Fuzzy maximal covering location models for fighting dengue

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***Abstract*—The global incidence of dengue has grown dramatically in recent decades and about half of the world's population is now at risk. The proper deployment of mobile health teams is one of the tools available to control the mosquitoes population responsible of the illness transmission. The team deployment problem can be casted as a maximal coverage location problem with uncertainty due to the nature of the information available. Our aim is to propose and solve two fuzzy maximal covering location problems in the context of dengue control.**

We manage imprecision at the level of the number of teams available and at the coverage distance they may cover. Both cases are modeled through a fuzzy constraint. Computational experiments, using real data from Estado de Guerrero in México, have been conducted to analyze how the proposed models can be solved and what kind of information can be obtained to help a potential decision maker in taking a more informed decision.

I. Introduction

According to the World Health Organization[1] dengue is a viral infection causing flu-like illness, and occasionally develop into a potentially lethal complication called severe dengue. Dengue is transmitted by the Aedes aegypti mosquito. Its global incidence has grown dramatically in recent decades and about half of the world's population is now at risk. Figure 1 shows countries or areas where dengue has been reported. This mosquito, shown in Figure 2, also transmits other infections like Yellow Fever, Zika and Chikungunya.

As shown in Fig. 1, dengue is found in tropical and subtropical climates worldwide, mostly in urban and semi-urban areas. Severe dengue is a leading cause of serious illness and death among children in some Asian and Latin American countries. There is no specific treatment for dengue/ severe dengue, but early detection and access to proper medical care lowers fatality rates below 1%. Hence, dengue prevention and control depends on effective vector control measures.

Nowadays, the main method to control or prevent the transmission of dengue virus is to combat vector mosquitoes by means of personal, family and community protective measures as well as by including innovative measures to provide adequate health care and treatment. In this context, Mobile Health or Sanitary Teams (MHT), composed by medical doctors and other specialized assistants, play a key role as they are the bridge connecting the community with the experts.

[1] http://www.who.int/mediacentre/factsheets/fs117/en/

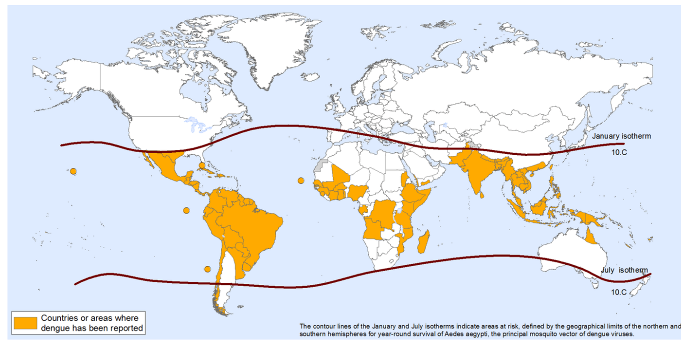


Fig. 1. Countries or areas where dengue has been reported

Fig. 2. Aedes aegypti mosquito

A proper deployment of the MHT is crucial to cover as much population as possible.

In mathematical terms, this situation can be casted as a Maximal Covering Location Problem (MCLP), which is one of the most used covering problems in the context of the location theory.

As stated in [1], Covering Location Problems (CLP) are usually addressed considering that the problem's parameters are precisely defined; i.e with neither uncertainty nor imprecision. However, in many real situations like the one considered in this contribution, uncertainty or imprecision can be present in some parameters, like the demand generated at nodes, the distance between the facilities and the demand nodes, the coverage distance, the server capacity, the installation cost, the number of facilities, etc. The location of MHT clearly fits such scenario.

In this context, and having in mind both the need of tools to help in the control of dengue transmission and the need of models that better address the information available, the aim of this contribution is to show how, new models of MCLP for locating MHT, properly addressing imprecision or vagueness, can be obtained and solved, thus providing new solutions for helping in the control of dengue transmission.

This paper is organized as follows. Section II presents the mathematical definition of MCLP. Then the proposed fuzzy models are presented. These models require some transformations in order to be solved, which are described in Section IV. Using information of diagnosed dengue cases in the city of Acapulco, México, in Section V we show the application of the proposed models. Conclusions and further work are outlined in Section VI.

II. Maximal Covering Location Problem Definition

The MCLP was originally proposed by Church and ReVelle [2] and later reconsidered in [3]. Several applications of the MCLP were recently reviewed in [1]. The problem considers a set of demand nodes and a set of facilities to be located. These facilities can be located within the same demand nodes. Each demand node has an associated value, which represent its level of importance (for example, the demand generated at the node). The objective of the MCLP is to find the best locations for a fixed number of facilities in order to maximize the demand covered. A demand node is considered covered if one facility exists, which can provide the required service at a coverage distance or travel time lower or equal than a predefined threshold. Once the problem is solved, some nodes can remain uncovered.

We describe here first the maximal covering location problem, where all the parameters are precisely known and defined.

Sets

- i, I index and set of the demand nodes.
- j, J index and set of potential locations for the facilities.
- N_i $\{j \in J | d_{ij} \leqslant S\}$ the set of potential facility locations that can cover the node i within the time or distance S, d_{ij} is the distance between the node i and the potential location for the facility j.

Input parameters

- p the number of facilities to be located.
- S the maximum allowed time or distance to respond to a request.
- w_i value that represent the demand associated to node i.

Decision variables

- x_j 1 if a facility is located at the node j, 0 otherwise.
- y_i represent the coverage of node i, 1 if node is covered $(\exists_j | x_j = 1 \wedge j \in N_i)$, 0 otherwise.

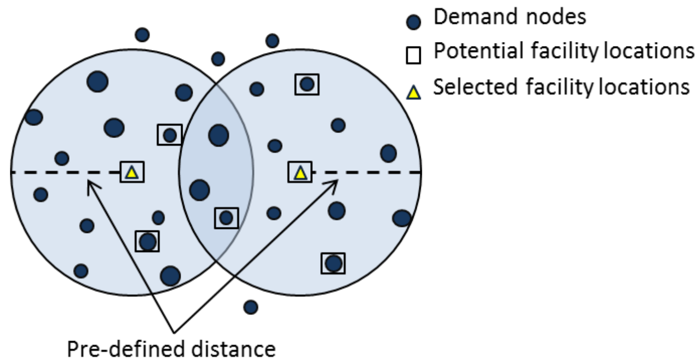


Fig. 3. A feasible solution of the MCLP with two facilities, seven potential facility locations, and twenty eight demand nodes.

Mathematical model

$$\max Z = \sum_{i \in I} w_i y_i \quad (1)$$

subject to

$$\sum_{j \in N_i} x_j \geqslant y_i \quad \forall i \in I \quad (2)$$

$$\sum_{j \in J} x_j = p \quad (3)$$

$$x_j = \{0, 1\} \quad \forall j \in J \quad (4)$$

$$y_i = \{0, 1\} \quad \forall i \in I \quad (5)$$

The objective function (1) aims at maximizing the demand covered by the set of established facilities. Constraint 2 states that one or more facilities will be located within the distance or travel time pre-defined S from the demand node i. Constraint 3 ensures that the number of facilities to be located is p. Finally, decision variables x_j and y_i are considered as binary.

As an example, Figure 3 shows a feasible solution for a MCLP with two facilities, which cover twenty four demand nodes. The size of the nodes indicates its demand. It should be noted that three demand nodes are covered by the two facilities and there are five uncovered nodes.

III. Towards Fuzzy Maximal Covering Location Problems

As we stated before, gathering the exact information to solve the previous model is far from trivial or even impossible. Even if, instead of precise values we have imprecise ones, the problems need to be solved. The usefulness of fuzzy sets here is obvious: it is somehow trivial to model parameters like the demand as fuzzy numbers. However, we may end with very nice theoretical models for which, there are no algorithms available to solve them.

A review of models and methods on fuzzy variants of MCLP has been recently presented in [4].

In the specific case of the maximal covering location problem in the context of MHT, we will present and solve two models: 1) posing a fuzzy restriction on the coverage distance of a MHT, thus assuming that a MHT is allowed to cover some extra distance; and 2) posing a fuzzy restriction on the number of MHT available. This situation reflects the fact that decision makers may state that "we have budget for around 10 MHT" which at the end may result in (let's say) 8, 9, 10, 11 or 12 MHT.

A. Model 1: fuzzy restriction on the coverage distance

In the MCLP, a demand node is covered by a facility if the distance between them is less than a given standard o coverage distance S. The coverage distance is usually considered exactly known. Thus, for example, nodes that are within a distance of 2 kilometers from a facility are fully covered, while those nodes that are at a distance of 2.05 kilometers are not covered. It is clear that this rigidity of the distance will lead to inappropriate and unrealistic solutions for the problem under study and it would be more reasonable to consider that a node is covered if there exist a facility at approximately 2.0 kms.

In mathematical terms this flexibility can be modeled using a fuzzy constraint.

In our particular model, we consider the following fuzzy constraint in the construction of the previously defined set N_i,

$$N_i = \{j \mid d_{ij} \leq_f S\} \tag{6}$$

where $\leq_f$ implies that the distance condition in 6 could be partially satisfied. It should be noted that although d_{ij} and S do not appear directly in the formulation of the problem, both values are included in the definition of the sets N_i.

Thus, the membership function that represents the satisfaction degree of the constraint (6), is the following piece-wise linear function:

$$\mu(d_{ij}) = \begin{cases} 1 & \text{if } d_{ij} \leqslant S, \\ 1 - \frac{d_{ij}-S}{\tau} & \text{if } S < d_{ij} \leqslant S+\tau, \\ 0 & \text{if } d_{ij} > S+\tau \end{cases} \tag{7}$$

where $\tau \in \mathbb{R}$ is the maximum tolerance allowed by the decision maker.

Hence the first Fuzzy MCLP model is defined as:

$$\begin{aligned} \max Z &= \sum_{i \in I} w_i y_i \\ \text{subject to} & \\ \sum_{j \in N_i} x_j &\geqslant y_i \ \forall i \in I \\ \sum_{j \in J} x_j &= p \\ x_j &= \{0,1\} \quad \forall j \in J \\ y_i &= \{0,1\} \quad \forall i \in I \\ N_i &= \{j \mid d_{ij} \leq_f S\} \ \forall i \in I \end{aligned} \tag{8}$$

B. Model 2: fuzzy restriction on the number of facilities

In general, the number of facilities to deploy is usually fixed and well known. However, experience shows that many times, a given level of coverage can be attained with a lower value of p, or some redundant coverage can be reached using a higher p value.

When considering the deployment of MHT, experts are more comfortable stating that the number of MHT is "around p" or "between v_1 and v_2".

In the context of MCLP, and recalling (3), this situation can be modeled with the help of a fuzzy constraint as follows:

$$\sum_{j \in J} x_j =_f p$$

where $=_f$ implies that we should deal with a fuzzy restriction whose meaning was explained before.

Now, the corresponding fuzzy MCLP model is

$$\begin{aligned} \max Z = \sum_{i \in I} w_i y_i \\ \sum_{j \in N_i} x_j \geq y_i \ \forall i \in I \\ \sum_{j \in J} x_j =_f p \ \forall i \in I \\ x_j, y_i \in \{0,1\} \forall i \in I, j \in J \end{aligned} \tag{9}$$

In order to solve the problem, we should consider that (9) is the intersection of the following two fuzzy constraints:

$$\sum_{j \in J} x_j \leq_f p \tag{10}$$

and

$$\sum_{j \in J} x_j \geq_f p \tag{11}$$

IV. Solution approach

To the best of our knowledge, there are still no tools to solve the above models in their fuzzy formulation. One way to go is to transform the fuzzy models into classical (crisp) models (e.g. [5], [6]) as the Parametric Approach [7] does.

The original fuzzy problem is converted into a set of crisp problems which in turn can be solved by means of exact or approximate algorithms. This approach has been successfully used in other relevant applications (e.g. [8], [9]), it does not present hard difficulties and, remarkably, it provides a fuzzy solution which is often very informative for decision makers.

The approach consists of two phases. Firstly, it transforms the fuzzy problem into several crisp problems using α-cuts, where the parameter $\alpha \in [0,1]$ represents the decision maker's satisfaction degree about the accomplishment of the fuzzy constraint or fuzzy value. As a consequence, for each $\alpha \in [0,1]$ considered, a classical MCLP (α-MCLP) is obtained.

Secondly, each of these problems is solved by classical optimization techniques. The results obtained for the different α values generate a set of solutions, which can be integrated by using the Representation Theorem for fuzzy sets. Thus, we can say that the solution provided by the Parametric Approach is a solution of our fuzzy MCLP extension.

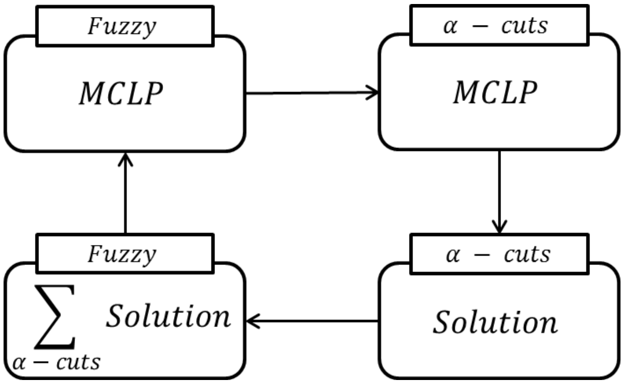


Fig. 4. Parametric approach applied to the fuzzy MCLP.

Figure 4 illustrates the process.

We will solve the two models previously proposed, using the parametric approach. The complete models are outlined below.

A. MCLP with a fuzzy restriction on coverage distance.

Consider the model described in III-A. In this case, taking into account the membership function in (7), the equation describing the α-cuts of the sets from (8) are given as:

$$N_i = \{j | d_{ij} \leq S + \tau(1-\alpha)\} \quad (12)$$

where $\alpha \in [0,1]$ and τ is the tolerance level allowed by the decision maker. Hence using this transformation, for each value of α, our fuzzy model leads to a crisp MCLP that can be solved by conventional optimization methods.

B. MCLP with a fuzzy restriction on the number of facilities

Let us focus now the model described in III-B. The fuzziness here comes from the fuzzy constraint in (9). As we stated before, the satisfaction of this fuzzy constraint is equivalent to the satisfaction of the fuzzy constraints (10) and (11) .

Let us suppose that decision maker allows violations up to the values $p + \Delta^r$ and $p - \Delta^l$ respectively. Being $P = \sum_{j \in J} x_j$, then (10) and (11) may be defined by the following membership functions:

$$\mu^r(P) = \begin{cases} 1 & \text{if } P \leq p, \\ 1 - \frac{P-p}{\Delta^r} & \text{if } p \leq P \leq p + \Delta^r, \\ 0 & \text{if } P > P + \Delta^r \end{cases} \quad (13)$$

and

$$\mu^l(P) = \begin{cases} 1 & \text{if } P \geq p, \\ 1 - \frac{p-P}{\Delta^l} & \text{if } p - \Delta^l \leq P \leq p, \\ 0 & \text{if } P < P - \Delta^l \end{cases} \quad (14)$$

respectively.

Then, the corresponding α-cuts

$$\mu^r(P) \geq \alpha, \;\; \mu^l(P) \geq \alpha, \;\; \forall \alpha \in [0,1]$$

become

$$\sum_{j \in J} x_j \leq p + \Delta^r(1-\alpha) \quad (15)$$

$$\sum_{j \in J} x_j \geq p - \Delta^l(1-\alpha) \quad (16)$$

So in this case the auxiliary α-MCLP models are formulated as:

$$\max Z = \sum_{i \in I} w_i y_i$$
$$\sum_{j \in N_i} x_j \geq y_i \; \forall i \in I$$
$$\sum_{j \in J} x_j \leq p + \Delta^r(1-\alpha) \; \forall j \in J \quad (17)$$
$$\sum_{j \in J} x_j \geq p - \Delta^l(1-\alpha) \; \forall j \in J \quad (18)$$
$$x_j, y_i \in \{0,1\} \; \forall i \in I, j \in J$$

which, for each value $\alpha \in [0,1]$ leads to a MCLP with an interval constraint given by (17) and (18). This introduces some novelty in this kind of problems as, to the best of our knowledge, has not been considered in the specialized literature. Nevertheless, the existence of this interval constraint will not increase the complexity of the (metaheuristic) solving algorithm.

V. Application Example

In this section we will show the usefulness of the proposed models using data from the city of Acapulco, Estado de Guerrero, in México.

Departing from demand points associated with diagnosed dengue cases, we solve the MHT location problem under the two models considered.

A. Data Information

We depart from 2050 demand points, each corresponding with a dengue case registered in Acapulco city during the period 2003-2008. It should be noted that the date of the data is irrelevant for the purposes of this research. Nowadays, data is collected more systematically, for example including the GPS coordinates, which will make easier for us to process it.

These records were grouped in 100×100 meters squares (named "cuadras"). Each square is a demand point, where the value of the demand represents the number of dengue cases inside. At the end, we've got 505 squares (demand points). Then, a single case is randomly selected to determine the coordinates (location) of the square.

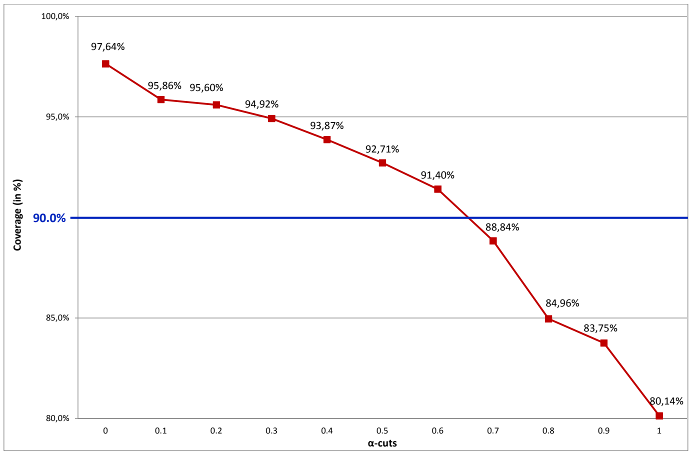


Fig. 5. Coverage variation in terms of the α-cuts. $\alpha = 1$ corresponds to the fully restricted constraint while $\alpha = 0$ is the fully relaxed.

B. Experiments and Results

Case 1: Fuzzy constraint on the coverage distance

The following assumptions are taken

- The number of MHT to locate is $p = 40$.
- The coverage distance is $S = 400$ meters.
- The distance between a demand point and a MHT is calculated using the geographical distance.
- Every demand point is a potential location for the MHTs.
- We do not consider location costs as we assume that the MHTs are homogeneous.

In the fuzzy constraint (12) we consider $\alpha \in [0, 0.1, \ldots, 1]$, thus obtaining 11 crisp location problems to solve. Also, we set $\tau = 200$ which represents a 50% of tolerance over S when $\alpha = 0$. In other words, S is relaxed up to 600 meters.

Each problem is solved using a metaheuristic called Iterated Local Search that is fully described in [10]. Due to the stochastic nature of the method, 30 independent runs are performed over each problem and the best solution (maximal coverage) found is recorded. The experiments were conducted on a notebook with an Intel Core i5 at 2.4 GHz running Windows 7 64-bit.

The change in coverage in terms of the α-cut considered is shown in Fig. 5. Recall that $\alpha = 0$ is the more relaxed problem while $\alpha = 1$ is the case with no relaxation.

As it is expected, the more relaxed the constraint is, the bigger is the coverage. Nevertheless, a potential decisor can observe that there are many ways to obtain a given level of desirable coverage. Let's suppose this level is 90%. It is clear that with no relaxation on the distance ($\alpha = 1$), the maximal coverage with $p = 40$ MHT is around 80%. However, if he assume that a MHT can cover some extra distances ($\alpha = 0.6$), then up to 90% of coverage can be reached. So, the decisor have two alternatives at hand: increase the number of MHT, or pay some extra budget to each allocated MHT to extend their coverage area.

It is also important to analyze the solutions not only from the point of view of the coverage attained, but also from how the MHTs are distributed. As we stated before, as the level of α decrease, the coverage increase (the coverage distance is increased). However such coverage increase is not only due to the increase in the coverage distance, but also due to the relocation of the MHT.

Figure 6 shows three solutions over the Acapulco city map. Each figure corresponds (from top to bottom) to $\alpha = 1$, $\alpha = 0.5$ and $\alpha = 0$ and the coverage obtained is also shown.

When $\alpha = 1$ (top figure) the MHT are located in the areas of bigger demand (dengue cases), leaving uncovered many others. When $\alpha = 0.5$, which implies that a MHT covers 500 meters (instead of 400) we can observe that some MHT are relocated in previously uncovered areas (like the ones in the top left and bottom right of the map). Finally, when the constraint is fully relaxed (implying a coverage distance of up to 600 meters), the relocation now allows the further coverage of other areas, like the one above the small island in the bottom left of the map, or in the coast at the top.

The cost of attaining such coverage keeping the constraint fixed (as in the top figure), would lead to a clear increase in the number of MHTs.

Case II: fuzzy constraint on the number of MHT

In this case, the demand points are the same as those in the previous case.

We set $p = 40$ in the constraints (17) and (18) , allowing for a maximum violation of up 10%, thus $\Delta^l = \Delta^r = 4$. For each $\alpha \in [0, 0.1, \ldots, 1]$ the bounds in the corresponding constraints (17) and (18) are shown in Table I. As $p \in \mathbb{N}$, values are rounded up for the lower bound while are rounded down for the upper bound.

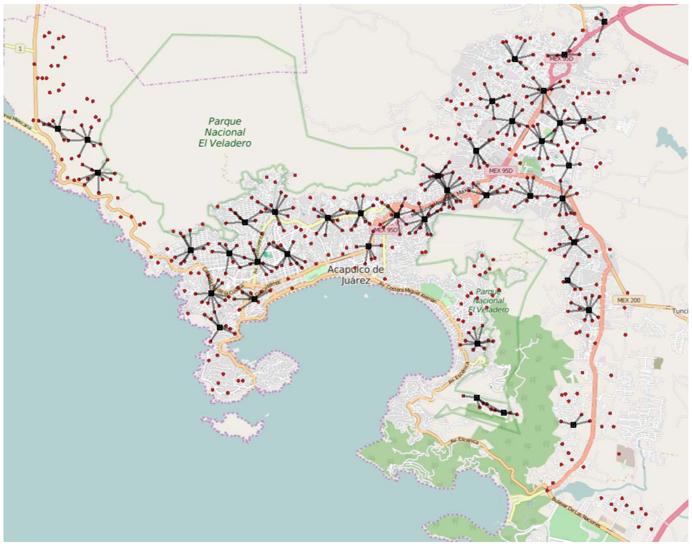
The coverage distance is set as $S = 400$. Again, each value of α leads to a crisp problem that needs to be solved.

For operational purposes, instead of solving the problems for every α, we solve it for every $p \in [36, 44]$ using the Iterated Local Search metaheuristic. The solutions are then assigned to the corresponding α-problems. As expected each solution may belong to more than one α-problem. For example, when $\alpha = 1$ we just need to consider $p = 40$. The solution obtained is also a feasible one for every $\alpha < 1$.

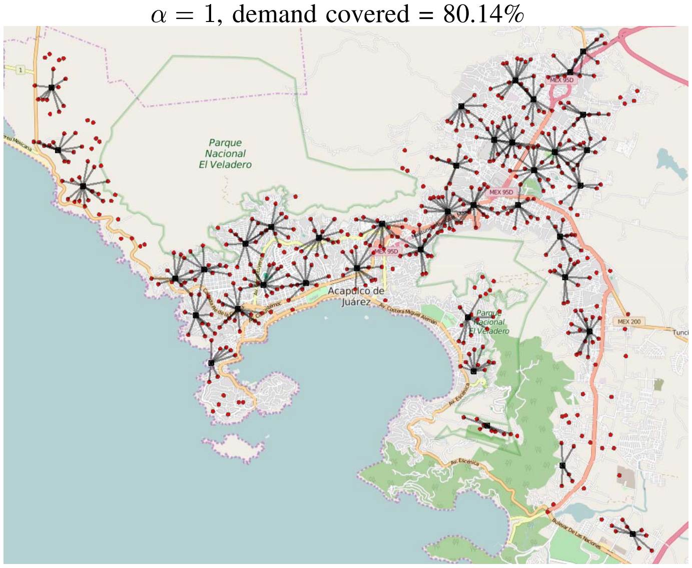
Again, the metaheuristic is executed 30 times per each value of $p \in [36, 44]$ and the best solutions are recorded. In a post processing stage, the best and worst solutions solutionsfor each value of α are kept.

The results are displayed in Fig. 7, where the range of potential coverage values for each α is shown. When $\alpha = 1$, the problem solved is the "classic" one with $p = 40$ and a coverage percentage of 80%.

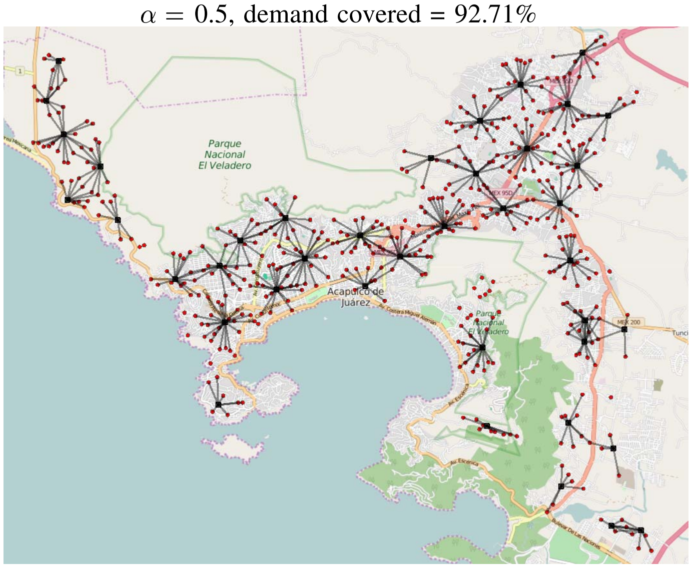
As the constraint is relaxed (from right to left in the plot), a wide set of alternatives can be considered by a potential decisor. Let's consider the case when $\alpha = 0.2$, where p varies between 37 and 43. The plot indicates that a coverage percentage between 76% and 82% can be attained.



$\alpha = 1$, demand covered = 80.14%



$\alpha = 0.5$, demand covered = 92.71%



$\alpha = 0$, demand covered = 97.64%

Fig. 6. Best potential locations for 40 MHT according to three different values of α. As α decreases, the problem is relaxed allowing the coverage or more areas using the same number of MHTs.

α	$p - \Delta^l(1-\alpha)$	$p + \Delta^r(1-\alpha)$
0	36.0 (36)	44.0 (44)
0.1	36.4 (36)	43.6 (44)
0.2	36.8 (37)	43.2 (43)
0.3	37.2 (37)	42.8 (43)
0.4	37.6 (38)	42.4 (42)
0.5	38.0 (38)	42.0 (42)
0.6	38.4 (38)	41.6 (42)
0.7	38.8 (39)	41.2 (41)
0.8	39.2 (39)	40.8 (41)
0.9	39.6 (40)	40.4 (40)
1	40.0 (40)	40.0 (40)

TABLE I
BOUNDS FOR FUZZY CONSTRAINT IN THE NUMBER OF MHT. FOR THE LOWER BOUND, VALUES ARE ROUNDED UP WHILE ARE ROUNDED DOWN FOR THE UPPER BOUND.

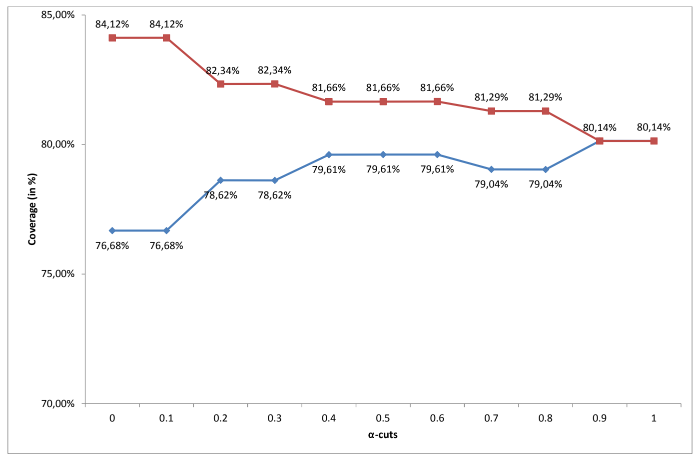


Fig. 7. Ranges of coverage attained for each α value.

VI. CONCLUSIONS

Fighting dengue and other illnesses transmitted by the Aedes Agypty mosquito are of crucial interest in the context of public health. The proper deployment of mobile health teams is one of the tools available to control the transmission of the mosquito. In this work we consider the team deployment problem as a maximal coverage location problem with uncertainty due to the nature of the information available.

We proposed two fuzzy maximal covering location problems, considering flexibility at the level of the number of teams available and at the coverage distance they may cover. Both cases are modeled through a fuzzy constraint.

Despite the theoretical interest of the models, we show how they can be managed and solved. The application example, using diagnosed dengue cases from Estado de Guerrero, México, allows to show the type of information that can be obtained.

Now, a software tool including the models and solving methods is already available and being tested by public health managers in México.

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