I. INTRODUCTION

Home Health Care (HHC) covers a wide range of services that are provided at the homes of the patients. HHC mainly addresses the needs of the patients who are over aged, who have disabilities and/or who have chronic diseases. HHC includes services such as nursing, medical visits, home life aids, psychological support, old people assistance, house cleaning etc. The demand for HHC services is growing rapidly due to the congestion of hospitals, ageing of population, and economic factors. Therefore, effective planning of HHC operations has become very important. There are a number of studies related to staff planning, scheduling and routing for HHC operations such as [1,2]. Each of these papers tackles a variation of the problem with different properties and assumptions.

This paper deals with the vehicle routing problem of the home health care crew where the crew size is limited and different types of care are provided within specific time windows. We refer to this problem as the Crew Constrained Home Care Routing Problem with Time Windows (CC-HCRPTW). CC-HCRPTW is motivated by a real life case of a company that provides such services to a district municipality in Istanbul. This company routes nurses and home health aides (will refer to as aides in the remainder of the paper) in vehicles that carry at most two people excluding the driver from a central office on a daily basis. It can be argued that it would be more efficient if one of the home care crew could drive the car. Yet this is not feasible in a city like Istanbul where parking is a huge problem and the driver needs to stay in the car while the patients give their service. It is the policy of the company that each patient is visited exactly once in a day. The patients to be visited in any day are finalized in the evening of the previous day.

The current practice of the company is to have one nurse and one aide in each vehicle, in spite of the fact that some patients do not need both a nurse and an aide. In fact, the services to be provided to the patients can be categorized in two major groups. The first group includes the services such as nursing, vaccination, blood sugar measurement, blood pressure measurement, etc. These services are provided by a nurse. The second group includes old people assistance, home life aids, bathing, etc. These are provided by an aide. On the other hand, some of the patients are in need of both of these services; hence, they should be serviced by a nurse and an aide. It is also possible that the service that a patient requires must be provided by two people simultaneously, e.g. the patient cannot move but needs a bath.

The patients may require any type of service depending on their health conditions. The company has limited number of nurses and aides. Although the current practice of the company is to have a nurse and an aide in each vehicle, it is possible that a vehicle carries just a nurse or just an aide. In such a case, the vehicle can only serve the patients that require the corresponding type of services. If a vehicle carries both a nurse and an aide, it can satisfy the requirements of any patient whereas a vehicle carrying either a nurse or an aide can satisfy the requirements of certain patients depending on their needs.

In this paper, we introduce and study the problem of minimizing the total distance the vehicles travel while satisfying the particular needs of the patients in the required time windows when the number of nurses and aides are limited. To the best of our knowledge, this particular vehicle routing problem (VRP) variant has not been studied in the literature. The remainder of the paper is organized as follows: Section II provides a brief
review of the related literature. Section III describes the problem and presents its mathematical programming formulation. Section IV develops a Variable Neighborhood Search algorithm to solve it. Section V designs the computational study and discusses the results of the experiments. Finally, Section VI provides the concluding remarks and future research outlook.

II. RELATED LITERATURE

There is a vast amount of research related to the Vehicle Routing Problem with Time-Windows (VRPTW) and its variants but the literature on HCC routing is rather scant. In [3], a daily scheduling problem is addressed as a multi-depot vehicle routing problem with time windows and connections between visits by using a multi-criteria objective. In [1], the authors propose an integrated approach that jointly addresses: (i) the compatibility of the skills, (ii) not violating the time windows, and (iii) the determination of the routes daily. By introducing a concept called pattern, which specifies possible schedule for skilled visits, the assignment, scheduling and routing decisions are jointly addressed. The objective proposed in this model is mainly related to the operator utilization. In [4], not only the scheduling and routing of home health care nursing is studied, but also a spatial decision support system is developed. In [5], the travel time and the waiting time of the patients are minimized for an application in Sweden. The problem is solved by using a set partitioning model and a decision support system called LAPS CARE is developed. [6] presents a problem with nurses having different skills and a heuristic to solve it. Here, the objective is to minimize a weighted sum of the total travel time plus a sum of several penalties like the violation of patients’ preferences or of time windows. The developed heuristic consists of two phases: (i) building a set of patients to be served by each nurse and (ii) finding an optimal sequence for each set of patients.

In [7], the routing problem of the HHC workers is formulated as a Multiple Travelling Salesman Problem with Time Windows (MTSPTW). The objective is to minimize the total travelling cost while not violating the time windows constraints, and synchronized (some cares requires more than one worker) and disjunctive (some workers cannot work at the same time) services constraints. The model is tested by solving randomly generated instances using a commercial solver. In the problem presented in this paper, one or more workers with different skills may be assigned to each route. If a crew of workers covering all skills is assigned to each vehicle, the problem becomes MTSPTW. Our problem has a resemblance with technician routing problem where technicians with different skill levels are considered (see e.g. [8]). Yet a typical assumption in technician routing problem is that a technician with a certain skill level can be assigned to any task that requires lower skill levels. In our problem, we have two types of crew that perform different types of tasks. Thus our problem has some flavors of the VRPTW and the Technician Routing Problem. Our problem can be considered within the context of the Resource Constrained Vehicle Routing Problem introduced in [9]. This problem is more general in many aspects but the time window constraints are not included. A general review on human resources scheduling and routing can be found in [10]. In [11], a similar problem is considered where different types of services are required by the patients. Different from our problem, the services can be provided separately at different times of the day.

III. PROBLEM DESCRIPTION AND MODEL

A. Problem Definition

We are given a set of patients and a central office. The patients are classified as type 1, type 2 or type 3, where type 1 patients need a nurse, type 2 patients need an aide, and type 3 patients need both. The service time for a patient depends on the type of the patient. Each patient is assigned a time window that describes the earliest and latest time to start the service at that patient. The time window constraint is not only due to better quality of service but because some tasks like injection or blood taking must be performed at a certain time of the day. A vehicle is referred to as type 1, type 2 or type 3, if it carries a nurse, a home health aide, or both, respectively. As mentioned before, a type 3 vehicle can serve all patients where as a type 1 (2) vehicle can only serve type 1 (2) patients. Each vehicle starts its tour at the central office, serves a set of patients, and returns to the central office before the shift ends. We assume that the numbers nurses and aides available are limited. In this respect, we have two types of resources that are both limited.

The goal is to determine the type of vehicles and route the vehicles such that each patient receives the service she requires within her time window and the total distance traveled is minimized. We use the distance minimization objective because the vehicles are provided by a third-party company and are charged by the total trip distance.

Figure 1 illustrates the problem on an example. The solution in Figure 1(a) utilizes three vehicles, each carrying both a nurse and an aid since each vehicle visits either a type 3 patient or at least one type 1 and one type 2 patients. So, three nurses and three aides are required in total. In Figure 1(b), the same service can be provided with again three vehicles but less crew. An aide is assigned to Route 1 (Route 2) as all the patients are type 2 (type 1) patients. A nurse and an aide are assigned to Route 3 since that vehicle serves all type of patients. So, the patients are served by two nurses and two aides in this solution, saving two personnel compared to the solution depicted in Figure 1(b).
B. Mathematical Model

The set of patients is denoted by $V = \{1, ..., N\}$. Vertices 0 and $N + 1$ denote the depot and every route starts at 0 and ends at $N + 1$. The sets including the depot are denoted as $V_0 = V \cup \{0\}$ and $V_{N+1} = V \cup \{N + 1\}$. The set containing all of the nodes is denoted as $V_{0,N+1} = V \cup \{0\} \cup \{N + 1\}$. Thus, the complete directed graph of this problem is denoted as $G = (V_{0,N+1}, A)$ with the set of arcs $A = \{(i,j) | i,j \in V_{0,N+1}, i \neq j\}$. Each arc is associated with distance $d_{ij}$ and travel time $t_{ij}$. Each patient $i \in V$ is of type $r_i$, where $r_i \in \{1,2,3\}$ has a service time $s_i$ and time window $[e_i, l_i]$. The time window states that the earliest time to start the care of patient $i$ is $e_i$ and the latest time to start the care of patient $i$ is $l_i$. The start time of service from depot and the latest time to arrive at the depot at the end of the services are denoted with time window $[e_0, l_0]$. The set of patients of type $r$ is denoted as $n_r$ and $n_{r,0} = n_r \cup \{0\}$. If a nurse (aide) is assigned to a vehicle, it is called a type 1 (type 2) vehicle. If a nurse and an aid are both assigned to a vehicle, it is called a type 3 vehicle. The binary decision variable $x_{ijr}$ takes value of 1 if arc $(i,j)$ is traversed by a vehicle of type $r$, and 0 otherwise. The decision variable $q_i$ keeps track of the arrival time to the vertex $i$. The available number of nurses and aides are denoted as $h_1$ and $h_2$, respectively, and referred as the crew (resource) constraints.

The objective function (1) minimizes the total distance traveled. Constraints (2)-(4) make sure that the care is provided to the patient exactly once by a vehicle that has the appropriate personnel. Type 1 care is provided by a vehicle of type 1 or type 3 in Constraints (2) whereas type 2 care is provided by a vehicle of type 2 or type 3 in Constraints (3). Constraints (4) ensure that type 3 care is given by only a vehicle of type 3. Constraints (5) enforce that the number of outgoing arcs equals to the number of incoming arcs at each vertex other than the depot. Constraints (6) ensure the time feasibility of the arcs leaving the patients and the depot. Constraints (7) enforce the time windows of the patients and the depot. Constraints (6) and (7) eliminate the sub-tours by maintaining the schedule feasibility with respect to time considerations. Constraint (8) and (9) make sure that the crew
assigned to the vehicles does not exceed the available number of nurses and aids, respectively. Constraints (10) define the binary decision variables. Finally, Constraints (11) are the non-negativity restrictions of the decision variables.

The model can be easily modified to handle other relevant objective functions. If the objective function is to minimize the total number of health care workers, it is formulated as (12). If the objective function is to minimize the total number of vehicles, it is formulated as (13).

\[
\min \sum_{j \in V_{N+1}} (x_{0j1} + x_{0j2} + 2x_{0j3}) \quad (12)
\]

\[
\min \sum_{j \in V_{N+1}} \sum_{r \in R} x_{0jr} \quad (13)
\]

**IV. SOLUTION METHODOLOGY**

VNS is an effective metaheuristic method introduced by [12]. It applies local search on different neighborhood structures in an attempt to explore the solution space without getting stuck in local optima. It has been successfully applied to a variety of VRPs including VRPTW [13-15].

Our VNS first determines an initial solution using Solomon’s well-known I1 algorithm [16] without considering vehicle capacity limitation. If the initial solution is not feasible with respect to the crew constraints we apply a repair method called as crashing algorithm. The algorithm determines the route servicing the least number of patients and tries to eliminate (crash) it by inserting the patients along that route to other existing routes. If all the patients cannot be feasibly inserted in the other routes, it continues with the remaining routes in the same manner. If the resulting solution is still infeasible we penalize the violation in the objective function as follows:

\[
f_{VNS}(S) = f(S) + p_1V_{h_1}(S) + p_2V_{h_2}(S) \quad (14)
\]

\(f_{VNS}(S)\) is the total cost of solution \(S\), \(f(S)\) is the total distance, \(V_{h_1}(S)\) and \(V_{h_2}(S)\) are the violations in resources \(h_1\) and \(h_2\), respectively, and \(p_1\) and \(p_2\) are penalty factors associated with violation in these resources, respectively.

Given a predefined set of neighborhood structures \(K_n\) and the initial solution \(S\), VNS generates a neighboring solution \(S^*\) using the local search (LS). LS basically determines the best move in all possible neighborhood structures \(n = 1, \ldots, n_{max}\) using the local search (LS). LS basically determines the best move in all possible neighborhood structures \(n = 1, \ldots, n_{max}\) and applies the one which yields the largest reduction in the total cost. If \(S^*\) improves the current best solution \(S\), \(S\) is updated with \(S^*\). If no improving move exists, it performs a shaking procedure by removing \(r\) routes randomly from the solution and inserting the removed patients into the existing or newly created routes. This repair mechanism consists of a probabilistic version of the initialization algorithm where the patient to be inserted to a route is selected randomly between the two best fitting patients. If \(S^*\) does not improve after \(k_{max}\) shaking iterations and the search gets stuck in a local minimum we restart the search by re-initializing \(S\) using a randomized version of the initialization algorithm where the patient to be inserted to a route in determined randomly from the two best fitting patients. At any iteration, when \(S^*\) is better than the best-so-far solution \(S^\star\), we update \(S^\star\). If \(S^*\) does not improve after \(t_{max}\) re-starts and the search gets stuck in a global minimum, we terminate the VNS.

---

**Algorithm 1 Overview of the VNS Algorithm**

1: \(K_n \leftarrow \text{set of VNS neighborhood structures } n = 1, \ldots, n_{\text{max}}\)
2: \(k \leftarrow 0\)
3: \(t \leftarrow 0\)
4: \(S^\star \leftarrow \emptyset\)
5: \(S \leftarrow \text{GenerateInitialSolution()}\)
6: while \(t < t_{\text{max}}\) do
7: if \(\neg \text{feasible}(S)\) then
8: \(S \leftarrow \text{MakeFeasible}(S)\)
9: end if
10: for \(n = 1, \ldots, n_{\text{max}}\) do
11: \(S_n \leftarrow \text{LocalSearch}(S, n)\)
12: end for
13: \(S^* \leftarrow \min\{\text{cost}(S_n)\}\)
14: if \(\text{cost}(S^*) < \text{cost}(S)\) then
15: \(S \leftarrow S^*\)
16: else if \(\text{cost}(S) < \text{cost}(S^*)\) then
17: \(S^* \leftarrow S\)
18: end if
19: end if
20: \(k \leftarrow k + 1\)
21: if \(k < k_{\text{max}}\) then
22: \(S \leftarrow \text{Shake}(S)\)
23: else
24: \(k \leftarrow 0\)
25: \(t \leftarrow t + 1\)
26: \(S \leftarrow \text{RandomRestart()}\)
27: end if
28: end if
29: end while
30: return \(S^*\)

In LS, we utilize three neighborhood structures. Relocate removes a patient from its position on the route and inserts it on another edge on the same or on another route. In Exchange two patients on the same or on different routes are swapped. 2-opt eliminates two edges and reconnects the two resulting paths. Note that the first two LS operators consider both intra-route and inter-route moves whereas the last is inter-route only.
The pseudo-code of the VNS algorithm is given in Algorithm 1.

V. COMPUTATIONAL STUDY

A. Experimental Design

To assess the performance of VNS on CC-HCRPTW we used a selection of 25-node Solomon instances that we can solve using CPLEX and adapted them to our problem. We chose two instances of each problem class (R1, R2, RC1, and RC2), namely R103, R108, R201, R210, RC101, RC105, RC201, and RC205. We excluded C1 and C2 type instances because our preliminary analyses showed that they all involved only one or two routes in the optimal solutions and were far from providing any insights as they could be easily solved to optimality. We used the same coordinates and time windows, and ignored the demands. In order to use this data in our problem, we needed to assign each customer a type and corresponding service time. For each instance, we randomly generated new data in three different groups with the following three care types: (i) the patients are equally likely to be of each of the three types (33.3% for each type); (ii) the probability of a patient being type 1 or type 2 is 40% percent and type 3 is 20%; and (iii) the probability of a patient being type 1 is 60%, and type 2 or type 3 is 20%. We refer to these categories as G1, G2 and G3, respectively. We generated two instances of each setting, thus six instances for each Solomon problem and a total of 48 instances. The service times of the instances are set as 10 minutes for care of type 1, 40 minutes for care of type 2, and 45 minutes for care of type 3.

We also had to determine the number of nurses and aides for each instance. This is not a straightforward task because if we set the crew constraints too tight we may end up with infeasible problem instances. On the other hand, if these constraints are too loose, the instances may no longer become challenging examples.

In order to determine meaningful crew sizes, we solved the problems using CPLEX with two different objective functions by relaxing the crew constraints (8) and (9). We first solved the model to minimize the total distance, which provided us a guideline to find the number of nurses and aides. Intuitively (but not theoretically), these serve as “upper bounds” for our crew sizes $h_1$ and $h_2$. This is because in a typical instance (not always), minimizing total distance and minimizing total personnel are conflicting objectives. In the same manner, we also solved the model which minimizes the total number of crew without the crew constraints and obtained lower bounds for our crew sizes. Then, we determine four different crew settings for each instance following these lower and upper bounds for the nurses and aids. In the first, we set the crew sizes $h_1$ and $h_2$ equal to the optimal number of vehicles achieved when total distance traveled is minimized. In the second, both $h_1$ and $h_2$ are set equal to the average of the number of nurses and aides need (rounded up to integer) when the objective function is to minimize the total number of crew. We refer to these two data types with loose and tight crew constraints L and T, respectively. For the third and fourth settings, each of $h_1$ and $h_2$ is determined between the two values set above. These provide medium tight (or medium loose) crew constraints and we refer to these instances as M1 and M2 types. The total number of problem instances $48 \times 4 = 192$; however, we omitted three instances which were infeasible. At the end, we obtained 189 test instances.

The VNS algorithm was coded in C++ and all tests were performed on an Intel Xeon E5 processor with 3.30 GHz speed and 64 GB RAM, running on a single thread using 64-bit Windows 7 operating system. The parameter setting is as follows: $p_1 = p_2 = 1000$, $k_{\text{max}} = 20$, and $t_{\text{max}} = 200$. The number of routes $r$ to be removed in the shaking procedure is determined randomly using a uniform distribution between 40% and 80% of the total number of routes in the solution rounded to the nearest integer. We performed five runs of VNS and reported the performance of the best solutions obtained and the average solutions obtained in the next section.

B. Results

We first solved the instances using CPLEX by setting the run time limit to 2 hours. CPLEX was able to obtain the optimal solution of 175 instances out of 192. For the remaining 17 instances, we used the best solution found in 2 hours as benchmark.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Crew Constraints Types</th>
<th>L</th>
<th>T</th>
<th>M1</th>
<th>M2</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>1.25</td>
<td>3.36</td>
<td>2.58</td>
<td>2.45</td>
<td>2.38</td>
<td></td>
</tr>
<tr>
<td>R2</td>
<td>2.59</td>
<td>2.28</td>
<td>0.69</td>
<td>1.99</td>
<td>1.88</td>
<td></td>
</tr>
<tr>
<td>RC1</td>
<td>0.92</td>
<td>1.88</td>
<td>1.32</td>
<td>2.44</td>
<td>1.73</td>
<td></td>
</tr>
<tr>
<td>RC2</td>
<td>0.70</td>
<td>2.10</td>
<td>4.99</td>
<td>2.10</td>
<td>2.48</td>
<td></td>
</tr>
<tr>
<td>All</td>
<td>1.36</td>
<td>2.39</td>
<td>2.49</td>
<td>2.27</td>
<td>2.12</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Problem</th>
<th>Crew Constraints Types</th>
<th>L</th>
<th>T</th>
<th>M1</th>
<th>M2</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>1.81</td>
<td>4.15</td>
<td>3.24</td>
<td>3.40</td>
<td>3.13</td>
<td></td>
</tr>
<tr>
<td>R2</td>
<td>4.16</td>
<td>5.21</td>
<td>4.14</td>
<td>4.18</td>
<td>4.41</td>
<td></td>
</tr>
<tr>
<td>RC1</td>
<td>0.98</td>
<td>2.89</td>
<td>1.53</td>
<td>2.79</td>
<td>2.15</td>
<td></td>
</tr>
<tr>
<td>RC2</td>
<td>1.97</td>
<td>8.35</td>
<td>7.09</td>
<td>4.29</td>
<td>5.43</td>
<td></td>
</tr>
<tr>
<td>All</td>
<td>2.23</td>
<td>4.27</td>
<td>3.60</td>
<td>5.17</td>
<td>3.78</td>
<td></td>
</tr>
</tbody>
</table>
Our VNS algorithm was able to find better solution than CPLEX in 4 instances and achieved the same solution in 44 instances. In 3 instances, it found solutions which violated either the nurse or the aid resource constraint by one additional worker. In our analysis, we only include the instances for which a feasible solution was obtained and omit the other three.

Table I reports the average percentage gaps between the best solutions found by VNS and CPLEX solutions whereas Table II shows the average percentage gaps between the average solutions found by VNS and CPLEX solutions. Let us recall that we take five runs of VNS for each instance. The first column identifies the problem types whereas columns “L”, “T”, “M1”, and “M2” indicate the tightness of the crew constraints in the problems. The gap is calculated as follows:

\[
\text{\%Gap} = \frac{z_{\text{VNS}} - z_{\text{CPLEX}}}{z_{\text{CPLEX}}} \times 100
\]

The results show that VNS performs better when the crew constraints are loose, which is expected. On the other hand, we do not observe any significant difference in the performance with respect to the other tightness cases. The overall average gaps corresponding to the best and average solutions are 2.12% and 3.78%, respectively. Note that in 8 instances the average gaps are larger than 10%, which have some impact on this performance. Comparing the results in Tables I and II, we observe that we can obtain relatively robust results from different runs of the VNS in R1 and RC1 instances.

Table III and IV summarizes the results with respect to three different data groups we created. We observe that VNS performed significantly better in G1 data where the number of type 1, type 2, and type 3 patients are equal. On the other hand, the performance of VNS deteriorates in G2 and G3 groups which include more type 2 and type 3 patients. Note that these patients require significantly more service time 40 and 45 minutes, respectively, compared to 10 minutes for type 1. Similar to our previous observations, a comparison of Tables III and IV reveal that the results obtained by VNS are more robust in R1 and RC1 instances.

In Table V, we compare the computational times of the VNS and CPLEX. As expected, VNS takes more time to solve type 2 instances where the time windows are wider, hence, the solution space is larger. The run time for CPLEX is not as robust as VNS and significantly longer. On the average, the computation time of CPLEX is 100 times more than that of VNS.

In sum, although VNS provides fairly good results fast it can be improved to obtain better, near-optimal solutions, for type-2 instances in particular. This may be achieved at the expense of additional computational effort.

### VI. Conclusion

In this study, we presented the Crew Constrained Home Care Routing Problem with Time Windows and formulated its mathematical programming model. Since the problem is NP-Hard, we developed a Variable Neighborhood Search algorithm and tested its performance against that of CPLEX on some small-size instances. For the experimental tests, we randomly generated a data set using Solomon’s benchmark problems and determined different personnel resource limitations. Our preliminary results showed that VNS is able to find fairly good results fast as compared to the results given by CPLEX. On the other hand, it may struggle finding a feasible solution when the resource constraints are too tight.
For this study, we have created feasible instances with 25 patients by solving the model with different objectives. Creating challenging and interesting instances with larger number of instances is a relevant problem. The final goal here should be creating a set of benchmark problems for a more general framework that considers different types of limited resources.

Further research on this topic may focus on improving the effectiveness of the VNS algorithm using other neighborhood structures and novel search mechanisms, particularly to enhance the performance on R2 and RC2 problems where the time-windows are wider and the planning horizon is longer. The hybridization of VNS with Tabu Search and Simulated Annealing may be considered as an alternative method to escape from the local optima and better explore the solution space. In this study, we presented some initial test results, the performance of the algorithm should be evaluated on larger instances.

Finally, the problem has several interesting extensions which deserve further investigation, e.g. vehicle routing with synchronization when the personnel do not necessarily travel in the same vehicle to service a patient requiring type 3 service (see [17,18]), vehicle routing with split deliveries when type 3 service may be provided by the nurse and the aid at different times as well as the time-dependent and stochastic VRP variants of the problem.

REFERENCES