2015 IEEE Symposium Series on Computational Intelligence

Winning in Retail Market Games: Relative Profit and Logit Demand

Jasper Hoogland
CWI Amsterdam and TU Delft
Email: hoogland@cwi.nl

Mathijs M. de Weerdt
TU Delft
Email: M.M.deWeerdt@tudelft.nl

Han La Poutré
CWI Amsterdam and TU Delft
Email: han.la.poutre@cwi.nl

***Abstract*—We examine retailers that maximize their relative profit, which is the (absolute) profit relative to the average profit of the other retailers. Customer behavior is modelled by a multinomial logit (MNL) demand model. Although retailers with low retail prices attract more customers than retailers with high retail prices, the retailer with the lowest retail price, according to this model, does not attract all the customers. We provide first and second order derivatives, and show that the relative profit, as a function of the retailer's own price, has a unique local maximum. Our experiments show that relative profit maximizers "beat" absolute profit maximizers, i.e. they outperform absolute profit maximizers if the goal is to make a higher profit. These results provide insight into market simulation competitions, such as the Power TAC.**

I. Introduction

In some situations, retailers (also referred to as firms) have the incentive to "win", by making a higher profit than other retailers, rather than to maximize their own profit. In the literature, several reasons for this are suggested. First, managers of firms may be rivalrous by nature [1]. Second, firms with an incentive to maximize their profit in the long term may sacrifice their profit in the short term, in order to gain a larger market share [2]. This way, other firms are pushed out of the market and new firms are prevented from entering, eventually to the benefit of the firm's profit. Another possibility is that managers are paid according to their performance relative to other managers [1], [2]. Finally, in tournaments where the participants are ranked by their profit (e.g. [3], [4]), the incentive is to make more profit than others, rather than to maximize one's own profit.

If the goal is to win, firms are known to perform better if they maximize their relative profit instead of their (absolute) profit [5]. Relative profit is the absolute profit minus the average absolute profits of the other firms (possibly weighted by some factor). Though relative profit maximization is not necessarily the same as winning, it is intuitive that relative profit maximizers, by taking into account the other firms' profits, outperform absolute profit maximizers if the goal is to win.

In this paper, we study relative profit maximizers in retail markets. To model customer behavior, we use the multinomial logit (MNL) demand model [6], which is among the most widely used stochastic demand models (see Section II). It has the desired property that customers do not necessarily buy a product at the lowest possible price, though the probability of buying a product is higher if its price is lower. There are several reasons why a customer may buy a more expensive product, despite the availability of a cheaper alternative. They may do this either intentionally, e.g. if they believe the more expensive product is better, or unintentionally, e.g. if they are unaware of the cheaper alternative. In addition, the MNL model has the mathematically elegant property that the ratio between the probabilities of choosing among two products is independent of the probabilities of buying other products.

Due to its desired properties, MNL demand is a useful model to study in combination with relative profit. In particular, an interesting application of MNL demand and relative profit maximization is the Power TAC competition [4]. The participants of this tournament create retailer agents that trade electricity in a simulation of a smart grid electricity market. Customers in this simulation behave according to the MNL model. The winner of the competition is the retailer with the highest profit, so participants have the incentive to create relative profit maximizers.

To our knowledge, relative profit maximizers have not yet been studied in combination with the MNL demand model, despite its desired properties. The goal of this paper is to provide this connection. The next section describes previous work on relative profit and MNL demand. In Section III, we formally describe the MNL demand model, and we introduce the concept of relative profit. In Section IV, we give the first and second order derivatives of the relative profit with respect to the own price, and we show that relative profit has a unique local maximum. We also show that, unlike absolute profit maximizers, a relative profit maximizer may, under certain conditions, have a retail price lower than the cost price. Furthermore, we show this retail price may be part of a Nash equilibrium. We give necessary and sufficient conditions under which a relative profit maximizer is more competitive than an absolute profit maximizer in the same situation. As opposed to what previously has been assumed, conditions exist under which a relative profit maximizer is less competitive. Section V describes and discusses the experiments we performed in order to examine the performance of relative profit maximizers if the goal is making more profit than other retailers. The results suggest that relative profit maximizers outperform absolute profit maximizers in all settings. Finally, in Section VI, we summarize and conclude.

II. Related Work

Though relative profit maximization has not been studied as extensively as absolute profit maximization, a considerable number of papers have been published over the years. Bishop

978-1-4799-7560-0/15 $31.00 © 2015 IEEE
DOI 10.1109/SSCI.2015.250

IEEE computer society

[7] suggests that relative profit maximization as a "warfare" strategy is a plausible outcome of duopoly market games where the firms compete on quantity. Shubik and Levitan [8] describe "beat-the average" games. Lundgren [1] suggests that collusion can be prevented by installing incentives to maximize relative profit. Donaldson and Neary [9] show that, if relative profit incentives are imposed, efficiency is achieved in a socialist industry. Matsumura et al. [10] examine relative profit maximizers in the context of R&D expenditure.

Several papers have shown that, if the goal is to have a higher profit than other firms, relative profit maximizers outperform absolute profit maximizers. Jones [3] describes a "classroom game" where firms compete on quantity, and the firm with the highest profit wins. In this game, successful players aim for relative profit maximization. In an evolutionary setting, Schaffer [5] and Vega-Redondo [11] show that, in Cournot-models, a firm outperforms another firm by unilaterally deviating from the Cournot equilibrium. By doing so, the firm decreases its own profit, but decreases the other firm's profit even more. It is shown that games with only relative profit maximizers result in the Bertrand equilibrium, in which firms sell their products at the cost price. Riechmann [2] analyzes a Cournot model in which the firms are a mix of absolute and relative profit maximizers, and shows that the equilibrium lies between the Cournot and Bertrand equilibria.

All the work mentioned so far uses Cournot models, in which the market price is determined by the total output of all firms. In such models, all firms with a non-zero output sell their products at this price. However, in many situations this is unrealistic, because firms can have different retail prices and still have a non-zero output. A few exceptions, though, exist in the literature. First, Appendix C in the paper by Lundgren [1] analyzes a case with heterogeneous products with different prices. It is shown that relative profit maximizers are more competitive than absolute profit maximizers, provided that both firms have a positive price-cost margin. Besides this conclusion, however, no extensive analysis is provided. Second, Nakamura and Saito [12] use a duopoly model allowing different prices, but their paper mainly focusses on capacity choice, which is not directly related to our problem. Finally, Satoh and Tanaka [13] analyze relative profit maximization for an oligopoly market that allows different prices. They show the existence of pure strategy equilibria, but they do not address the issue of outperforming other firms.

To our knowledge, our paper studies relative profit for MNL demand for the first time. This model, as opposed to Cournot models, has the desired property that firms can have different prices and still have a non-zero output. For this model, we derive some properties that have not been reported for other models, such as the possibility of prices lower than the cost price and situations in which relative profit maximizers are less competitive than absolute profit maximizers. Furthermore, we study the performance of relative profit maximizers if the goal is to outperform each other.

Despite their absence in the relative profit literature, MNL demand models [6] are common models for customer behavior in the economics literature. In most of this research, firms are assumed to be absolute profit maximizers [14]–[20]. Some research has been conducted, though, on firms with different objective functions (e.g. risk averse firms [21]).

III. Retail Market Model

Here we describe the MNL demand model [6], which expresses the demand of customers as a function of the retail prices of the retailers in the market. We also introduce the concept of relative profit. Though more research has been conducted on absolute profit, in some situations, relative profit maximizers outperform absolute profit maximizers. In particular, if retailers have the goal to have a higher profit than other retailers, then relative profit maximizers perform better than absolute profit maximizers.

A. MNL Demand

The MNL demand model is a stochastic choice model, which means the choice of a customer is sampled from a probability distribution. A customer chooses from a finite set of products or chooses not to buy any of them. The latter is called the exit option, and it means that the customer either buys elsewhere or does not buy at all. The MNL model assigns a probability to every option. The model can also be used to model a group of customers with the same choice model. In that case, the MNL demand is interpreted as the proportion of customers that buy the product. In our model, every retailer sells a single type of product. These products are homogeneous, i.e. all products have the same value to the customer, and preferences are only determined by the retail prices. Although cheaper products are bought more often than more expensive products, customers do not only buy the cheapest product. Furthermore, the ratio between the demand of two products is independent of the demand of other products.

We now give an expression for the MNL demand. Let n be the number of retailers and let $\vec{p} = \langle p_1, \ldots, p_n \rangle$ be the vector of retail prices of the retailers' products. The probability $q_i(\vec{p})$ that a customer buys from retailer i is

$$q_i(\vec{p}) = \frac{e^{-\lambda p_i}}{e^{-\lambda p_0} + \sum_{j=1}^{n} e^{-\lambda p_j}}, \tag{1}$$

where $\lambda > 0$ and $p_0 > 0$ are parameters of the model. Note that $0 < q_i(\vec{p}) < 1$. The parameter λ determines the amount of stochasticity. In the extreme case of $\lambda = 0$, the customer's choice has a uniform distribution, regardless of the retail prices. The other extreme, $\lambda = \infty$, is the deterministic case, where the customer buys the cheapest product with probability 1. The parameter p_0 specifies the price of the exit option. Depending on the application, customers that choose the exit option buy the product elsewhere, or they do not buy it at all.

B. Absolute and Relative Profit

Here, we give a formal definition of relative profit. We assume each retailer i has a constant marginal cost c_i. The absolute profit π_i of retailer i is

$$\pi_i(\vec{p}) = (p_i - c_i) q_i(\vec{p}). \tag{2}$$

The relative profit $\rho_i^\alpha(\vec{p})$ of retailer i is defined as

$$\rho_i^\alpha(\vec{p}) = \pi_i(\vec{p}) - \alpha \sum_{1 \le k \le n, k \ne i} \pi_k(\vec{p}). \tag{3}$$

The parameter α specifies the degree of relativity. It can have any real value (even less than 0 for colluding retailers). In this paper we assume $\alpha \geq 0$. Three special cases are to be distinguished. First, if $\alpha = 0$, then $\rho_i^\alpha(\vec{p})$ coincides with the absolute profit. Second, if $\alpha = \frac{1}{n-1}$ then $\rho_i^\alpha(\vec{p})$ is the retailer's own profit minus the average profit of the other retailers. If the goal is to maximize relative profit with $\alpha = \frac{1}{n-1}$, then the game is zero-sum, because $\sum_{i=1}^n \rho_i^{\frac{1}{n-1}}(\vec{p}) = 0$. Finally, if $\alpha = 1$, then an increase of another retailer's profit has the same impact on $\rho_i^\alpha(\vec{p})$ as a decrease by the same amount of the retailer's own profit. In some papers, the definition of relative profit is restricted to $\alpha = \frac{1}{n-1}$ (e.g. [1], [13]), while others use the same definition as ours (e.g. [10], [12]). For a more general version of relative profit, we refer to Koçkesen [22].

IV. Analysis

Here we give some useful properties of the MNL demand model, the absolute profit, and the relative profit. The derivative of the demand $q_i(\vec{p})$ with respect to p_i is

$$\frac{\partial q_i}{\partial p_i} = -\lambda q_i(\vec{p})(1 - q_i(\vec{p})). \tag{4}$$

Note that $\frac{\partial q_i}{\partial p_i} < 0$, so the higher the retail price, the lower the probability that the customer buys from i. Furthermore, for $k \neq i$, the derivative of $q_k(\vec{p})$ with respect to p_i is

$$\frac{\partial q_k}{\partial p_i} = \lambda q_k(\vec{p}) q_i(\vec{p}).$$

This implies $\frac{\partial q_k}{\partial p_i} > 0$, so the higher the retail price of one retailer, the higher the probability that the customer buys from another retailer.

The derivative of the absolute profit with respect to p_i is

$$\frac{\partial \pi_i}{\partial p_i} = q_i(\vec{p})\Big[1 - \lambda(p_i - c_i) + \lambda \pi_i(\vec{p})\Big].$$

The derivative of the profit of retailer k with respect to the price p_i of another retailer $i \neq k$ is

$$\frac{\partial \pi_k}{\partial p_i} = \lambda q_i(\vec{p}) \pi_k(\vec{p}).$$

Hence, the first derivative of the relative profit is

$$\frac{\partial \rho_i^\alpha}{\partial p_i} = q_i(\vec{p})\Big[1 - \lambda(p_i - c_i) + \lambda \rho_i^\alpha(\vec{p})\Big]. \tag{5}$$

The second derivative of the relative profit of retailer i is

$$\frac{\partial^2 \rho_i^\alpha}{\partial p_i^2} = \lambda\Big[(2q_i(\vec{p}) - 1)\frac{\partial \rho_i^\alpha}{\partial p_i} - q_i(\vec{p})\Big]. \tag{6}$$

The following proposition shows that the relative profit has exactly one maximum.

Proposition 1: Let $0 \leq \alpha \leq 1$. Then, the relative profit $\rho_i^\alpha(\vec{p})$ has exactly one maximum for p_i, viz. for which $\frac{\partial \rho_i^\alpha}{\partial p_i} = 0$.

Proof: First, we show a stationary point exists, then we show this point is a unique local maximum.

By substitution of Equations (2) and (3) into Equation (5), any p_i satisfying

$$\lambda(p_i - c_i)(1 - q_i(\vec{p})) = 1 - \lambda\alpha \sum_{1 \leq k \leq n, k \neq i} \pi_k(\vec{p})$$

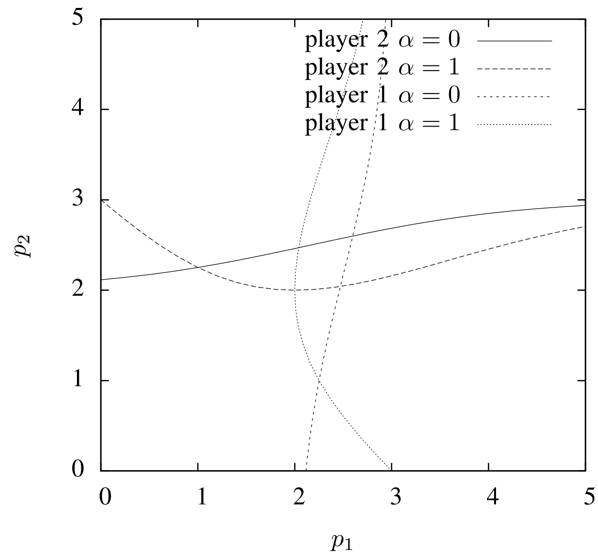


Fig. 1. This diagram shows the best responses of both players in 2 player games. The parameters of the games are $p_0 = 3$ and $\lambda = 1$. The cost price is $c_i = 1$ for both players i. For each player, the best response is shown as a function of the other player's price for absolute ($\alpha = 0$) and relative ($\alpha = 1$) profit.

is a stationary point of the relative profit. The left hand side of this equation approaches 0 if $p_i \to -\infty$, and approaches ∞ if $p_i \to \infty$. The right hand side approaches 1 if $p_i \to -\infty$, and approaches some real number if $p_i \to \infty$. Since both sides of the equation are continuous, there is at least one p_i that satisfies the equation, and hence is a stationary point of the relative profit.

By Equation (6), $\frac{\partial \rho_i^\alpha}{\partial p_i} = 0$ implies

$$\frac{\partial^2 \rho_i^\alpha}{\partial p_i^2} = -\lambda q_i(\vec{p}) < 0,$$

so any stationary point is a local maximum. Moreover, since all stationary points are local maxima, there is at most one local maximum. ■

Figure 1 shows the best responses for two retailers in duopoly games. Nash equilibria exist where the graphs of retailers 1 and 2 intersect.

For Cournot games, it has been reported that relative profit maximizers with $\alpha = \frac{1}{n-1}$ result in the Bertrand equilibrium [5], [11], i.e. they have retail prices equal to the cost price. This means equilibrium prices are lower for relative profit maximizers than for absolute profit maximizers. The next proposition shows a similar property for MNL demand duopoly markets for retailers with the same cost prices.

Proposition 2: For duopoly markets ($n = 2$) with $p_0 = \infty$ in which the two retailers are relative profit maximizers with the same $\alpha_i = \alpha$ and the same cost price $c_i = c$ for $i \in \{1, 2\}$, a symmetric Nash equilibrium exists such that, for both retailers $i \in \{1, 2\}$,

$$p_i - c = \frac{2}{\lambda(1 + \alpha)}. \tag{7}$$

Proof: Since $p_1 = p_2$, it holds that $q_1(\vec{p}) = q_2(\vec{p}) = \frac{1}{2}$. Therefore, by Equation 5, it holds that

$$\frac{\partial \rho_i^\alpha}{\partial p_i} = 0 \Leftrightarrow (p_i - c) - \pi_i(\vec{p}) + \alpha \pi_k(\vec{p}) = \frac{1}{\lambda}$$
$$\Leftrightarrow p_i - c = \frac{2}{\lambda(1+\alpha)}$$

for $i, k \in \{1, 2\}, i \neq k$. Hence, by Proposition 1, Equation (7) for $i \in \{1, 2\}$ is a Nash equilibrium. ■

From Proposition 2, under the conditions assumed by this proposition (e.g. the same cost price for both retailers), it follows that equilibrium prices are lower for relative profit maximizers ($\alpha > 0$) than for absolute profit maximizers ($\alpha = 0$), because p_i decreases with respect to α. Unlike Cournot games, however, the retail prices are still higher than the cost price. The retail prices only approach the cost price for high values of λ.

Under different conditions, though, relative profit maximizers have retail prices that are even lower than the cost price. For example, if a relative profit maximizer has a cost price much higher than the cost price of another retailer, then the relative profit maximizer may accept a negative profit, in order to prevent the other retailer from having a high market share and, consequently, a high profit. Moreover, there are games for which a Nash equilibrium exists such that the retail price of one of the retailers is lower than the cost price. This is shown by the following proposition.

Proposition 3: There are games with relative profit maximizers, in which a Nash equilibrium exists such that one of the retailers has a retail price lower than the cost price.

Proof: We give an example of such a game with two relative profit maximizers with $\alpha = 1$. Let the parameters of the MNL demand model be $\lambda = 1$ and $p_0 = \infty$, and let the cost prices be $c_1 = 5$ and $c_2 = 1$. Then, $p_1 = p_2 = 4$ is a Nash equilibrium. By Equation (5), if $p_2 = 4$, then $p_1 = 4$ is a stationary point of the relative profit $\rho_1^\alpha(\vec{p})$ of retailer 1, and by Proposition 1, it is the optimal value. Similarly if $p_1 = 4$, then $p_2 = 4$ maximizes the relative profit $\rho_2^\alpha(\vec{p})$ of retailer 2. Thus, $p_1 = p_2 = 4$ is a Nash equilibrium. In this equilibrium, the retail price $p_1 = 4$ of retailer 1 is lower than the cost price $c_1 = 5$. ■

A retailer is more competitive than another retailer if it has a lower retail price than the other retailer in the same situation. Retailer 1 in Proposition 3 is more competitive than an absolute profit maximizer, because absolute profit maximizers never have retail prices lower than the cost price. In addition, for $n = 2$ retailers and $\alpha = 1$, Lundgren has shown that a relative profit maximizer is more competitive than an absolute profit maximizer, provided that both retailers have positive price-cost margins [1] (Appendix C). For MNL demand and arbitrary $n \geq 2$ and $\alpha > 0$, it can be shown that a relative profit maximizer is more competitive than an absolute profit maximizer if the sum of the profits of the other retailers is positive. However, if this sum is negative, then the opposite holds. In that case a relative profit maximizer is less competitive than an absolute profit maximizer. Both cases are covered by Proposition 4. First, we prove the following property of MNL demand.

Property 1: The sign of $\sum_{k \neq i} \pi_k(\vec{p})$ does not depend on p_i.

Proof: By Equations (1) and (2), the sum of the profits of the other retailers $k \neq i$ is

$$\sum_{1 \leq k \leq n, k \neq i} \pi_k(\vec{p}) = \sum_{1 \leq k \leq n, k \neq i} (p_k - c_k) q_k(\vec{p})$$
$$= \frac{1}{\sum_{j=0}^n e^{-\lambda p_j}} \sum_{1 \leq k \leq n, k \neq i} (p_k - c_k) e^{-\lambda p_k}. \tag{8}$$

Though $\sum_{k \neq i} \pi_k(\vec{p})$ depends on p_i, whether or not it is positive is independent of p_i. This holds, because $\frac{1}{\sum_{j=0}^n e^{-\lambda p_j}}$ is always positive, so the sign of $\sum_{k \neq i} \pi_k(\vec{p})$ only depends on $\sum_{k \neq i} (p_k - c_k) e^{-\lambda p_k}$, which is independent of p_i. ■

Due to Property 1, we can speak of $\sum_{k \neq i} \pi_k(\vec{p}) > 0$ regardless of p_i.

Proposition 4: Given the prices $p_1, \ldots, p_{i-1}, p_{i+1}, \ldots, p_n$ of the other retailers, the retail price of retailer i is lower for a relative profit maximizer (with $\alpha \geq 0$) than for an absolute profit maximizer if and only if the sum of the absolute profits of the other retailers is positive.

Proof: Let p_i^a be the retail price of an absolute profit maximizer ($\alpha = 0$), let $\vec{p}^a = \langle p_1, \ldots, p_{i-1}, p_i^a, p_{i+1}, \ldots, p_n \rangle$ be the retail price vector with $p_i = p_i^a$, and let $q_i^a = q_i(\vec{p}^a)$ be the demand given this price. Similarly, let p_i^r be the retail price of a relative profit maximizer for some $\alpha > 0$, let $\vec{p}^r = \langle p_1, \ldots, p_{i-1}, p_i^r, p_{i+1}, \ldots, p_n \rangle$ be the retail price vector with $p_i = p_i^r$, and let $q_i^r = q_i(\vec{p}^r)$ be the demand given this price. Thus, our goal is to prove that

$$p_i^r < p_i^a \Leftrightarrow \sum_{1 \leq k \leq n, k \neq i} \pi_k(\vec{p}) > 0. \tag{9}$$

By Proposition 1, $\frac{\partial \rho_i^\alpha}{\partial p_i} = 0$ for $p_i = p_i^r$, and $\frac{\partial \rho_i^0}{\partial p_i} = 0$ for $p_i = p_i^a$. Therefore, by substitution of Equations (2) and (3) into Equation (5), it holds that

$$\lambda(p_i^r - c_i)(1 - q_i^r) = 1 - \lambda\alpha \sum_{1 \leq k \leq n, k \neq i} \pi_k(\vec{p}^r), \tag{10}$$

with $\vec{p}^r = \langle p_1, \ldots, p_{i-1}, p_i^r, p_{i+1}, \ldots, p_n \rangle$, and, by choosing $\alpha = 0$,

$$\lambda(p_i^a - c_i)(1 - q_i^a) = 1. \tag{11}$$

Note also that $p_i^a > c_i$ holds by Equation (11) and $0 < q_i^a < 1$. First, we assume $p_i^r \leq c_i$. Then, $p_i^r < p_i^a$ and, by Equation (10), $\sum_{k \neq i} \pi_k(\vec{p}) > 0$, so Equation (9) follows from $p_i^r \leq c_i$.

We now assume $p_i^r > c_i$. Then, for p_i in the interval between p_i^r and p_i^a, both $(p_i - c_i)$ and $(1 - q_i(\vec{p}))$ are positive and increase monotonically with respect to p_i (By Equation (4)), so their product $(p_i - c_i)(1 - q_i(\vec{p}))$ also increases monotonically with respect to p_i in this interval. Therefore,

$$p_i^r < p_i^a \Leftrightarrow (p_i^r - c_i)(1 - q_i^r) < (p_i^a - c_i)(1 - q_i^a).$$

Furthermore, by equations (11) and (10), it holds that

$$\sum_{1 \leq k \leq n, k \neq i} \pi_k(\vec{p}) > 0 \Leftrightarrow (p_i^r - c_i)(1 - q_i^r) < (p_i^a - c_i)(1 - q_i^a).$$

Therefore, Equation (9) follows from $p_i^r > c_i$.

Thus, regardless of $p_i^r > c_i$, Equation (9) holds, so Proposition 4 has been proven. ■

V. EXPERIMENTS

We performed experiments in order to examine the performance of relative profit maximizers if their goal is to win, i.e. to outperform each other. We evaluate retailers by computing their relative performance S, which we define as the retailer's profit divided by the highest profit of the other retailers: $S = \frac{\pi_i}{\max_{k \neq i} \pi_k}$. In addition to expressing whether a retailer has the highest profit ($S > 1$), the relative performance expresses, in case of a victory, how much better a retailer is than the second best retailer, or, in case of a loss, how close the retailer is to the winner.

A. Settings

We analyzed retail market games with 3 players and with 4 players. We tested best response strategies for relative profit with α-values in the interval $[0, 1]$. For the 3 player games, we used step size 0.01 for α, resulting in 101 different relative profit maximizers. For the 4 player games, we used step size 0.02 for α, resulting in 51 different relative profit maximizers. We ran games for all combinations of retailer strategies. Since we examine relative profit maximizers, a strategy corresponds to a choice for α. Every game consisted of at most 30 iterations. In each iteration, the retailers choose the best-response retail price given the retail prices of the previous iteration. The game is terminated if the retail prices have converged before the last iteration. As parameter values, we chose $\lambda \in \{1, 2\}$, $p_0 \in \{2, 4\}$, and the same unit cost price $c_i = 1$ for all retailers i. The best response relative profit maximizers were implemented in Java using gradient ascent. As initial price for the gradient ascent algorithm, we chose $p_{init} = 2$. For each game, we recorded the number of iterations and, for each player, the final retail price and profit.

B. Results

In all games, the retail prices converged before the last iteration. This provides empirical evidence for the existence of a Nash equilibrium. We tried different values for the parameters λ and p_0, but this had no major impact on the results. Here, we present the results for the games with $\lambda = 1$ and $p_0 = 4$. For each game in the experiments, we computed the relative performance $\frac{\pi_i}{\max_{k \neq i} \pi_k}$. The results of the experiments are summarized in Figure 2. For each α, the best case and worst case relative performances over all games are shown.

In games with 3 players, the best case and worst case relative performances have a peak at $\alpha = 0.5$. Around this point, the worst case relative performance is approximately 1, so, at worst, these retailers are never far from having the highest profit. The highest relative performances were achieved in games against opponents with α-values close to either 0 or 1. In these games the profit was up to 5.95% higher than the opponents' profits. Figure 3 shows the relative performance of a retailer with $\alpha = 0.5$ against all combinations of opponent strategies.

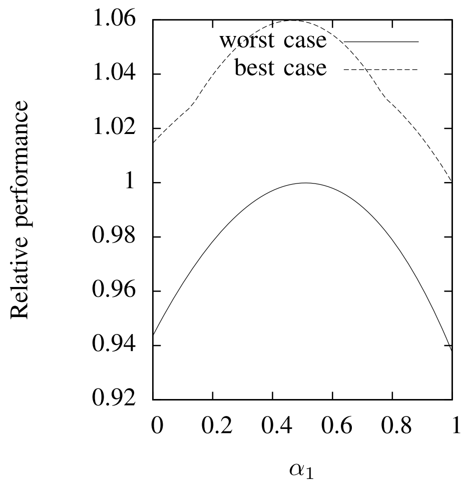
In games with 4 players, the best case and worst case relative performances have peaks around $\alpha = 0.32$ and $\alpha = 0.34$ respectively. For the retailer with $\alpha = 0.34$, the worst case relative performance is approximately 1, so, at worst, this retailer is never far from having the highest profit. The highest relative performances were achieved in games against opponent retailers with α-values close to 1, in which the relative profit maximizer with $\alpha = 0.32$ had a profit up to 8.72% higher than the opponents' profits. This is not a surprise, as these α-values are the furthest from the best performing retailers. High performance is also achieved in games against absolute profit maximizers, in which the relative profit maximizer with $\alpha = 0.32$ had a profit up to 3.50% higher than the opponents' profits.

The good performance of the retailer with $\alpha = 0.5$ in 3-player games and the retailers with $\alpha = 0.32, 0.34$ in 4-player games can be explained as follows. If the goal is to outperform other retailers, then it is a zero-sum game, and an increase of the retailer's absolute profit is equally beneficial as a decrease of all opponents' absolute profits by the same amount. This is achieved by optimizing the retailer's own profit minus the average profit of the other retailers, which is equivalent to maximizing relative profit with $\alpha = \frac{1}{n-1}$. Though extrapolation to games with $n > 4$ players seems likely, more experiments are required to prove this.

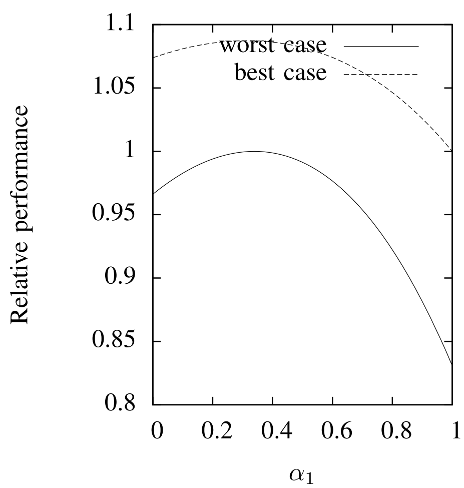
The worst case performances of the best strategies ($\alpha = \frac{1}{2}$ for 3 players and $\alpha = 0.32, 0.34$ for 4 players) were slightly lower than 1.0, so they did not win in all situations. At five digits after the comma (e.g. $S = 0.99995$ for $\alpha = \frac{1}{2}$ 3 player games, and 0.99996 for $\alpha = 0.34$ in 4 player games), we found that the relative performance was slightly less than 1 in some situations. In games in which this occurs, retailer 1 has $\alpha = \frac{1}{n-1}$, retailer 2 has an α-value slightly higher than $\frac{1}{n-1}$, and other retailers have α-values close to 0 or 1 (e.g. in the 3 player game with $\alpha_2 = 0.52$ and $\alpha_3 = 0$). These results can be explained as follows. In these games, only the strategies of retailers 1 and 2 are good enough to compete for the highest profit, and the other retailers can be ignored. Therefore, an α-value slightly higher than $\frac{1}{n-1}$ performs better than $\alpha = \frac{1}{n-1}$ in these cases.

VI. CONCLUSIONS

We analyze retailers that maximize their relative profit for the MNL demand model. We derive first and second order equations of the relative profit, and show that relative profit has a unique local maximum with respect to the retailer's own price. We also show that, under certain conditions, the optimal retail price of a relative profit maximizer is lower than the cost price. In addition, we give a necessary and sufficient condition in which a relative profit maximizer is more competitive than an absolute profit maximizer. This holds if and only if the sum of the profits of other retailers is positive. We performed experiments in order to examine the relation between relative profit maximization and winning. We simulated games with 3 and 4 players with different combinations of α-values in the range of $[0, 1]$. The results show that retailers with $\alpha = \frac{1}{2}$ have a high performance in games with 3 players, and retailers with $\alpha = \frac{1}{3}$ have a high performance in games with 4 players. Furthermore, by extrapolating to games with $n > 4$ retailers, these experiments suggest that retailers outperform others if



(a) 3 players



(b) 4 players

Fig. 2. Relative performance of retailer 1 for different values of α_1. Relative performance is defined as the retailer's profit divided by the maximum of the profits of the other retailers in the game. For each α_1, the graphs show the best and the worst relative performance of the retailer over all games.

they maximize the difference between their absolute profit and the average profit of the other retailers ($\alpha = \frac{1}{n-1}$).

An application of relative profit maximization to MNL demand is the Power TAC competition. The Power TAC is a smart grid electricity market simulation in which customers behave according to the MNL model, and participants have the incentive to outperform others rather than to maximize their own profit. In order to deal with the complexity of the simulation, participants agents have used computational intelligence techniques, such as reinforcement learning [23], [24], particle swarm optimization [25], and other adaptive strategies [26]–[28]. However, none have so far addressed the issue of relative profit maximization. The insights provided by our analysis can be used to enhance computational intelligence-based Power TAC agents.

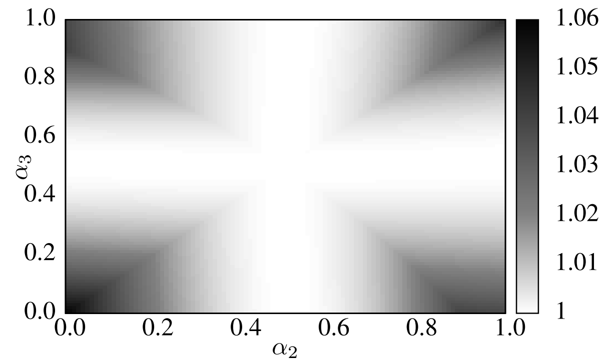


Fig. 3. This diagram shows the relative performance $\frac{\pi_1}{\max_{k \in 2,3} \pi_k}$ of retailer 1 with $\alpha_1 = \frac{1}{2}$ in games with 3 players. The axes show the α_j-values of the other retailers $j = 2, 3$. The diagram shows that retailer 1 has the highest relative performance in games in which the other retailers have α_j-values close to either 0 or 1.

REFERENCES

[1] C. Lundgren, "Using relative profit incentives to prevent collusion," *Review of Industrial Organization*, vol. 11, no. 4, pp. 533–550, 1996.

[2] T. Riechmann, "Mixed motives in a cournot game," *Economics Bulletin*, vol. 4, no. 29, pp. 1–8, 2006.

[3] M. Jones, "Note on oligopoly: Rival behavior and efficiency," *The Bell Journal of Economics*, pp. 709–714, 1980.

[4] W. Ketter, J. Collins, P. P. Reddy, and M. de Weerdt, "The 2015 power trading agent competition," *ERIM Report Series Reference No. ERS-2015-001-LIS*, February 2015.

[5] M. E. Schaffer, "Are profit-maximisers the best survivors?: A darwinian model of economic natural selection," *Journal of Economic Behavior & Organization*, vol. 12, no. 1, pp. 29–45, 1989.

[6] D. McFadden, "The revealed preferences of a government bureaucracy: Theory," *The Bell Journal of Economics*, pp. 401–416, 1975.

[7] R. L. Bishop, "Duopoly: Collusion or warfare?" *The American Economic Review*, pp. 933–961, 1960.

[8] M. Shubik and R. Levitan, *Market structure and behavior*. Harvard University Press Cambridge, MA, 1980.

[9] D. Donaldson and H. Neary, "Decentralized control of a socialist industry," *Canadian Journal of Economics*, pp. 99–110, 1984.

[10] T. Matsumura, N. Matsushima, and S. Cato, "Competitiveness and r&d competition revisited," *Economic Modelling*, vol. 31, pp. 541–547, 2013.

[11] F. Vega-Redondo, "The evolution of walrasian behavior," *Econometrica: Journal of the Econometric Society*, pp. 375–384, 1997.

[12] Y. Nakamura and M. Saito, "Capacity choice in a mixed duopoly: The relative performance approach," *Theoretical Economics Letters*, vol. 3, no. 02, p. 124, 2013.

[13] A. Satoh and Y. Tanaka, "Choice of strategic variables under relative profit maximization in asymmetric oligopoly," *Economics and Business Letters*, vol. 3, no. 2, pp. 115–126, 2014.

[14] S. P. Anderson and A. De Palma, "The logit as a model of product differentiation," *Oxford Economic Papers*, pp. 51–67, 1992.

[15] F. Bernstein and A. Federgruen, "A general equilibrium model for industries with price and service competition," *Operations research*, vol. 52, no. 6, pp. 868–886, 2004.

[16] D. Besanko, S. Gupta, and D. Jain, "Logit demand estimation under competitive pricing behavior: An equilibrium framework," *Management Science*, vol. 44, no. 11-part-1, pp. 1533–1547, 1998.

[17] A. Caplin and B. Nalebuff, "Aggregation and imperfect competition:

On the existence of equilibrium," *Econometrica: Journal of the Econometric Society*, pp. 25–59, 1991.

[18] W. Hanson and K. Martin, "Optimizing multinomial logit profit functions," *Management Science*, vol. 42, no. 7, pp. 992–1003, 1996.

[19] G. J. Werden and L. M. Froeb, "The effects of mergers in differentiated products industries: Logit demand and merger policy," *Journal of Law, Economics, & Organization*, pp. 407–426, 1994.

[20] W. R. Morrow and S. J. Skerlos, "On the existence of bertrand-nash equilibrium prices under logit demand," *arXiv preprint arXiv:1012.5832*, 2010.

[21] C. B. Resende, C. G. Heckmann, and J. J. Michalek, "Robust design for profit maximization under uncertainty of consumer choice model parameters using the delta method," in *ASME 2011 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference*. American Society of Mechanical Engineers, 2011, pp. 421–434.

[22] L. Koçkesen, E. A. Ok, and R. Sethi, "The strategic advantage of negatively interdependent preferences," *Journal of Economic Theory*, vol. 92, no. 2, pp. 274–299, 2000.

[23] J. Babic and V. Podobnik, "Adaptive bidding for electricity wholesale markets in a smart grid," in *Proceedings of the Workshop on Agent-Mediated Electronic Commerce and Trading Agent Design and Analysis (AMEC/TADA 2014) @ AAMAS 2014*. Paris, France: International Foundation for Autonomous Agents and Multiagent Systems (IFAAMAS), 2014, pp. 1–14.

[24] D. Urieli and P. Stone, "Tactex'13: A champion adaptive power trading agent," in *AAAI Conference on Artificial Intelligence*, 2014.

[25] E. Ntagka, A. Chrysopoulos, and P. A. Mitkas, "Designing tariffs in a competitive energy market using particle swarm optimization techniques," in *Agent-Mediated Electronic Commerce. Designing Trading Strategies and Mechanisms for Electronic Markets*. Springer, 2014, pp. 129–143.

[26] J. Hoogland and H. La Poutré, "An effective broker for the power tac 2014." Istanbul, Turkey: International Foundation for Autonomous Agents and Multiagent Systems (IFAAMAS), 2015.

[27] B. Liefers, J. Hoogland, and H. La Poutré, "A successful broker agent for power tac," in *Proceedings of the Workshop on Agent-Mediated Electronic Commerce and Trading Agent Design and Analysis (AMEC/TADA 2014) @ AAMAS 2014*. Paris, France: International Foundation for Autonomous Agents and Multiagent Systems (IFAAMAS), 2014, pp. 1–14.

[28] S. Özdemir and R. Unland, "Agentude: The success story of the power tac 2014's champion." Istanbul, Turkey: International Foundation for Autonomous Agents and Multiagent Systems (IFAAMAS), 2015.