An Enhanced Real Coded Approach for the Optimization of the Unit Commitment Problem

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***Abstract*—In the present study, a novel enhanced real-coded approach is proposed for the simultaneous optimization of Unit Commitment and Economic Dispatch. In the proposed method, the objective variables represent the power generated by each unit at each hour. By utilizing the operational characteristics of each unit, its states and generation levels during the scheduling period are optimized concurrently. Moreover, a repair mechanism, which stochastically adjusts the generation of the units, is implemented to ensure the feasibility of the derived operation schedules. The solver employed for the optimization of the Unit Commitment Problem using the proposed method is L-SHADE, which is a state-of-the-art adaptive algorithm based on Differential Evolution. The real-coded approach is tested on a set of power systems consisting of up to 100 thermal units, available in the literature. The experimental results indicate that the proposed method exhibits more robust solution distributions and lower minimum operation cost for large scale systems compared to other state-of-the-art methods.**

TABLE I
NOMENCLATURE

N	Number of generating units
T	Number of hours of the scheduling period, (h)
fes_{max}	Maximum number of function evaluations
i	Index of thermal generating units
t	Index of scheduling hour
a_i, b_i, c_i	Cost function coefficients of unit i, $(\$/h)$, $(\$/MWh)$ and $(\$/MWh^2)$ respectively
P_i^t	Power output of unit i at hour t, (MW)
$P_{i,max}$	Maximum power output of unit i, (MW)
$P_{i,min}$	Minimum power output of unit i, (MW)
u_i^t	State of unit i at hour t
f_i	Fuel cost if unit i, (\$)
ST_i^t	Start up cost of unit i at hour t, (\$)
Sh_i	Hot start up cost of unit i, (\$)
Sc_i	Cold start up cost of unit i, (\$)
P_D^t	System load demand at hour t, (MW)
P_R^t	System spinning reserve at hour t, (MW)
$T_{i,on}^t$	Continuously on time of unit i up to hour t, (h)
$T_{i,off}^t$	Continuously off time of unit i up to hour t, (h)
$T_{i,up}$	Minimum up time of unit i, (h)
$T_{i,down}$	Minimum down time of unit i, (h)
$T_{i,cold}$	Cold start hour of unit i, (h)

I. Introduction

The optimization of the Unit Commitment Problem (UCP) is of paramount importance for the operation planning of power systems. It determines the optimum scheduling of operating units and the amount of energy delivered by each unit over a certain timespan, in order the projected demand to be satisfied at minimum production cost. The derived generating schedule is also limited by certain constraints, related to the operation of the aggregate power system and the characteristics of each unit.

Mathematically, UCP is a complex, large-scale, non-linear optimization problem, containing binary and continuous variables. Its exact solution can be obtained by a complete enumeration of the possible states of the units. However, this approach cannot be implemented on realistic power systems due to the excessive computational time required [1]. For this reason, during the past years, researchers have proposed several alternative techniques to approximate the optimal solution of UCP in reasonable computational time. Such techniques include Priority List [2], Branch-and-Bound [3], Lagrangian Relaxation (LR) [4] and Mixed Integer Linear Programming [5] methods. Most of the above methods encounter difficulties when facing large scale instances of the UCP. Among them, LR is the most widely used. In this method, the Lagrangian function is decomposed to N smaller minimization problems. Subsequently, LR searches for the Lagrangian multipliers that maximize the dual problem. However, due to the non-convexity of the UCP, the maximization of the dual problem does not guarantee the feasibility of the primal [1].

More recently, meta-heuristics optimization methods were employed. Their global searching capacity and their ability to deal with non-linear constraints render them as attractive alternatives to UCP. Such methods include Genetic Algorithm (GA) [6], Evolutionary Programming (EP) [7], Simulated Annealing (SA) [8], Particle Swarm Optimization (PSO) ([9], [10]), Quantum-inspired Evolutionary Algorithm (QEA) [11], Binary Gravitational Search Algorithm (BGSA) [12] and Differential Evolution ([13], [14], [15]). Furthermore, several hybrid algorithms have been proposed. They combine meta-heuristics and deterministic methods, in order to exploit the distinct characteristics of each algorithm during the different stages of the optimization. In these methods, evolutionary algorithms are employed to update the Lagrangian multipliers ([16], [17]), are seeded with the solution vector of LR as initialization to further improve it [18] or are used to handle the matrices of both UCP sub problems, namely Unit Commitment (UC) and Economic Dispatch (ED) [19]. A common characteristic of the aforementioned approaches is that, at each iteration of the optimization procedure, the UC schedule is derived at first and then the optimization of ED occurs. No further changes in the state of the units occur during the optimization of ED. Thus, the optimization of ED is biased by the output of UC. Therefore, a means for mutually interacting information from the UC to the ED problem and vice versa during each iteration of the optimization procedure, would worth to be investigated.

In the present study, an Enhanced Real-Coded Approach (ERC) for the optimization of UCP using Evolutionary Algorithms (EA) is presented, where ED is not limited by the on/off schedule of UC. According to this approach, each solution vector consists of continuous variables, which correspond to the energy produced by each unit at each hour, P_i^t. The generation level and the minimum power output ($P_{i,min}$) of each unit are used to define the committed units during the scheduling period. Subsequently, a heuristic repair mechanism is applied on the solution vectors to ensure that they will satisfy the problem's constraints. This mechanism, searches for the units whose status should change and adjusts stochastically their generation output. Consequently, UC and ED are optimized simultaneously during each iteration. In order to optimize the UCP using ERC, L-SHADE is utilized, which is a state-of-the-art variant of DE. The UCP is optimized for power systems containing up to 100 units. The results indicate that the efficiency and the execution time of ERC using L-SHADE is competitive in comparison to other algorithms proposed for the optimization of UCP.

The rest of the paper is organized as follows: Section II presents the general formulation of the UCP problem. Section III describes the proposed solution procedure and L-SHADE. Finally, sections IV and V discuss the experimental results and the conclusions, respectively.

II. Formulation of the Problem

A. Objective function

The optimal solution of the UCP problem is the electricity production schedule, which minimizes the total operation cost. The latter is the sum of fuel and start-up cost of each unit over the planning horizon. Thus, the objective function of UCP is formulated as follows:

$$\underset{P_i^t, u_i^t}{\text{minimize}}\ OC = \sum_{t=1}^{T}\sum_{u=1}^{N}[f(P_i^t) + ST_i^t \cdot (1-u_i^{t-1})] \cdot u_i^t \quad (1)$$

where u_i^t indicates the units that are operational. Therefore, if $u_i^t = 1$, unit i delivers power to the grid at hour t. Otherwise, when $u_i^t = 0$, unit i is not committed.

The fuel cost is expressed as a quadratic function of the unit's power output. Thus :

$$f_i(P_i^t) = a_i + b_i \cdot P_i^t + c_i \cdot (P_i^t)^2 \quad (2)$$

The start-up cost depends on the time for which unit i is off, prior hour t, when it starts generating energy. A time-dependent start up-cost is used, calculated as follows [6]:

$$ST_i^t = \begin{cases} S_{h,i}, & \text{if } T_{i,down} \leq T_{i,off}^t \leq T_{i,down} + T_{i,cold} \\ \\ S_{c,i}, & \text{if } T_{i,down} + T_{i,cold} \leq T_{i,off}^t \end{cases} \quad (3)$$

where $T_{i,off}^t$ is the number of consecutive hours, for which unit i is off until hour t. Therefore :

$$T_{i,off}^t = \begin{cases} 0, & \text{if } u_i^t = 1 \\ \\ 1 + T_{i,off}^{t-1}, & \text{if } u_i^t = 0 \end{cases} \quad (4)$$

B. Constraints

1) **System power balance:** The electricity demand should be satisfied every hour of the planning horizon. This is expressed as:

$$\sum_{i=1}^{N} u_i^t \cdot P_i^t = P_D^t \quad \forall\ t \quad (5)$$

2) **System spinning reserve requirements:** The spinning reserve should be sufficient, in order a possible unit outage to be addressed without a significant drop of the system's frequency. Thus :

$$\sum_{i=1}^{N} u_i^t \cdot P_{i,max} \geq P_D^t + P_R^t \quad \forall\ t \quad (6)$$

3) **Generation limits of each unit:** Each operating unit's power output should lie within its generation limits. Thus:

$$P_{i,min} \leq P_i^t \leq P_{i,max} \quad \forall\ t \quad (7)$$

4) **Minimum up/down time of each unit:** When a unit begins generating energy, it should remain operational at least for a minimum number of consecutive hours, based on its minimum up time:

$$T_{i,on}^t \geq T_{i,up} \quad (8)$$

where $T_{i,on}^t$ is the consecutive on time of unit i until hour t. It is calculated as follows:

$$T_{i,on}^t = \begin{cases} 0, & \text{if } u_i^t = 0 \\ \\ 1 + T_{i,on}^{t-1}, & \text{if } u_i^t = 1 \end{cases} \quad (9)$$

Similarly, once unit i is decommitted, it should not be recommitted for a certain timespan, determined by $T_{i,down}$:

$$T_{i,off}^t \geq T_{i,down} \quad (10)$$

The initial status of each unit is also taken into account.

III. Description of the Proposed Method

A. Structure of the individuals

In the present research, each solution vector represents the energy produced by the units at each hour of the scheduling period: P_i^t, *i=1,...,N* and *t=1,...,T*. In order to determine the unit commitment schedule, the lower production limit ($P_{i,min}$) of each unit is used as a threshold value. When the power output of a unit is above its minimum output, it is considered committed. Otherwise, the unit does not deliver power to the grid. Thus :

$$u_i^t = \begin{cases} 0, & \text{if } 0 \leq P_i^t \leq P_{i,min} \\ \\ 1, & \text{if } P_{i,min} \leq P_i^t \leq P_{i,max} \end{cases} \tag{11}$$

The initial population is created randomly using the uniform distribution $U(0, P_{i,max})$.

By utilizing the above formulation and the repair mechanism described in the following section ED and UC interact during each iteration of the proposed method. Initially, the UC schedule is derived, by taking directly into account the energy generation of the units, represented by the continuous variables; a unit i, whose power output at hour t lies below the permissible limit (based on its characteristics), is considered to be decommited, thus $u_i^t = 0$. The opposite applies to the units, whose power output is above $P_{i,min}$. Then, ED is adjusted, when the proposed repair mechanism alters UC; the heuristic determines the units whose state should be changed and modifies stochastically their generation output. Notably, both the generation and the state of each power plant is represented with a single continuous variable by using this formulation.

B. Repairing mechanism description

The UCP is a large scale, highly constrained and combinatorial problem. Thus, the optimization algorithms may encounter difficulties in achieving feasible solutions. In this context, the utilization of repair mechanisms is suggested, to ensure that the derived generating schedules will satisfy the problem's constraints [20]. The heuristic adopted here is an '*always replacing approach*' [21], meaning that the repaired version of the individuals forms the population. It was initially implemented on binary parameter vectors, corresponding to the state of the units, in [4]. In this study, it is modified, by introducing two stochastic operators, which are applied on the continuous variables, representing the generating energy. Moreover, in contrast to the initial mechanism, the proposed one utilizes two Priority Lists, to sort the units; the first is employed during the changes of the on/off schedule, while the second during the repair of the power balance constraint. The heuristic procedure is described in the following subsections.

1) ***Spinning reserve constraint repairing:*** During the optimization, individuals may violate the requirement of sufficient spinning reserve. Therefore, the parameter vectors should be adjusted to satisfy the above constraint. The procedure employed, utilizes a Priority List based on the average production cost of the units when they operate at their average power output [4]:

$$M1_i = \frac{f_i(P_i)}{P_i}\Big|_{x_i \cdot Pmax_i} \tag{12}$$

where, x_i defines the level of production, which is used to calculate the average cost [4]. Thus, for $M1_i$:

$$x_i = \frac{1}{2} \cdot (1 + \frac{P_{i,min}}{P_{i,max}}) \tag{13}$$

When the reserve margin constraint is violated, the uncommitted units are sorted in ascending order, based on $M1$. Then, the cheapest units are committed until the constraint is satisfied. This part of the repair mechanism comprises of the following steps:

S.1: Set t=1.

S.2: Sort the uncommitted units based on their average load average cost ($M1$).

S.3: Calculate the spinning reserve violation:

$$R_t = \sum_{i=1}^{N} (u_i^t \cdot P_{i,max}) - P_D^t - P_R^t \tag{14}$$

S.4: If $R_t \geq 0$, go to Step 6

S.5: Commit uncommitted units, beginning from the one with the lowest $M1$ value, until the spinning reserve is satisfied. The commitment of the units is implemented using the following formula:

$$P_i^t = (1 + k_1 \cdot U(0,1)) \cdot P_{i,min} \tag{15}$$

S.6: If $t \leq T$ then $t = t + 1$ and go to Step 2. Otherwise, stop.

Step 5 is modified in this study; a unit is committed by increasing its power output above the predefined threshold ($P_{i,min}$). The increase is implemented stochastically, by the uniform distribution $U(0,1)$ and a user defined parameter k_1, which controls the amount of the energy added at $P_{i,min}$. Stochasticity is introduced to maintain the population's diversity, in an attempt to avoid the state of stagnation [22], which might occur if the output of the unit was set to a predefined value. Based on preliminary experiments, the value of k_1 should be in $[0, 0.05]$.

2) ***Minimum up and down time constraint repairing:*** The schedule formed after the implementation of the repair mechanism's first part may not satisfy the minimum up and down time constraints. Therefore, the state of the units, which violate the constraint, should be adjusted. The adjustment is carried out by stochastically increasing the power output of the units as in subsection III-B1. Briefly, the repair algorithm searches for the consecutive hours, where a unit changes its status from on to off. If the minimum up ($T_{i,up}$) or down time ($T_{i,down}$) is violated, the unit is committed for the following hours until the minimum up time constrained is satisfied [23]. The procedure is the following:

S.1: Set t=1.

S.2: Set i=1.

S.3: If $u_i^t = 0$ and $u_i^{t-1} = 1$ and $T_{i,on}^{t-1} < T_{i,up}$, then commit the unit at hour t using equation (15).

S.4: If $u_i^t = 0$ and $u_i^{t-1} = 1$ and $t + T_{i,down} - 1 \leq T$ and $T_{i,off}^{t+T_{i,down}-1} < T_{i,down}$, then commit the unit at hour t using equation (15).

S.5: If $u_i^t = 0$ and $u_i^{t-1} = 1$ and $t + T_{i,down} - 1 > T$ and $\sum_{j=t}^{T} u_i^j > 0$, then commit the unit at hour t using equation (15).

S.6: Update $T_{i,off}$ and $T_{i,on}$ using equations (4) and (9), respectively.

S.7: If $i < N$ then $i = i + 1$ and go to Step 4.

S.8: If $t < T$ then $t = t + 1$ and go to Step 3. Otherwise, stop.

*3) **Decommitment of excess units**:* Repairing the minimum up and down time constraints may cause excessive spinning reserve. As a result some units may operate at suboptimal output levels, increasing the total production cost. Therefore, the redundant units should be decommitted. In this procedure the priority list introduced in subsection III-B1 is utilized.

Decommitting a unit, impacts both the reserve margin and the minimum up and down time constraints. Thus, beginning from the units with the highest values of $M1$, the algorithm determines and decommits the units, which can be turned off without violating minimum up and down time and spinning reserve constraints, until no more units can be decommitted. This part of the repair mechanism consists of the following steps:

S.1: Set t=1.

S.2: Set i=1.

S.3: If unit i can be decommitted without violating the minimum up/down time and spinning reserve constraints, put it into an excess list $SS1$.

S.4: If $i < N$ then $i = i + 1$ and return to Step 3.

S.5: If the excess list is empty, go to Step 7.

S.6: Decommit the unit with the highest $M1$ in $SS1$ and eliminate it from the list. The decommitment is carried out using the following formula:

$$P_i^t = (k_2 \cdot U(0,1)) \cdot P_{i,min} \tag{16}$$

Update $T_{i,off}$, $T_{i,on}$ and the reserve margin using formulas (4), (9) and (14), respectively. Check if the next unit in $SS1$ can be decommited without violating spinning reserve and minimum up/down times constraints. Repeat this step until no unit can be decommited.

S.7: If $t < T$ then $t = t + 1$ and go to Step 2. Otherwise, stop.

Unit decommitment is carried out using equation (16); the energy output of the unit is stochastically decreased below the threshold value ($P_{i,min}$). Here, the user defined control parameter k_2, should lie in the interval $[0.1, 0.6]$, based on preliminary experiments.

*4) **Power balance constraint repairing**:* The power balance constraint may be violated, after the decommitment of the excessive units. Thus, the power output of the operating units should be adjusted, leading to the satisfaction of the power balance constraint.

Generally, as highlighted in [15], the average production cost of a unit is lower when it operates close to its maximum power output. Therefore, a thermal unit operating at lower output levels may burden the total operation cost, since additional units may be needed to satisfy the overall projected demand. Thus, it is desired, the committed units to generate power near their maximum power capacity. In this vein, the priority list utilized in this part of the repair mechanism is based on the average cost of the units at their highest power output ($P_{i,max}$). Thus:

$$M2_i = \frac{f_i(P_i)}{P_i}\bigg|_{x_i \cdot Pmax_i} \tag{17}$$

where

$$x_i = 1 \tag{18}$$

Initially, the violation of the demand constraint at each hour is calculated. If there is excessive energy, it is subtracted from the production of the most expensive committed units according to $M2$. However, if the energy produced by the operating units is not sufficient, then additional energy is distributed to the cheapest committed units based on $M2$, till the demand is satisfied. The procedure consists of the following steps:

S.1: Set t=1.

S.2: Create a list $SS2$ with the committed units at hour t.

S.3: Calculate the power balance constraint violation:

$$D^t = \sum_{i=1}^{N} u_i^t \cdot P_i^t - P_D^t \tag{19}$$

S.4: If $D^t \geq 0$ (*energy excess*), find the most expensive committed unit based on $M2$ in $SS2$. Calculate the maximum possible reduction in the unit's energy, in order the unit to remain committed:

$$DP_i^t = P_i^t - P_{i,min} \tag{20}$$

If $DP_i^t > D^t$ then update the output of that unit:

$$P_i^t = P_i^t - D^t \tag{21}$$

and go to Step 6. Otherwise, set $P_i^t = P_{i,min}$, erase the unit from $SS2$ and return to step 3.

S.5: If $D^t \leq 0$ (*energy shortage*), find the cheapest committed unit based on $M2$ in $SS2$. The maximum permissible increase in the unit's energy is:

$$DP_i^t = P_{i,max} - P_i^t \tag{22}$$

If $DP_i^t > |D^t|$, then update the output of that unit:

$$P_i^t = P_i^t + |D^t| \tag{23}$$

and go to Step 6. Otherwise, set $P_i^t = P_{i,max}$, erase the unit from $SS2$ and return to step 3.

S.6: If $t \leq T$ then $t = t + 1$ and go to step 1. Otherwise, stop.

C. L-SHADE

Differential Evolution [24] is a simple but effective algorithm for numerical optimization over continuous spaces. Since its efficiency depends on the adequate tuning of its control parameters (CR, F, NP), self adaptive parameter mechanisms for DE have been developed. In this context, Success History Based Parameter Adaptation for Differential Evolution with Linear Population Size Reduction (L-SHADE) was proposed in [25]. L-SHADE extends SHADE [26], by utilizing a linear function in order to continuously decrease the population size. Moreover, L-SHADE uses a historical memory M_{CR} and M_F, where sets of CR and F values, that produced successful offspring, are stored. In each generation, the new parameters CR and F of each individual are generated by directly sampling the parameter space close to one of the aforementioned stored pairs. L-SHADE also utilizes an external archive A, to maintain the diversity of the population. The parent vectors, which were replaced in the population by their offspring, are stored in A. They are candidates to participate in the creation of the mutant vectors of the following generations. The mutation strategy used in L-SHADE is the current-to-pbest/1, initially presented in [27].

Difficulties arising in the setting of CR and F for real world problems are avoided by utilizing self adaptive DE [28]. Moreover, the Linear Population Size Reduction (LPSR) employed in L-SHADE may enhance the optimization of UCP. In the initial stages of the optimization, the exploration of the search space calls for a large population size. While the population evolves and an adequate on/off schedule is found, the enhanced exploitation ability of L-SHADE, due to the increased number of function evaluations dedicated to the best individuals, may lead to further improvement of the energy dispatch. Nevertheless, a comparison of the performance of simple and self adaptive EA for ERC may constitute a field of future research.

IV. Numerical Results

In this section the performance of ERC is evaluated. The method has been applied to power systems of 10, 20, 40, 60 and 100 thermal units for a 24 hour scheduling period. The data of the examined 10-unit system can be found in [6]. The aforementioned system is replicated 2, 4, 6, and 10 times, respectively, to form the systems of higher number of units. The projected demand has been accordingly multiplied for each of the systems. The spinning reserve is assumed to be equal to 10% of the load demand. The algorithm was developed on an Intel $i7$ with 4 processors at $3.07GHz$ and $8\ GB\ RAM$, using Parallel Computing Toolbox of Matlab $R2012b$. L-SHADE code has been retrieved from R. Tanabe's home page[1]. The experimental evaluation of the algorithm is divided into two case studies. Initially a parameter sensitivity analysis has been performed. Subsequently, the performance of ERC is benchmarked against other methods applied for the optimization of the UCP.

[1]https://sites.google.com/site/tanaberyoji/home

A. Parameter Sensitivity Analysis

A series of experiments has been implemented to analyse the sensitivity of the performance of ERC combined with L-SHADE (ERC-L-SHADE) to the maximum number of function evaluations (fes_{max}) and the initial population size (NP_{init}). L-SHADE has two additional user defined parameters, i.e. the size of the external archive $|A|$ and p, which is the parameter used to adjust the greediness of the current-to-pbest/1 mutation scheme. The aforementioned variables are assigned constant values, $|A| = 1.4 \cdot NP$ and $p = 0.11$, as proposed in [25]. For each combination of the examined parameters 30 runs have been executed to verify the consistency of the proposed method. In Table II the best, worst and average cost and the coefficient of variation (CV) are presented. Values of CV lower than $10^{-3}\%$ are considered equal to zero.

As shown in Table II, the method achieves solution distributions of adequate robustness for the examined combinations of parameter values. By examining CV and the average cost, the differences observed are not pronounced for the different pairs of parameters. However, a small improvement in the algorithm's performance is noted, when fes_{max} increases for constant NP_{init}; the best, average and worst cost found decrease slightly with increasing fes_{max}, indicating that the method has converged even when $fes_{max} = 25000$. Moreover, in the majority of the cases CV is negligible or has a small value, revealing that ERC-L-SHADE derived robust results even for the larger power systems.

Regarding the power systems of 10 and 20 thermal units, the results are similar for the majority of the examined combinations. However, for the systems of 40, 60 and 100 units the algorithm derived the minimum best operation cost when (fes_{max}, NP_{init}) equals (50000, 400), (25000, 400) and (100000, 400) respectively. In this context, the results for (50000, 400) (in boldface) are used for the comparison of ERC with the benchmark algorithms. For these parameter values, ERC-L-SHADE achieved both adequate statistic results and best solutions of high quality, while the computation time needed remained moderate.

B. Performance Comparison

In order to validate the performance of optimizing simultaneously UC and ED, ERC-L-SHADE is compared to other approaches, in which UC and ED are optimized sequentially. The proposed approach is compared to a Memetic Algorithm (MA) method [18], a Simulated Annealing (SA) approach [8], two Particle Swarm Optimization algorithms ([9], [29]), a Quantum Inspired Evolutionary algorithm (QEA) [11] and a Binary Gravitational Search Algorithm (BSGA) [12]. The first method utilizes MA for the UC, while ED is carried out by a non-linear optimization program. In the second approach, SA is employed to optimize the UC combined with a dynamic economic dispatch method. In the remaining benchmark methods EAs are utilized for the UC, while Lamda-iteration method [1] optimizes ED. The results of these algorithms are available in the literature. The methods are compared based

TABLE II
ANALYSIS OF fes_{max} AND NP_{init} EFFECT ON THE PERFORMANCE OF ERC-L-SHADE

		NP_{init} = 100				NP_{init} = 200				**NP_{init} = 400**				NP_{init} = 800			
fes_{max}	Units	Best Cost ($)	Max. Cost ($)	Avg. Cost ($)	CV (%)	Best Cost ($)	Max. Cost ($)	Avg. Cost ($)	CV (%)	Best Cost ($)	Max. Cost ($)	Avg. Cost ($)	CV (%)	Best Cost ($)	Max. Cost ($)	Avg. Cost ($)	CV (%)
25000	10	563,938	564,091	564,022	0.01	563,938	564,065	564,019	0.01	563,960	564,120	564,016	0.01	563,964	564,108	564,021	0.01
	20	1,124,547	1,124,830	1,124,658	0.00	1,124,551	1,124,765	1,124,690	0.00	1,124,664	1,124,791	1,124,723	0.00	1,124,664	1,124,810	1,124,756	0.00
	40	2,246,656	2,247,019	2,246,868	0.00	2,246,776	2,247,060	2,246,957	0.00	2,246,681	2,247,144	2,246,988	0.00	2,246,965	2,247,187	2,247,070	0.00
	60	3,367,958	3,368,505	3,368,219	0.00	3,367,781	3,368,341	3,368,171	0.00	3,366,868	3,368,400	3,368,237	0.00	3,368,255	3,368,455	3,368,370	0.00
	100	5,612,126	5,612,837	5,612,436	0.00	5,612,042	5,612,660	5,612,448	0.00	5,612,256	5,612,805	5,612,484	0.00	5,612,348	5,612,884	5,612,634	0.00
50000	10	563,938	564,091	564,010	0.00	563,938	564,059	563,997	0.00	**563,938**	**564,059**	**563,975**	**0.00**	563,938	564,010	563,968	0.00
	20	1,124,531	1,124,731	1,124,604	0.00	1,124,484	1,124,722	1,124,574	0.00	**1,124,488**	**1,124,703**	**1,124,585**	**0.00**	1,124,569	1,124,734	1,124,641	0.00
	40	2,246,528	2,246,932	2,246,762	0.00	2,245,690	2,247,021	2,246,762	0.01	**2,245,118**	**2,246,888**	**2,246,709**	**0.01**	2,246,828	2,246,994	2,246,921	0.00
	60	3,367,747	3,368,265	3,368,013	0.00	3,367,572	3,368,287	3,368,019	0.00	**3,367,786**	**3,368,264**	**3,368,049**	**0.00**	3,367,874	3,368,326	3,368,155	0.00
	100	5,611,668	5,612,724	5,612,191	0.00	5,611,883	5,612,583	5,612,213	0.00	**5,610,688**	**5,612,493**	**5,612,197**	**0.01**	5,611,760	5,612,440	5,612,079	0.00
100000	10	563,959	564,064	564,003	0.00	563,938	564,059	563,982	0.00	563,938	564,010	563,970	0.00	563,938	564,009	563,956	0.00
	20	1,124,433	1,124,714	1,124,592	0.00	1,124,425	1,124,708	1,124,535	0.00	1,124,416	1,124,665	1,124,517	0.00	1,124,471	1,124,585	1,124,540	0.00
	40	2,246,374	2,246,869	2,246,699	0.00	2,246,269	2,246,869	2,246,648	0.00	2,246,535	2,246,838	2,246,658	0.00	2,246,724	2,246,938	2,246,850	0.00
	60	3,367,287	3,368,109	3,367,828	0.00	3,367,428	3,368,092	3,367,769	0.00	3,367,486	3,368,050	3,367,816	0.00	3,367,585	3,368,087	3,367,893	0.00
	100	5,611,648	5,612,575	5,612,076	0.00	5,611,145	5,612,540	5,611,974	0.00	5,610,579	5,612,537	5,611,980	0.01	5,611,760	5,612,440	5,612,079	0.00

TABLE III
COMPARISON OF THE EXPERIMENTAL RESULTS OF EACH METHOD

	N = 10					N = 20					N = 40				
	Best Cost ($)	Max. Cost ($)	Avg. Cost ($)	CV (%)	Avg. Time (s)	Best Cost ($)	Max. Cost ($)	Avg. Cost ($)	CV (%)	Avg. Time (s)	Best Cost ($)	Max. Cost ($)	Avg. Cost ($)	CV (%)	Avg. Time (s)
MA [18]	566,686	567,022	566,787	0.03	61	1,128,192	1,128,403	1,128,213	0.02	113	2,249,589	2,249,589	2,249,589	0.00	217
SA [8]	565,828	566,260	565,988	0.02	3	1,126,251	1,129,112	1,128,313	0.03	17	2,250,508	2,254,539	2,252,125	0.01	88
EPSO [9]	**563,938**	564,266	564,206	0.00	7	1,123,773	1,127,070	1,125,513	0.03	16	2,244,772	2,251,241	2,248,741	0.04	36
IBPSO [29]	563,977	565,312	564,155	0.03	27	1,125,216	1,125,730	1,125,448	0.02	55	2,248,581	2,249,302	2,248,875	0.01	110
QEA [11]	**563,938**	564,672	563,969	0.03	19	**1,123,607**	1,125,715	1,124,689	0.02	28	2,245,557	2,248,296	2,246,728	0.01	28
BGSA [12]	**563,938**	564,241	564,031	0.02	-	1,123,996	1,125,156	1,124,738	0.03	-	2,246,445	2,247,962	2,247,400	0.03	-
ERC-L-SHADE	**563,938**	564,059	563,975	0.00	46	1,124,488	1,124,703	1,124,585	0.00	92	2,245,118	2,246,888	2,246,709	0.01	177
ERC-L-SHADE & QP	**563,938**	**563,938**	**563,938**	0.00	-	1,124,291	**1,124,291**	**1,124,291**	0.00	-	**2,243,676**	**2,245,691**	**2,245,623**	0.00	-

	N = 60					N = 100				
	Best Cost ($)	Max. Cost ($)	Avg. Cost ($)	CV (%)	Avg. Time (s)	Best Cost ($)	Max. Cost ($)	Avg. Cost ($)	CV (%)	Avg. Time (s)
MA [18]	3,370,595	3,371,272	3,370,820	0.01	576	5,616,314	5,616,900	5,616,699	0.00	1,338
SA [8]	-	-	-	-	-	5,617,876	5,628,506	5,624,301	0.05	696
EPSO [9]	**3,364,250**	3,371,783	3,368,686	0.06	54	5,608,055	5,619,445	5,614,073	0.05	91
IBPSO [29]	3,367,865	3,368,779	3,368,278	0.01	172	5,610,293	5,612,265	5,611,181	0.02	295
QEA [11]	3,366,676	3,372,007	3,368,229	0.08	54	5,609,550	5,613,220	5,611,797	0.05	80
BGSA [12]	3,364,665	3,368,394	3,366,257	0.03	-	5,607,838	5,611,188	5,609,585	0.06	-
ERC-L-SHADE	3,367,486	3,368,050	3,367,816	0.00	254	5,610,688	5,612,493	5,612,197	0.01	383
ERC-L-SHADE & QP	3,366,019	**3,366,432**	**3,366,399**	0.00	-	**5,606,704**	**5,608,862**	**5,608,680**	0.00	-

on their best, maximum and average operation cost, and the CV of the experimental results. CV values lower than $10^{-3}\%$ are considered equal to zero. The average computational time needed by each method is also presented.

Moreover, a quadratic programming based economic dispatch has been applied to fine tune the results (ERC-L-SHADE & QP), as in [5], in order to examine whether ERC-L-SHADE obtains consistently the best commitment state. Since the solution vectors derived by ERC-L-SHADE are used as initial points, the quadratic programming algorithm converges rapidly, e.g. in approximately 2 seconds for the system of 100 thermal units. In Table III the simulation results of ERC-L-SHADE and ERC-L-SHADE & QP are compared to those of the selected methods. The best results in each category are highlighted.

The results of Table III indicate that the proposed method robustly derives solutions of high quality in terms of operation cost. Specifically, regarding the 10 thermal unit system, ERC-L-SHADE and ERC-L-SHADE & QP achieved the minimum cost, whilst the average cost and the maximum cost are the lowest among the examined methods. For the system of 20 thermal units, concerning the best solution found, ERC-L-SHADE outperforms MA, SA and IBPSO, while ERC-L-SHADE & QP provided the most consistent results compared to all the examined methods, since the dispersion of its solutions is negligible. Examining the system of 40 units, ERC-L-SHADE falls short of EPSO, which achieves lower minimum cost. Nevertheless, the distribution of the solutions of ERC-

TABLE IV

OPTIMAL GENERATION SCHEDULE OF THE 40 UNITS POWER SYSTEM

		Hours																							
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
Units	1	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00
	2	245.00	295.00	388.75	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	310.00	260.00	360.00	455.00	455.00	455.00	455.00	432.50	345.00
	3	0	0	0	0	0	0	0	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	0	0	0
	4	0	0	0	0	0	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	0	0	0
	5	0	0	0	40.00	57.50	27.50	45.00	30.00	103.75	162.00	162.00	162.00	162.00	103.75	30.00	25.00	25.00	25.00	30.00	162.00	103.75	98.75	0	0
	6	0	0	0	0	0	0	0	0	20.00	33.00	73.00	80.00	33.00	20.00	0	0	0	0	0	41.75	20.00	20.00	0	0
	7	0	0	0	0	0	0	0	0	0	25.00	25.00	25.00	25.00	0	0	0	0	0	0	0.00	0	0	0	0
	8	0	0	0	0	0	0	0	0	0	10.00	10.00	43.00	10.00	0	0	0	0	0	0	10.00	0	0	0	0
	9	0	0	0	0	0	0	0	0	0	0	10.00	10.00	0	0	0	0	0	0	0	10.00	0	0	0	0
	10	0	0	0	0	0	0	0	0	0	0	0	10.00	0	0	0	0	0	0	0	0.00	0	0	0	0
	11	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00
	12	245.00	295.00	388.75	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	310.00	260.00	360.00	455.00	455.00	455.00	455.00	432.50	345.00
	13	0	0	0	0	0	0	0	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	0	0	0
	14	0	0	0	0	0	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	0	0	0
	15	0	0	0	40.00	57.50	27.50	45.00	30.00	103.75	162.00	162.00	162.00	162.00	103.75	30.00	25.00	25.00	25.00	30.00	162.00	103.75	98.75	0	0
	16	0	0	0	0	0	0	0	0	20.00	33.00	73.00	80.00	33.00	200.00	0	0	0	0	0	41.75	20.00	20.00	0	0
	17	0	0	0	0	0	0	0	0	0	25.00	25.00	25.00	25.00	0	0	0	0	0	0	0	0	0	0	0
	18	0	0	0	0	0	0	0	0	0	10.00	10.00	43.00	10.00	0	0	0	0	0	0	10.00	0	0	0	0
	19	0	0	0	0	0	0	0	0	0	0	10.00	10.00	0	0	0	0	0	0	0	10.00	0	0	0	0
	20	0	0	0	0	0	0	0	0	0	0	0	10.00	0	0	0	0	0	0	0	0	0	0	0	0
	21	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00
	22	245.00	295.00	388.75	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	310.00	260.00	360.00	455.00	455.00	455.00	455.00	432.50	345.00
	23	0	0	0	0	0	0	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	0	0	0
	24	0	0	0	0	0	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	0	0
	25	0	0	0	40.00	57.50	27.50	45.00	30.00	103.75	162.00	162.00	162.00	162.00	103.75	30.00	25.00	25.00	25.00	30.01	162.00	103.75	98.75	25.00	0
	26	0	0	0	0	0	0	0	0	20.00	33.00	73.00	80.00	33.00	20.00	0	0	0	0	0	41.75	20.00	20.00	0.00	0
	27	0	0	0	0	0	0	0	0	0	25.00	25.00	25.00	25.00	0	0	0	0	0	0	0	0	0	0	0
	28	0	0	0	0	0	0	0	0	0	10.00	10.00	43.00	10.00	0	0	0	0	0	0	10.00	0	0	0	0
	29	0	0	0	0	0	0	0	0	0	0	10.00	10.00	0	0	0	0	0	0	0	10.00	0	0	0	0
	30	0	0	0	0	0	0	0	0	0	0	0	10.00	0	0	0	0	0	0	0	0	0	0	0	0
	31	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00
	32	245.00	295.00	388.75	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	455.00	31.00	260.00	360.00	455.00	455.00	455.00	455.00	432.50	345.00
	33	0	0	0	0	0	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	0	0	0
	34	0	0	0	0	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	130.00	0	0
	35	0	0	25.00	40.00	57.50	27.50	45.00	30.00	103.75	162.00	162.00	162.00	162.00	103.75	30.00	25.00	25.00	25.00	29.99	162.00	103.75	98.75	25.00	0
	36	0	0	0	0	0	0	0	0	20.00	33.00	73.00	80.00	33.00	20.00	0	0	0	0	0	41.75	20.00	20.00	0	0
	37	0	0	0	0	0	0	0	0	25.00	25.00	25.00	25.00	25.00	25.00	0	0	0	0	0	25.00	25.00	25.00	0	0
	38	0	0	0	0	0	0	0	0	0	10.00	10.00	43.00	10.00	0	0	0	0	0	0	10.00	0	0	0	0
	39	0	0	0	0	0	0	0	0	0	0	10.00	10.00	0	0	0	0	0	0	0	10.00	0	0	0	0
	40	0	0	0	0	0	0	0	0	0	0	0	10.00	0	0	0	0	0	0	0	0	0	0	0	0
	Sum	2800.00	3000.00	3400.00	3800.00	4000.00	4400.00	4600.00	4800.00	5200.00	5600.00	5800.00	6000.00	5600.00	5380.00	4800.00	3921.00	4000.00	4400.00	4800.00	5600.00	5200.00	4400.00	3600.00	3200.00

L-SHADE exhibited lower dispersion than those of EPSO. Moreover, ERC-L-SHADE & QP attained the minimum best cost, indicating that an appropriate generating schedule is found by ERC-L-SHADE, which can be slightly improved using a fine tuning. The same holds for the system of 100 units, where ERC-L-SHADE & QP is superior to all the algorithms both in terms of best solution quality and robustness of the solutions' distribution. However, concerning the system of 60 units ERC-L-SHADE and ERC-L-SHADE & QP falls short of BSGA and EPSO, regarding the minimum operation cost. For illustration purposes, the generation schedule of the 40 units system derived by ERC-L-SHADE & QP, is presented in Table IV.

The average execution time of the proposed methodology is shown in Figure 1. The slope of the linear segments becomes less steep while the number of units increases, indicating that the parallel CPU configuration may accelerate the procedure, when the size of the power system increases.

As observed, in most of the examined power systems, the proposed methodology achieved the minimum cost, demonstrating the efficiency of the proposed algorithm. Thus, the system operators might be benefited by the potential cost savings on a daily basis, which emerge especially in large scale systems. Moreover, the dispersion of the solutions derived from ERC is lower in all systems, indicating the consistency and reliability of the proposed method for optimizing the UCP. However, the average execution time is longer (in the order of seconds) compared to some of the benchmark methods. Nevertheless, UCP is a day ahead scheduling problem, thus, a slight compromise in terms of execution time might be

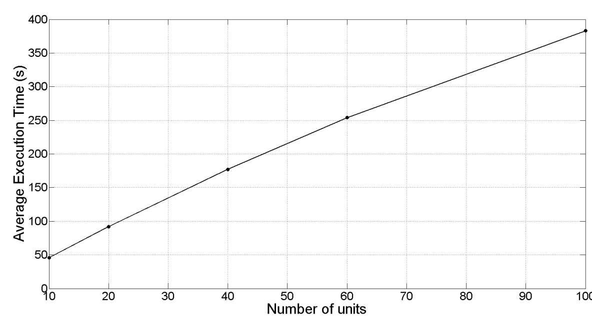

Fig. 1. Average execution time of the proposed methodology

acceptable by the system operators, in order to obtain higher cost savings.

V. Conclusion

In the present research, a novel real coded approach for the optimization of the UCP was presented. In ERC, the objective variables are the power output of the thermal units. The UC schedule of the units is derived by comparing the power outputs of each unit with its minimum permissible generation. Thus, in ERC the state and the generation of each unit are represented using a single variable. Moreover, a stochastic heuristic procedure is applied on the parameter vectors in order to ensure their feasibility; it modifies the states of the units and adjusts stochastically their generation output. L-SHADE, a state of the art adaptive DE, was employed to optimize UCP using the proposed method. A sensitivity analysis of ERC-L-SHADE was carried out, revealing that the method exhibits adequate robustness for a wide range of maximum function evaluations and different initial population sizes. The efficiency of the method was demonstrated through a comparison with some state-of-the-art approaches for UCP optimization. The results indicate that the proposed method is competitive in terms of minimum operation cost and solutions' consistency. Moreover, when the solutions derived by ERC-L-SHADE are fine tuned by a quadratic programming economic dispatch, the minimum cost found outperforms those of the examined methods, especially in power systems with high number of units. Therefore, optimizing UCP using ERC may derive adequate commitment schedules, leading to decreased operation cost. Finally, a further analysis of the impact of parameters such as k_1 and k_2 on the performance of ERC may constitute a field of future research.

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