. Entries, succeeded by a star, are statistically significantly better (according to the rank-sum test) when compared to all other algorithms of the table, with $p = 0.05$ or $p = 10^{-k}$ when the number k following the star is larger than 1, with Bonferroni correction by the number of instances.

#FEs/D	0.5	1.2	3	10	50	#succ
f1	*6.3e+1*:24	*4.0e+1*:42	*1.0e-8*:43	*1.0e-8*:43	*1.0e-8*:43	15/15
JADE	22(10)	20(3)	486(9)	486(11)	486(11)	15/15
JADEb	18(9)	17(2)	388(19)	388(25)	388(23)	15/15
DE-F-AU	49(12)	46(12)	850(33)	850(48)	850(53)	15/15
RL-SHAD	11(1)	**7.1**(0.5)$^{\star 2}$	∞	∞	∞ *2000*	0/15
PSO-cf	**10**(4)	10(3)	328(16)	328(22)	328(26)	15/15
DMSPSO	17(5)	15(4)	**135**(3)	**135**(3)	**135**(3)	15/15
UPSO	27(13)	24(5)	452(10)	452(14)	452(13)	15/15

#FEs/D	0.5	1.2	3	10	50	#succ
f2	*4.0e+6*:29	*2.5e+6*:42	*1.0e+5*:65	*1.0e+4*:207	*1.0e-8*:412	15/15
JADE	1.3(0.9)	1.8(2)	30(3)	19(1)	73(4)	15/15
JADEb	1.2(1)	2.5(2)	20(3)	12(1)	59(4)	15/15
DE-F-AU	1.5(1)	4.9(7)	55(11)	32(3)	126(9)	15/15
RL-SHAD	1.3(1)	**1.3**(3)	**6.1**(0.7)$^{\star 4}$	**4.2**(2)$^{\star 4}$	∞ *2000*	0/15
PSO-cf	1.5(1)	1.6(1)	16(5)	11(2)	**50**(4)	15/15
DMSPSO	1.7(3)	2.9(1)	20(5)	15(2)	120(3)	15/15
UPSO	**0.97**(0.6)	3.6(4)	27(5)	16(2)	64(1)	15/15

#FEs/D	0.5	1.2	3	10	50	#succ
f3	*6.3e+2*:33	*4.0e+2*:44	*1.6e+2*:109	*1.0e+2*:255	*2.5e+1*:3277	15/15
JADE	5.0(2)	16(5)	27(3)	28(8)	6.8(0.7)	15/15
JADEb	5.3(4)	12(4)	21(5)	21(4)	**5.4**(0.4)	15/15
DE-F-AU	13(5)	35(17)	59(11)	342(252)	2181(1670)	4/15
RL-SHAD	5.2(4)	**6.4**(0.6)	**5.1**(0.9)$^{\star 4}$	**5.9**(3)$^{\star 3}$	8.9(17)	1/15
PSO-cf	**3.1**(2)	7.3(2)	20(4)	16(4)	57(107)	8/15
DMSPSO	5.7(3)	12(5)	20(4)	19(4)	6.0(4)	15/15
UPSO	7.2(5)	19(5)	28(7)	20(6)	861(778)	1/15

#FEs/D	0.5	1.2	3	10	50	#succ
f4	*6.3e+2*:22	*4.0e+2*:91	*2.5e+2*:250	*1.6e+2*:332	*6.3e+1*:1927	15/15
JADE	20(12)	13(3)	10(1)	14(1)	8.6(0.4)	15/15
JADEb	15(8)	9.5(3)	7.2(1)	10(2)	5.4(0.8)	15/15
DE-F-AU	41(22)	30(15)	24(4)	50(20)	809(187)	14/15
RL-SHAD	12(2)	**3.6**(0.6)$^{\star 4}$	**1.9**(0.4)$^{\star 4}$	**4.2**(3)$^{\star 3}$	7.4(9)	2/15
PSO-cf	**11**(6)	7.6(0.5)	5.9(2)	10(2)	**4.6**(2)	15/15
DMSPSO	12(9)	8.7(1)	6.8(1)	10(1)	4.7(2)	15/15
UPSO	30(10)	17(5)	12(1)	15(3)	101(234)	9/15

#FEs/D	0.5	1.2	3	10	50	#succ
f5	*2.5e+2*:19	*1.6e+2*:34	*1.0e-8*:41	*1.0e-8*:41	*1.0e-8*:41	15/15
JADE	5.3(1)	11(4)	54(7)	54(10)	54(9)	15/15
JADEb	3.3(3)	6.7(2)	37(11)	37(8)	37(7)	15/15
DE-F-AU	7.0(4)	10(2)	54(9)	54(11)	54(26)	15/15
RL-SHAD	6.3(4)	7.9(0.4)	242(222)	242(234)	242(99)	3/15
PSO-cf	**2.4**(1)	**3.2**(0.1)$^{\star 2}$	**10**(3)$^{\star 4}$	**10**(2)$^{\star 4}$	**10**(4)$^{\star 4}$	15/15
DMSPSO	2.5(2)	4.7(2)	∞	∞	∞ *2e5*	0/15
UPSO	3.7(3)	5.2(1)	23(8)	23(8)	23(8)	15/15

#FEs/D	0.5	1.2	3	10	50	#succ
f6	*2.5e+5*:16	*6.3e+4*:43	*1.6e+4*:62	*1.6e+2*:353	*1.6e+1*:1078	15/15
JADE	13(15)	14(11)	15(4)	11(3)	10(0.8)	15/15
JADEb	8.2(4)	8.7(4)	8.8(4)	11(6)	20(10)	15/15
DE-F-AU	16(13)	18(7)	16(11)	29(9)	20(1)	15/15
RL-SHAD	8.0(8)	6.3(0.7)	5.1(1)	**3.9**(3)	∞ *2000*	0/15
PSO-cf	**5.1**(3)	**3.7**(1)$^{\star 2}$	**3.6**(1)*	8.3(3)	**10**(2)	15/15
DMSPSO	14(14)	17(7)	18(4)	5.9(3)	11(3)	15/15
UPSO	15(16)	13(12)	14(12)	13(3)	14(3)	15/15

#FEs/D	0.5	1.2	3	10	50	#succ
f7	*1.0e+3*:11	*4.0e+2*:39	*2.5e+2*:74	*6.3e+1*:319	*1.0e+1*:1351	15/15
JADE	1.8(3)	10(5)	11(4)	8.3(2)	**4.8**(0.8)	15/15
JADEb	2.2(1)	8.1(4)	9.1(1)	6.8(1)	119(371)	13/15
DE-F-AU	3.0(5)	14(7)	16(9)	11(2)	5.2(0.5)	15/15
RL-SHAD	2.0(0.8)	5.6(2)	**4.1**(0.3)*	**3.5**(3)	∞ *2000*	0/15
PSO-cf	**1.3**(3)	**5.5**(5)	6.0(2)	4.4(2)	173(370)	7/15
DMSPSO	4.3(4)	7.1(6)	6.2(2)	5.2(0.9)	10(5)	15/15
UPSO	2.5(0.8)	10(6)	9.3(3)	8.5(2)	91(122)	12/15

#FEs/D	0.5	1.2	3	10	50	#succ
f8	*4.0e+4*:19	*2.5e+4*:35	*4.0e+3*:67	*2.5e+2*:231	*1.6e+1*:1470	15/15
JADE	29(8)	21(6)	25(6)	17(2)	12(2)	15/15
JADEb	22(8)	16(4)	19(2)	14(3)	**11**(5)	15/15
DE-F-AU	54(21)	42(15)	47(8)	30(3)	18(5)	15/15
RL-SHAD	14(0.6)	**8.2**(0.7)	**7.2**(2)$^{\star 3}$	22(16)	∞ *2000*	0/15
PSO-cf	**13**(10)	10(4)	13(3)	**12**(4)	18(16)	15/15
DMSPSO	23(9)	16(5)	18(3)	18(3)	21(12)	15/15
UPSO	33(15)	24(7)	28(4)	19(3)	19(7)	15/15

#FEs/D	0.5	1.2	3	10	50	#succ
f9	*1.0e+2*:357	*6.3e+1*:560	*4.0e+1*:684	*2.5e+1*:756	*1.0e+1*:1716	15/15
JADE	13(1)	**10**(0.8)	9.1(0.5)	10(0.7)	36(4)	15/15
JADEb	13(5)	14(21)	13(17)	13(6)	**24**(5)	15/15
DE-F-AU	21(2)	15(1)	14(1)	16(1)	26(5)	15/15
RL-SHAD	20(18)	∞	∞	∞	∞ *2000*	0/15
PSO-cf	**10**(1)	20(2)	18(1)	17(38)	125(92)	11/15
DMSPSO	15(2)	11(0.1)	**8.9**(0.0)	**8.0**(0.0)	111(0.1)	15/15
UPSO	14(1)	11(1)	10(2)	11(2)	818(758)	2/15

#FEs/D	0.5	1.2	3	10	50	#succ
f10	*1.6e+6*:15	*1.0e+6*:27	*4.0e+5*:70	*6.3e+4*:231	*4.0e+3*:1015	15/15
JADE	20(24)	21(9)	22(15)	27(5)	14(2)	15/15
JADEb	11(12)	16(12)	20(13)	34(11)	17(7)	15/15
DE-F-AU	17(17)	24(8)	23(8)	17(2)	**7.7**(0.5)	15/15
RL-SHAD	12(6)	8.9(3)	**5.9**(2)*	**10**(7)	∞ *2000*	0/15
PSO-cf	**10**(7)	**8.4**(6)	11(6)	17(11)	86(32)	14/15
DMSPSO	16(9)	14(6)	11(4)	17(6)	94(67)	15/15
UPSO	17(12)	19(14)	17(7)	25(12)	80(34)	15/15

#FEs/D	0.5	1.2	3	10	50	#succ
f11	*4.0e+4*:11	*2.5e+3*:27	*1.6e+2*:313	*1.0e+2*:481	*1.0e+1*:1002	15/15
JADE	**1.2**(2)	3.6(2)	21(22)	28(15)	92(508)	14/15
JADEb	2.1(2)	4.2(3)	59(21)	111(257)	281(499)	12/15
DE-F-AU	1.8(1)	4.5(7)	8.1(5)	**7.7**(0.7)	**7.3**(0.5)	15/15
RL-SHAD	2.1(5)	3.8(3)	11(17)	60(133)	∞ *2000*	0/15
PSO-cf	2.3(2)	**2.9**(2)	20(18)	38(20)	118(42)	15/15
DMSPSO	3.2(3)	4.3(4)	**2.4**(0.5)	13(21)	182(0.0)	15/15
UPSO	3.1(2)	4.2(3)	117(101)	195(72)	∞ *2e5*	0/15

#FEs/D	0.5	1.2	3	10	50	#succ
f12	*1.0e+8*:23	*6.3e+7*:39	*2.5e+7*:76	*4.0e+6*:209	*1.0e+1*:1042	15/15
JADE	16(7)	20(6)	22(2)	15(1)	19(4)	15/15
JADEb	17(5)	16(4)	15(3)	11(1.0)	23(18)	15/15
DE-F-AU	33(19)	40(7)	41(8)	29(2)	27(7)	15/15
RL-SHAD	11(0.7)	**7.5**(0.5)	**5.0**(0.4)$^{\star 4}$	**6.1**(5)	∞ *2000*	0/15
PSO-cf	**9.3**(7)	8.4(4)	9.1(2)	7.5(1)	105(144)	10/15
DMSPSO	14(11)	15(7)	14(2)	12(1)	**8.0**(3)	15/15
UPSO	24(10)	22(6)	21(6)	15(2)	16(1)	15/15

#FEs/D	0.5	1.2	3	10	50	#succ
f13	*1.6e+3*:28	*1.0e+3*:64	*6.3e+2*:79	*4.0e+1*:211	*2.5e+0*:1724	15/15
JADE	13(10)	18(3)	26(3)	38(3)	**11**(6)	15/15
JADEb	14(3)	15(4)	21(3)	30(4)	68(56)	15/15
DE-F-AU	24(11)	35(6)	47(5)	58(4)	12(0.9)	15/15
RL-SHAD	8.8(3)	**5.2**(0.2)*	**7.4**(13)$^{\star 2}$	∞	∞ *2000*	0/15
PSO-cf	**6.2**(4)	7.9(1)	14(4)	**25**(5)	107(203)	8/15
DMSPSO	12(7)	14(2)	20(3)	31(4)	29(78)	14/15
UPSO	19(8)	20(3)	29(7)	44(4)	40(146)	12/15

#FEs/D	0.5	1.2	3	10	50	#succ
f14	*2.5e+1*:15	*1.6e+1*:42	*1.0e+1*:75	*1.6e+0*:219	*6.3e-4*:1106	15/15
JADE	25(12)	22(6)	18(6)	17(2)	**19**(1)	15/15
JADEb	23(12)	16(5)	14(6)	13(1)	20(4)	15/15
DE-F-AU	51(36)	36(8)	33(6)	28(3)	20(2)	15/15
RL-SHAD	17(6)	**7.7**(0.7)	**5.8**(2)	31(33)	∞ *2000*	0/15
PSO-cf	**13**(7)	8.6(4)	8.2(3)	**8.9**(3)	34(7)	15/15
DMSPSO	19(8)	12(5)	11(1)	13(1)	172(0.1)	15/15
UPSO	35(19)	24(10)	20(11)	19(3)	62(8)	15/15

#FEs/D	0.5	1.2	3	10	50	#succ
f15	*6.3e+2*:15	*4.0e+2*:67	*2.5e+2*:292	*1.6e+2*:846	*1.0e+2*:1671	15/15
JADE	13(9)	12(4)	5.8(1)	5.8(2)	11(2)	15/15
JADEb	11(5)	8.5(5)	4.7(1)	6.5(1)	18(7)	15/15
DE-F-AU	19(19)	22(7)	10(1)	7.1(1)	43(37)	15/15
RL-SHAD	12(7)	**4.4**(0.8)	**1.9**(1)	**3.2**(4)	18(50)	1/15
PSO-cf	**6.3**(5)	5.4(1)	3.0(2)	4.6(2)	23(32)	13/15
DMSPSO	13(7)	7.1(2)	4.1(0.9)	3.6(0.9)	**3.5**(0.1)	15/15
UPSO	19(16)	13(3)	6.3(1)	4.8(1)	4.9(3)	15/15

#FEs/D	0.5	1.2	3	10	50	#succ
f16	*4.0e+1*:26	*2.5e+1*:127	*1.6e+1*:540	*1.6e+1*:540	*1.0e+1*:1384	15/15
JADE	2.4(2)	44(59)	33(11)	33(5)	24(6)	15/15
JADEb	3.8(2)	84(81)	78(44)	78(30)	60(37)	15/15
DE-F-AU	4.1(5)	83(133)	3076(2441)	3076(2339)	∞ *2e6*	0/15
RL-SHAD	3.8(6)	**3.8**(1)*	**2.8**(3)$^{\star 3}$	**2.8**(2)$^{\star 3}$	**6.8**(7)$^{\star 3}$	3/15
PSO-cf	4.6(4)	20(16)	12(2)	12(5)	44(74)	12/15
DMSPSO	**2.3**(2)	7.1(4)	13(6)	13(8)	22(12)	15/15
UPSO	4.1(5)	13(5)	7.2(2)	7.2(2)	29(39)	15/15

#FEs/D	0.5	1.2	3	10	50	#succ
f17	*1.6e+1*:11	*1.0e+1*:63	*6.3e+0*:305	*4.0e+0*:468	*1.0e+0*:1030	15/15
JADE	**4.4**(5)	7.8(5)	4.5(2)	5.8(2)	**7.4**(2)	15/15
JADEb	8.2(6)	9.3(6)	5.5(6)	6.4(4)	14(14)	15/15
DE-F-AU	25(39)	23(11)	11(3)	12(2)	13(3)	15/15
RL-SHAD	5.5(6)	**3.9**(0.5)	**2.2**(2)	11(10)	∞ *2000*	0/15
PSO-cf	6.9(10)	5.9(5)	3.0(2)	4.8(2)	116(296)	10/15
DMSPSO	8.7(2)	6.0(3)	3.8(1)	**4.4**(0.6)	11(6)	15/15
UPSO	14(18)	12(4)	5.6(1)	8.2(3)	374(189)	6/15

#FEs/D	0.5	1.2	3	10	50	#succ
f18	*4.0e+1*:116	*2.5e+1*:252	*1.6e+1*:430	*1.0e+1*:621	*4.0e+0*:1090	15/15
JADE	3.8(2)	5.0(1.0)	6.3(1)	7.2(1)	**8.0**(2)	15/15
JADEb	3.5(3)	4.1(1)	5.0(2)	6.3(2)	9.0(7)	15/15
DE-F-AU	9.4(5)	10(3)	11(2)	11(0.8)	10(1)	15/15
RL-SHAD	**2.0**(0.5)	**2.2**(0.4)	4.2(4)	11(17)	∞ *2000*	0/15
PSO-cf	2.4(2)	2.7(1)	**3.3**(2)	**4.5**(2)	52(139)	12/15
DMSPSO	3.0(2)	3.8(0.7)	4.5(1)	5.6(1)	13(7)	15/15
UPSO	5.5(4)	6.3(1)	8.9(10)	35(23)	151(145)	9/15

#FEs/D	0.5	1.2	3	10	50	#succ
f19	*1.6e-1*:2.5e5	*1.0e-1*:3.4e5	*6.3e-2*:3.4e5	*4.0e-2*:3.4e5	*2.5e-2*:3.4e5	3/15
JADE	∞	∞	∞	∞	∞ *1e6*	0/15
JADEb	∞	∞	∞	∞	∞ *1e6*	0/15
DE-F-AU	∞	∞	∞	∞	∞ *2e6*	0/15
RL-SHAD	∞	∞	∞	∞	∞ *2000*	0/15
PSO-cf	∞	∞	∞	∞	∞ *2e5*	0/15
DMSPSO	**12**(8)	∞	∞	∞	∞ *2e5*	0/15
UPSO	∞	∞	∞	∞	∞ *2e5*	0/15

#FEs/D	0.5	1.2	3	10	50	#succ
f20	*1.6e+4*:38	*1.0e+4*:42	*2.5e+2*:62	*2.5e+0*:250	*1.6e+0*:2536	15/15
JADE	10(5)	15(7)	27(0.9)	42(7)	10(1)	15/15
JADEb	7.8(3)	11(4)	20(3)	17(6)	**3.5**(0.9)	15/15
DE-F-AU	20(11)	28(10)	45(7)	1269(1370)	1048(1115)	10/15
RL-SHAD	7.1(0.9)	6.9(0.8)	**8.8**(5)*	**7.4**(4)$^{\star 2}$	12(7)	1/15
PSO-cf	**6.2**(3)	**6.6**(3)	14(5)	12(2)	7.9(0.2)	14/15
DMSPSO	7.3(2)	7.9(3)	15(2)	21(5)	7.1(4)	15/15
UPSO	13(6)	16(5)	28(4)	15(2)	16(40)	13/15

#FEs/D	0.5	1.2	3	10	50	#succ
f21	*6.3e+1*:36	*4.0e+1*:77	*4.0e+1*:77	*1.6e+1*:456	*4.0e+0*:1094	15/15
JADE	32(14)	24(6)	24(6)	7.5(2)	22(38)	15/15
JADEb	26(9)	19(10)	19(8)	11(23)	33(67)	15/15
DE-F-AU	46(31)	39(24)	39(15)	13(4)	289(459)	13/15
RL-SHAD	9.1(0.7)	**6.6**(3)	**6.6**(2)	**4.5**(4)	**6.1**(9)	4/15
PSO-cf	**8.9**(2)	7.2(3)	7.2(3)	70(220)	124(229)	9/15
DMSPSO	18(9)	18(9)	18(6)	7.4(13)	32(93)	14/15
UPSO	26(11)	19(9)	19(8)	6.1(3)	18(4)	14/15

#FEs/D	0.5	1.2	3	10	50	#succ
f22	*6.3e+1*:45	*4.0e+1*:68	*4.0e+1*:68	*1.6e+1*:231	*6.3e+0*:1219	15/15
JADE	27(12)	29(16)	29(15)	36(87)	100(428)	14/15
JADEb	17(9)	19(5)	19(4)	54(199)	52(124)	15/15
DE-F-AU	51(19)	56(16)	56(12)	27(10)	1100(822)	9/15
RL-SHAD	**7.7**(1)	**7.2**(2)*	**7.2**(1)*	**7.2**(5)	**2.5**(2)	8/15
PSO-cf	327(1114)	221(741)	221(4)	222(434)	145(165)	8/15
DMSPSO	19(6)	20(8)	20(8)	10(4)	29(82)	13/15
UPSO	25(9)	25(5)	25(5)	12(4)	29(123)	13/15

#FEs/D	0.5	1.2	3	10	50	#succ
f23	*6.3e+0*:29	*4.0e+0*:118	*2.5e+0*:306	*2.5e+0*:306	*1.0e+0*:1614	15/15
JADE	**1.6**(2)	8.4(11)	52(28)	52(62)	131(65)	15/15
JADEb	2.8(1)	16(23)	153(117)	153(108)	673(472)	10/15
DE-F-AU	2.2(2)	10(10)	239(164)	239(340)	2385(1182)	7/15
RL-SHAD	2.7(3)	**5.4**(4)	48(26)	48(26)	∞ *2000*	0/15
PSO-cf	3.5(5)	10(9)	34(29)	34(14)	164(104)	8/15
DMSPSO	2.2(4)	5.5(5)	**10**(5)	**10**(5)	53(23)	15/15
UPSO	5.4(5)	13(6)	28(11)	28(17)	**20**(17)	15/15

#FEs/D	0.5	1.2	3	10	50	#succ
f24	*2.5e+2*:208	*1.6e+2*:918	*1.0e+2*:6628	*6.3e+1*:9885	*4.0e+1*:31629	15/15
JADE	6.9(1)	7.0(3)	7.5(2)	10(2)	10(3)	15/15
JADEb	7.2(1)	8.9(5)	13(3)	36(82)	58(57)	6/15
DE-F-AU	14(2)	11(4)	52(66)	967(1053)	304(208)	3/15
RL-SHAD	**3.0**(2)	5.7(13)	∞	∞	∞ *2000*	0/15
PSO-cf	3.5(2)	7.3(8)	5.1(6)	15(8)	42(37)	2/15
DMSPSO	4.1(1)	**4.1**(2)	**1.8**(0.5)	**4.0**(0.9)	**5.2**(3)	14/15
UPSO	8.6(3)	6.7(3)	2.6(0.3)	11(16)	91(81)	1/15

TABLE III: Expected running time (ERT in number of function evaluations) divided by the respective best ERT measured during BBOB-2009 in dimension 20. The ERT and in braces, as dispersion measure, the half difference between 90 and 10%-tile of bootstrapped run lengths appear for each algorithm and run-length based target, the corresponding best ERT (preceded by the target Δf-value in *italics*) in the first row. #succ is the number of trials that reached the target value of the last column. The median number of conducted function evaluations is additionally given in *italics*, if the target in the last column was never reached. Entries, succeeded by a star, are statistically significantly better (according to the rank-sum test) when compared to all other algorithms of the table, with $p = 0.05$ or $p = 10^{-k}$ when the number k following the star is larger than 1, with Bonferroni correction by the number of instances.

2) PSO variants: In general, UPSO excels relatively on multi-modal and weakly structured multi-modal functions. As the problem dimensionality increases, χPSO climbs the performance ladder, especially on separable functions. While DMSPSO enjoys a similar performance on multi-modal functions, it greatly suffers on separable functions with higher dimensions (e.g., f_5, see Table III).

Overall, JADE shows a robust performance on all dimensions. Nevertheless, PSO variants appear to perform better than DE counterparts with increasing problem dimensionality.

IV. Conclusion

This paper provides an extensive evaluation and comparison of eight DE variants and eight PSO variants on the noiseless BBOB testbed under limited number of function evaluations. Based on the results, no algorithm is sutiable for all problems. Nevertheless, JADE is shown to be performing well. Moreover, the following remarks can be done:

Algorithms Suitability: JADE , χPSO, and UPSO are suitable for moderate and ill-conditioned functions, separable functions, and multi-modal functions, respectively.

Algorithms Rectifications: In general, *dimensionality* imposes a challenge on all DE algorithms (except perhaps for JADE). DMSPSO, being one of the top performers on multimodal functions, should be redesigned to consider functions of linear slope (e.g., f_5). Tuning parameters shows a promising direction in boosting the algorithms' performance (e.g., RLSHADE).

Data, Code, and Future Benchmarking: The data of these experiments will be made available on the BBOB webpage [11]. Furthermore, a repository for PSO variant codes and their data will be hosted online at `https://sites.google.com/site/psobenchmark/`.

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