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Order Routing and Arbitrage Opportunities in a Multi-Market Trading Simulation

Andrew Todd, Peter Beling and William Scherer
Department of Systems and Information Engineering
University of Virginia, Charlottesville, VA 22904-4747, USA
Email: {aet6cd, pb3a, wts}@virginia.edu

***Abstract*—In this paper we introduce a simple model of multi-market trading. An identical security trades on two independent trading platforms. Prices and quotes are connected only by the strategic behavior of traders. The experimental design varies the degree to which traders monitor and act on information from both markets. We report on the degree of integration between the two markets as measured by the availability of arbitrage opportunities and the percentage of volume that trade throughs better quotes. Finally, we discuss the limits of integration with respect to our modeling assumptions.**

I. Introduction

The last day of open outcry trading for futures in Chicago was July 6th, 2015 [1]. At the time, trading floor volume had fallen to only 1% of total futures trading. The closing of the trading floor in Chicago is symbolic of a tidal change in financial markets in the United States and around the world. All aspects of trading have been transformed by technology. Understanding the resulting changes in market structure is of critical importance to market participants and regulators.

Financial markets are no longer dominated by large quasi-monopolistic exchanges. Instead, traders operate in a highly competitive and geographically distributed multi-market system. Both the buy- and sell-side of the markets monitor a multitude of market feeds and information sources, which they rely upon to manage their trading operations. The number of venues increases the complexity of automated trading. Whereas before brokers need only decide on how to trade optimally in a single venue, now they must decide how best to route orders to a large number of potential exchanges, which may vary widely with respect to the trading platform, fees and rebates, liquidity and transparency.

In this paper, we investigate a simple model of multi-market trading in an attempt to establish a relationship between basic market order routing and the relative integration of otherwise independent markets. In the model, an identical security trades on two independent but identical trading platforms. Prices and quotes at the two venues are connected only by the strategic behavior of traders. The experimental design varies the degree to which traders act on information from both markets. We study the integration of the two markets as measured by the existence of arbitrage opportunities. Finally, we discuss the limits of integration with respect to our modeling assumptions.

II. Limit Order Book

A fundamental ingredient of our model is the limit order book. A limit order book is an implementation of a continuous double auction. Traders may submit and cancel limit orders and submit market orders to trade at the best available price. Limit orders, which are instructions to buy or sell a certain quantity of a security at a given price, are queued and executed in price-time priority. Market orders execute immediately at the best available price. The price of new limit orders is governed by a minimum price increment known as the tick size. Quantities are restricted to be a multiple of a specified lot size. The number of lots available to execute at a specific price is the market depth at that price. The state of limit order book at time t can be given by the depth at each price, which is sufficient to calculate the best bid and ask prices denoted $b(t)$ and $a(t)$, respectively.

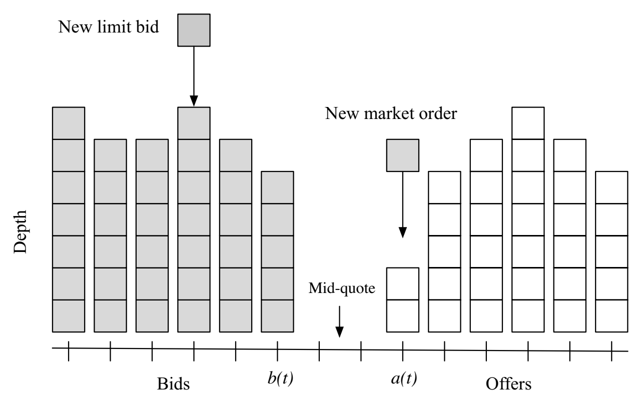

Fig. 1. Limit order book diagram. A new limit order to buy arrives at price $b(t) - 2$. A new market order to buy arrives and will execute at the the best ask price, $a(t)$.

Figure 1 contains a snapshot of a hypothetical limit order book. The current price as defined by the mid-quote is $0.5(a(t) - b(t))$. The spread is $a(t) - b(t)$. A new market order to buy executes at the best ask price $a(t)$. The depth available at that price will be reduced from two lots to one lot. A new limit bid arrives at price $b(t) - 2$ increasing the depth from seven lots to eight lots. The state of the limit order book will evolve as new limit orders and market orders are submitted and existing limit orders are canceled. In this paper, we study a multi-market system in which the dynamics of two independent order books are linked by the strategic behavior of traders.

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III. Multi-market Trading

Many countries have opened the door for competition in their financial markets. It is increasingly common for securities to trade on a number of local exchanges. Securities also trade in other countries and in different currencies. Competition among exchanges can benefit investors, but market fragmentation underlies a number controversial practices related to high frequency trading and other strategies exploiting the specific details of market structure [2]. Understanding how all of these individual markets are linked together is critical for our ability to assess the quality and function of the overall market [3].

In a multi-market setting, traders must determine how attractive each venue is for trading. The optimal order execution problem is widely studied. Originally formulated in a single market, it has been extended to a multi-market setting where the decision-maker must consider a number of additional factors, such as price and depth, fees and rebates and execution probability. Orders that are not appropriately routed may trade through better prices. Order routing, however, is not the only form of strategic behavior that links independent markets. Some market participants specifically monitor markets for the purpose of profiting from price dislocations, i.e arbitrage opportunities. In fact, there have been calls to abandon the continuous limit order book due to the inefficiencies arising from inter-market arbitrage [4], [5].

An opportunity for instantaneous profit exists if there exists a bid price greater than an ask price. The size of the arbitrage opportunity is constrained by the depth available at each price. In a two-market setting, markets may become locked or crossed [6]. An arbitrage opportunity exists when markets are crossed, i.e. $b_1(t) > a_2(t)$ or $b_2(t) > a_1(t)$. Markets are locked if $b_1(t) = a_2(t)$ or $b_2(t) = a_1(t)$ (See Figure 2).

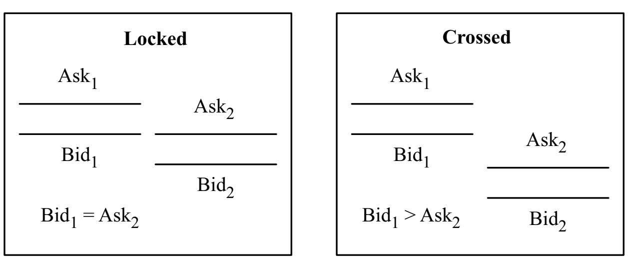

Fig. 2. A locked market occurs when the best bid in one market is equal to the best ask in another market. Markets are crossed whenever the bid price in one market exceeds that of an offer price in another market.

IV. Model

This paper employs the zero-intelligence method for investigating markets wherein the mechanics of the market are carefully implemented and sophisticated models of strategic behavior are replaced by simple models of aggregate order flow. The approach highlights the effects of the market mechanism and provides a means for testing hypotheses regarding the sources of certain stylized facts of financial time series. In this model, we extend the idea of the mechanism beyond the limit order book to include a simple implementation of order routing. In other words, we treat the two-market system with order routing as a single mechanism.

The model is based on a long line of literature that treats the arrival of limit order book events as a statistical process. Specifically, the model is an adaptation and extension of Smith et al (2003) to two independent limit order books [7], [8]. Limit orders and cancelations arrive independently at each market. Market order also arrive independently at each market, but with a certain probability are routed to the market with superior quotes. The probability of a market order being routed away from its“home market” is varied as part of the experimental design.

The model of order flow is a major assumption and is chosen due to a number of attractive features. First, it is simple to describe and implement. Second, it performs admirably when fit to data and even outperforms more complicated models in its class [9], [10]. Finally, the model and its variations have been widely studied.

The model has five parameters. First, a parameter v, which governs the discrete set of prices relative to the opposite quote at which new limit orders may arrive. The available prices for new limit orders to buy are $\{a(t) - v - 1, \ldots, a(t) - 1\}$. The available prices for new limit orders to sell are $\{b(t) + 1, \ldots, b(t) + v + 1\}$. New limits orders of unit quantity arrive at each available price at a rate of ℓ per unit time (making the total limit order arrival rate $2v\ell$). New market orders (also of unit quantity) arrive on each side of the book in each market at a rate of m per unit time. However, market orders may be routed away from their "home" market to a market with superior quotes with probability s. Outstanding limit orders are canceled at rate c per unit time.

The model can be thought of a collection of birth-death processes that are linked by the mechanics of the limit order book and the mechanics of the order routing algorithm. When a single order is left on either the buy or sell side of a market, the cancelation and market rates are set to zero, which prevents the book from becoming completely empty and ensures that the set of available prices for new limit orders is well defined.

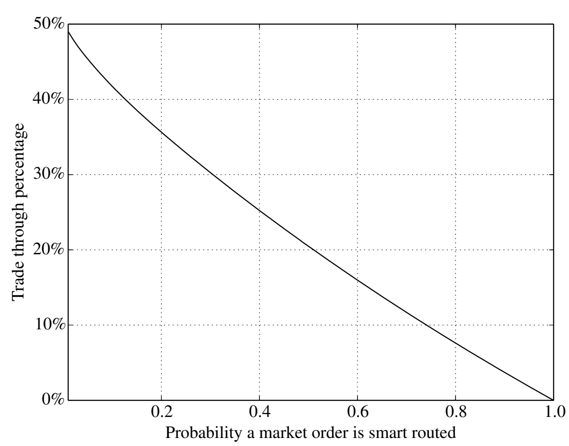

Fig. 3. The percentage of volume that trades through as a function of the probability that market orders are routed.

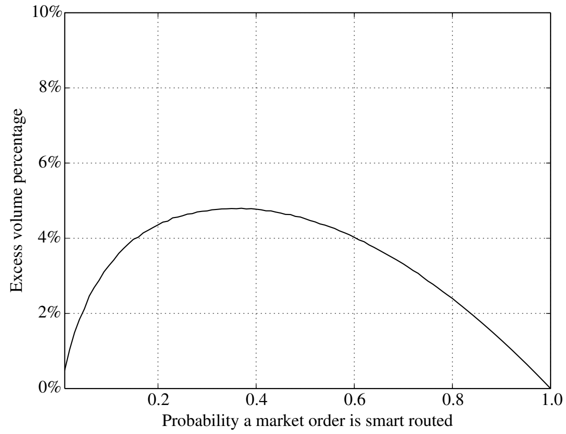

Fig. 4. The percentage of volume in excess of what is routed that executes at the best market, i.e. $100\frac{(1-s)}{2} - \theta$ where θ is the percentage of volume that trades through better quotes.

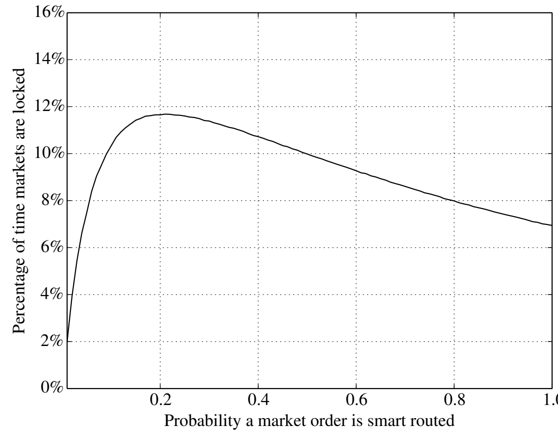

Fig. 6. The percentage of time markets are locked as a function of the probability that market orders are routed.

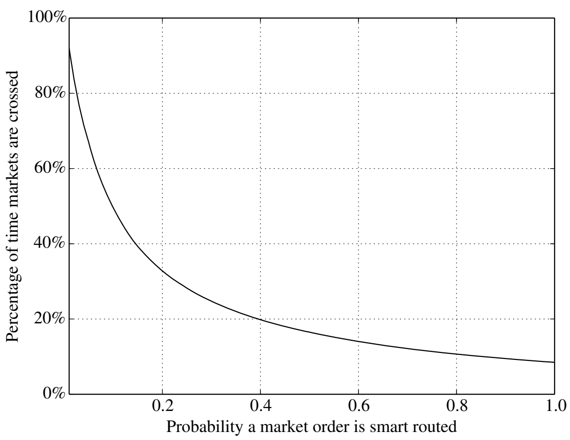

Fig. 5. The percentage of time markets are crossed a function of the probability that market orders are routed.

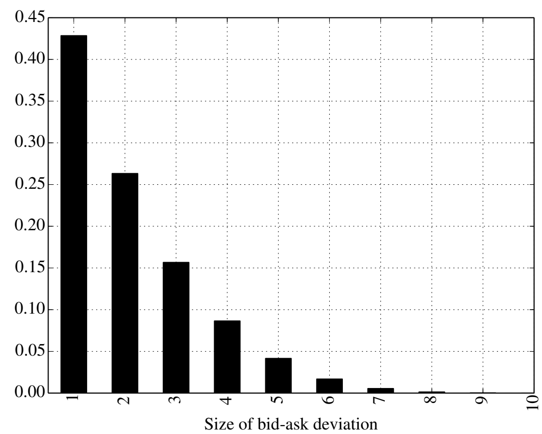

Fig. 7. The density of bid-ask deviations conditional on being in a crossed state ($s = 1$). A bid-ask deviation is defined as $b_1(t) - a_2(t)$ or $b_2(t) - a_1(t)$.

V. Results and Discussion

Results are estimated by simulation using parameters $v = 10, \ell = 1, m = 5, c = 0.2$. The values of the parameters are chosen according to Gatheral and Oomen (2010), who find that they generate a price series with properties typical of a DJ30 stock [11]. The probability of "smart order routing" is varied between 0.01 and 1 by increments of 0.01. As seen in Figure 3, market orders that trade through better quotes decreases linearly, as expected. Figure 4 shows the percentage of volume in excess of what is routed that executes at the best market. Figure 5 and Figure 6 show the percentage of time that markets are either crossed or locked as the probability that market orders are routed varies. Interestingly, the percentage of time that the market is in an arbitrage state drops quickly to below 25% with only approximately 30% of market orders routing to the better quotes. However, even with all market orders being routed optimally the markets are still either crossed or locked over 15% of the time (8.5% crossed, 7% locked).

Figure 7 depicts the density of bid-ask deviations (defined as $b_1(t) - a_2(t)$ or $b_2(t) - a_1(t)$) conditional on the market being in a crossed-state. Surprisingly, even with all market orders being routed to the best quotes, bid-ask deviations are 5 ticks or greater approximately 6.5% of the time that the market is in a crossed state. The mechanics of trading are such that better market integration may only be achieved by the strategic behavior of market participants employing limit orders.

Clearly market order routing is not the only form of strategic behavior linking markets. Arbitrageurs actively seek to exploit price dislocations. Market participants employing limit orders, whether they be market makers or passive traders, also monitor the state of the markets and modify their limit orders accordingly. Future work includes extensions to the model that also captures the aggregate behavior market participants employing limit orders.

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