Evolutionary Multitasking in Combinatorial Search Spaces: A Case Study in Capacitated Vehicle Routing Problem

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***Abstract*—Multifactorial optimization (*MFO*) is a new paradigm proposed recently for evolutionary multi-tasking. In contrast to traditional evolutionary optimization approaches, which focus on solving only a single optimization problem at a time, *MFO* was proposed to solve multiple optimization problems simultaneously. It is contended that the concept of evolutionary multi-tasking provides the scope for implicit knowledge transfer of useful traits across different but related problem domains, thereby enhancing the evolutionary search for problem-solving. With the aim of evolutionary multi-tasking, multifactorial evolutionary algorithm (*MFEA*) was proposed in [1], and demonstrated efficient multi-tasking performances on several problem domains, including continuous, discrete, and the mixtures of continuous and combinatorial tasks. To solve different problems, the design of unified solution representations and effective problem specific decoding operators are required in *MFEA*. In particular, the random-key unified representation and the sorting based decoding operator were presented in *MFEA* for multi-tasking in the context of vehicle routing problem. However, problems such as ineffective solution representation and decoding are existed in this unified representation, which would deteriorate the multi-tasking performance of *MFEA*. Taking this cue, in this paper, we propose an improved *MFEA* (*P-MFEA*) with a permutation based unified representation and a *split* based decoding operator. To evaluate the efficacy of the proposed *P-MFEA*, comparison against the traditional single task evolutionary search paradigm on 12 multi-tasking capacitated vehicle routing problems is presented and discussed.**

I. Introduction

Evolutionary algorithms (EAs) are adaptive search approaches that take inspirations from the principles of natural selection and genetics [2, 3]. They have been shown to be suitable for solving nonlinear, multi-modal, and discrete NP-hard problems effectively. In the last decades, EAs have been successfully applied to solve a variety of real world optimization problems, manifesting continuous optimization [4–6], combinatorial optimization [7, 8], constrained optimization [9, 10], etc. However, despite the success that EAs enjoyed, it is worth noting that most EAs are designed to solve a single problem in a single run, which appears to be inefficient in today's cloud computing era, where multiple optimization problems are required to be solved simultaneously.

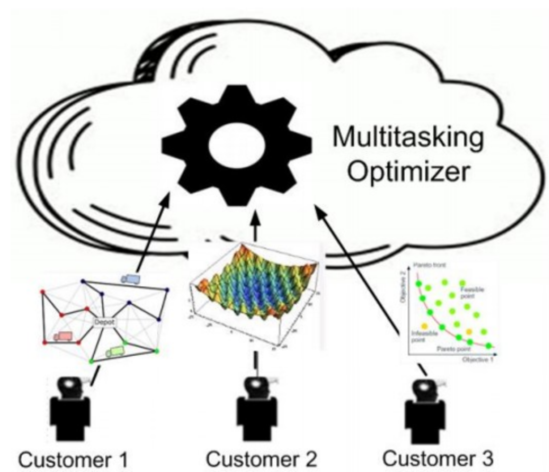


Fig. 1. Multifactorial Optimization Paradigm for Evolutionary Multi-tasking.

To this end, recently, Gupta *et al.* proposed a multifactorial optimization (*MFO*) paradigm for evolutionary multi-tasking on single objective and multi-objective optimization [1, 11, 12]. In contrast to traditional evolutionary search paradigm, as illustrated in Fig. 1 *MFO* contends to conduct evolutionary search on multiple concurrently search spaces corresponding to different tasks or optimization problems, each possessing a unique function landscape. Further, in [1], a detailed design of evolutionary algorithm for *MFO* (i.e., *MFEA*), one that is capable of multi-tasking across multiple optimization problems was also presented. With the introduction of suitable decoding operators for different optimization problems, *MFEA* has demonstrated efficient multi-tasking performance in problem domains including continuous, discrete, and the mixtures of continuous and combinatorial tasks. It is worth noting that the decoding operator plays the role of translating the solutions from the unified search space to some problem specific solution space, the designs of an unified solution representation and corresponding problem specific decoding operator represent two key factors in defining the performance of *MFEA* for evolutionary multi-tasking.

Vehicle routing problem (VRP), which represents the cornerstone of optimization for distribution networks, is one of the most important practical problems of operational research [13, 14]. To consider *MFO* of multiple VRPs simultaneously, a random-key based unified representation together with a sorting based decoding operator have been proposed in *MFEA* [1]. However, there are several drawbacks of this unified representation and corresponding decoding operator for solving multi-tasking VRPs. In particular, the simple random-key representation do not represent VRP solutions effectively. For example, solutions $\{0.1, 0.2, 0.3\}$ and $\{0.5, 0.6, 0.7\}$ represent two unique solutions in the unified search space, but both translate to a common VRP solution $\{1, 2, 3\}$ when using the simple ascending sorting decoding scheme. Further, the evolutionary search of the random-key solutions does not consider the VRP structures of the decoded routing solutions, which may deteriorate the multi-tasking performance of *MFEA* for multiple VRPs.

In this paper, we present a permutation-based multifactorial evolutionary algorithm (*P-MFEA*) for solving multi-tasking VRPs. Particularly, instead of using a random-key representation to construct the unified search space in *MFEA*, a permutation based unified representation which is able to describe the customers in VRPs directly is considered. Based on this unified permutation representation, we subsequently present a *split* based decoding operator, which takes the VRP constraints into consideration and optimizes the number of vehicles during solution translation from unified space to routing space, simultaneously. Further, empirical studies on 12 capacitated VRP multi-tasking problems with instances drawn from the existing commonly used *CMT* benchmarks proposed by Christofides *et al.* [15] are conducted to investigate the efficacy of the proposed *P-MFEA* for solving multi-tasking VRPs.

The rest of this paper is organized as follows: Section II begins with a brief introduction of *MFO*. The mathematical description of the capacitated VRP (CVRP), which serves as the routing problem domain for investigation in this paper, is also given in this section. The details of the proposed *P-MFEA*, with a permutation based unified representation and a *split* based decoding operator, are presented in Section III. Further, section IV investigates the performance of the *P-MFEA* on 12 multi-tasking CVRPs, which are extended from existing commonly used CVRP benchmarks. Lastly, the conclusions of this study and the possible directions for future works are given in section V.

II. Preliminary

This section first presents the brief introduction of the concept of multifactorial optimization, towards evolutionary multi-tasking. Next, the mathematical description of the capacitated vehicle routing problem (CVRP), which serves as the routing problem domain for investigation in this paper, is presented.

A. Multifactorial Optimization

Consider a situation wherein K optimization tasks are to be performed, multifactorial optimization (*MFO*) has been defined in [1] as an evolutionary multi-tasking paradigm that builds on the implicit parallelism of population-based search with the aim of finding the optimal solutions for each task simultaneously. In *MFO*, each task is treated as an additional factor influencing the evolution of a single population of individuals.

To compare population individuals in a multi-tasking environment, the following properties for every individual are also defined in [1]. Note that each individual in the population is encoded in a unified search space encompassing the search space of all the tasks, and can be decoded into a task-specific solution representation with respect to each of the K optimization tasks.

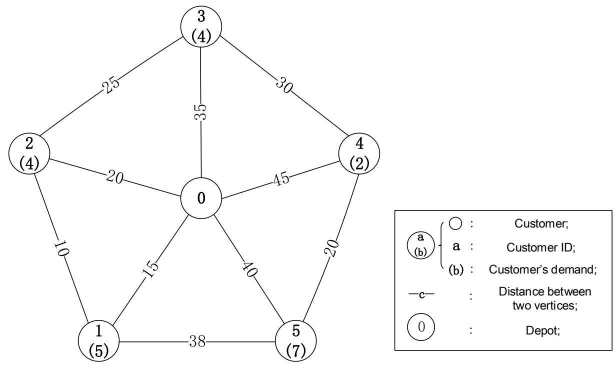
- *Factorial Cost:* The factorial cost f_p of an individual p denotes its fitness or objective value on a particular task T_i. For K tasks, there will be a vector with length K, in which each dimension gives the fitness of p on the corresponding task.
- *Factorial Rank:* The factorial rank r_p simply denotes the index of individual p in the list of population members sorted in ascending order with respect to their factorial costs on one specific task.
- *Scalar Fitness:* The scalar fitness φ_p of an individual p is defined based on its best rank over all tasks, which is given by $\varphi_p = \frac{1}{\min_{j\in\{1,\ldots,K\}} r_p^j}$.
- *Skill Factor:* The skill factor τ_p of individual p denotes the task, amongst all other tasks in *MFO*, on which p is most effective, i.e., $\tau_p = argmin\{r_p^j\}$, where $j \in \{1, \ldots, K\}$.

With the properties given above, performance comparison between solutions in *MFO* can be carried out based on scalar fitness φ. In particular, individual p_a is considered to dominate p_b in multifactorial sense simply if $\varphi_{p_a} > \varphi_{p_b}$. Therefore, suppose all the tasks are minimization problems, the definition of multifactorial optimality is given as [1]:

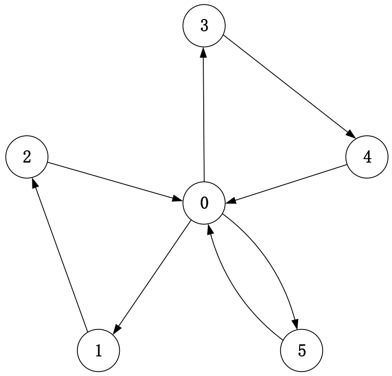
Multifactorial Optimality: An individual p^*, with a list of fitness or objective value $\{f_1^*, f_2^*, \ldots, f_K^*\}$ on K tasks, is considered optimum in multifactorial sense if and only if $\exists j \in [1, \ldots, K]$, such that $f_j^* \leq f(\mathbf{x}_j)$, where $\mathbf{x}_j$ denotes all the feasible solutions in the search space of task T_j.

B. Capacitated Vehicle Routing Problem

The capacitated vehicle routing problem (CVRP) is a fundamental problem in combinatorial optimization with wide-ranging applications in practice [14]. It is a problem to design a set of vehicle routes in which a fleet of capacitated vehicles must serve a given list of customers from a common depot. Formally, the CVRP can be described as a problem defined on an undirected graph $G = (V, E)$, where $V = \{v_0, v_1, \ldots, v_n\}$



(a) Graph of the CVRP



(b) A possible CVRP solution of Fig. 2(a)

Fig. 2. An example of CVRP.

is the vertex set and $E = \{e_{ij}\}$ denotes the arc between vertices v_i and v_j. Vertex v_0 represents the depot where k homogeneous vehicles with capacity Q are based. The remaining vertices denote the customers needed to be served. Each customer is associated with a non-negative demand $q_i \leq Q$. Further, each arc e_{ij} is assigned with a non-negative value $c_{ij} = c_{ji}$, which represents the travel distance or cost from $v_i(v_j)$ to $v_j(v_i)$. The objective of CVRP is thus to design a set of vehicle routes $R = \{\mathcal{C}_i\}$, $i = 1, \ldots, k$, with minimal travel cost such that (1) each route starts and ends at the depot v_0; (2) the total demand of each route must be less than or equal to the vehicle capacity Q; (3) each customer can only exists in one route. The overall travel cost of R is given by:

$$cost(R) = \sum_{i=1}^{k} c(\mathcal{C}_i) \tag{1}$$

where $c(\mathcal{C}_i)$ is the summarization of the travel cost c_{ij} incurred in route $\mathcal{C}_i$.

An illustrative example of CVRP is given in Fig. 2. As depicted, in Fig. 2(a), five customers labeled by number 1 to 5 need to be served, the integer value in the bracket gives the corresponding demand of each customer. The capacity of vehicle is 10, and the value associated with each edge denotes its corresponding travel cost. Fig. 2(b) presents one feasible solution of the CVRP given in Fig. 2(a), i.e., $R = \{\mathcal{C}_1, \mathcal{C}_2, \mathcal{C}_3\}$, $\mathcal{C}_1 = \{1, 2\}$, $\mathcal{C}_1 = \{3, 4\}$, and $\mathcal{C}_1 = \{5\}$. Based on Eq. 1, the total cost of R is given by $c(R) = c_{01} + c_{12} + c_{20} + c_{03} + c_{34} + c_{40} + c_{05} + c_{50} = 235$.

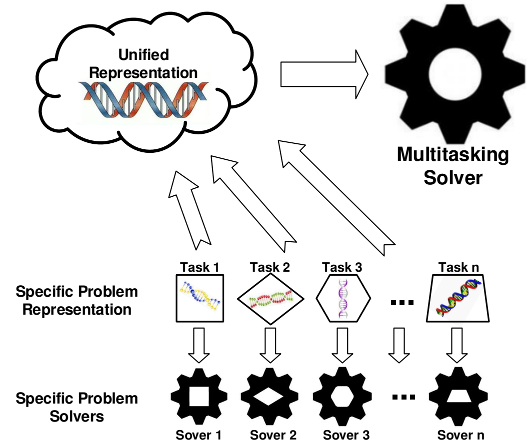


Fig. 3. Illustration of unified representation in MFEA. Domain-specific representations (depicted by different shapes) must be encoded within a unified representation scheme and the MFEA will directly operate on the unified representation for optimization on multiple tasks.

III. Permutation Based Multifactorial Evolutionary Algorithm for Solving VRPs

In this section, we present the details of the proposed permutation-based multifactorial evolutionary algorithm (*P-MFEA*) for solving multi-tasking VRPs. First of all, we recall the multifactorial evolutionary algorithm (*MFEA*) proposed in [1]. As outlined in Fig. 4, the work-flow of *MFEA* can be summarized as:

Step 1: Generate an initial population with NP individuals using random-key as unified representation. Note that, as illustrated in Fig. 3, an unified representation must be first defined for different tasks, and the MFEA will then directly operate on the individuals encoded with the unified representation.

Step 2: Evaluate each individual on all the tasks by calculating its *factorial cost* f_p, *factorial rank* r_p, *scalar fitness* φ_p and *skill factor* τ_p.

Step 3: Apply genetic operators, i.e., *assortative mating*, on the current population to generate an offspring population.

Step 4: Evaluate offspring individuals on selected tasks based on *vertical cultural transmission*.

Step 5: Update the scalar fitness φ_p and skill factor τ_p of individuals in both parent and offspring population.

Step 6: Select the fittest NP individuals from both parent and offspring population to survive for the next generation.

Step 7: If the stopping criteria are not met, then repeat **Step 3** to **Step 6**.

As afore discussed, to evaluate the individual on a specific task, a particular decoding operator is required to translate the individual from unified solution space to the task-specific solution space. For solving multi-tasking VRPs, in [1], a random-key unified representation together with a clustering and sorting based decoding operator were proposed. However,

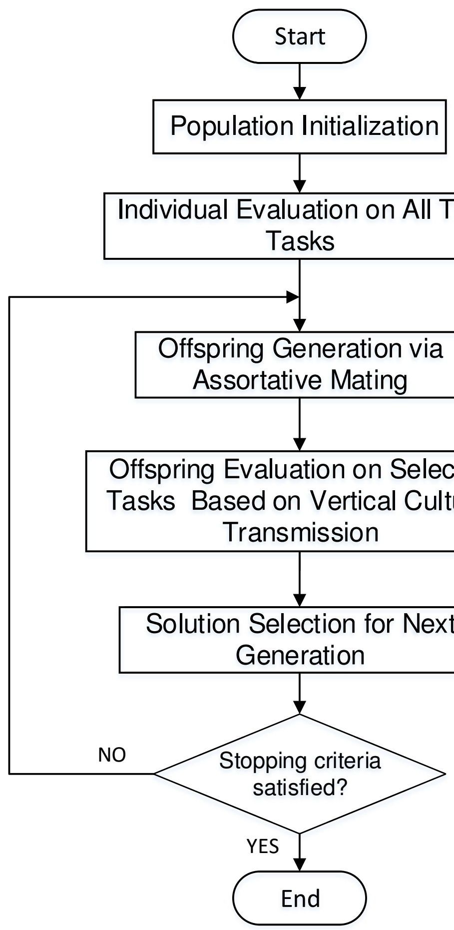


Fig. 4. The outline of *MFEA*.

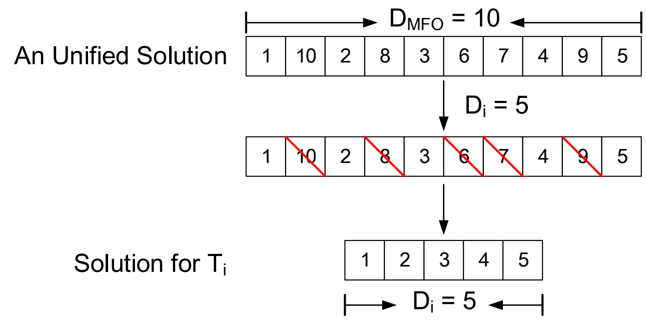


Fig. 5. An example of permutation based representation

as discussed in Section I, there are several drawbacks of this unified representation and decoding operator, which could deteriorate the search efficacy of *MFEA* for solving mutli-tasking VRPs. For instance, they cannot represent the VRP solutions effectively, the evolution of the unified solutions is ineffective, since it does not take the VRP structures of the decoded routing solution into consideration.

To tackle these problems, in the proposed *P-MFEA*, we first present a permutation based unified representation [16] to construct the unified solution space. Based on this unified representation, a *split* [17] based operator is proposed to take the VRP constraints into consideration and optimizes the number of vehicle routes simultaneously, in the solution decoding process.

A. *Permutation Based Unified Representation for* P-MFEA

Permutation based representation is commonly used to describe solutions in routing problems, such as vehicle routing [18, 19], arc routing [20, 21], etc. Take Fig. 2(b) as an example, the corresponding permutation based solution representation is $s = \{1, 2, 3, 4, 5\}$. Each dimension of s directly denotes the customer needs to be served, and the size of s is equal to the number of customers.

Assume that in K VRPs to be performed simultaneously, the dimensionality of the ith VRP is given by D_i. To construct the permutation based unified representation for multifactorial optimization, we first define the dimensionality of the unified search space as $D_{MFO} = \max D_i$, $i = 1, \ldots, K$. For the population initialization step, each individual is thus endowed with a vector of D_{MFO} different integer variables, lying within the fixed range of $[0, D_{MFO}]$. The jth dimension of the vector denotes the jth customer needs to be served. While addressing task T_i, which has dimension D_i less than D_{MFO}, we simply refer to the dimensions of values from 1 to D_i in the unified search space to represent customers in T_i. For instance, as depicted in Fig. 5, the dimension of the unified space is $D_{MFO} = 10$. If an unified solution is given as $\{1, 10, 2, 8, 3, 6, 7, 4, 9, 5\}$, the solution for T_i with dimension $D_i = 5$ is then obtained by removing the extra dimensions with values larger than 5.

B. *Split Based Decoding Operator for* P-MFEA

Based on the permutation based unified representation proposed above, this section presents a *split* [17] based decoding operator to translate solutions in the unified search space to VRP solution space.

In particular, the pseudo code of the *split* based decoding operator is given in Alg. 1. $\mathbf{F}$ and $\mathbf{P}$ are two vectors with size $N + 1$ (N is the number of customers in a given VRP). For a solution in the unified space S, $F(j)$ and $P(j)$ record the minimum cost of the routes in which the last customer is $S(j)$ and the predecessor vertex of $S(j)$, respectively. In Alg. 1, after the initialization of vector $\mathbf{F}$ and $\mathbf{P}$ (i.e., line 1-2), line 3-17 will enumerate all the feasible vehicle routes starting from $S(i)$ and store the routes with minimum cost accordingly. Line 18 will output the decoded CARP solution R by injecting the delimiter 0 into S based on the predecessors recorded in $P(N)$.

Further, the example presented in Fig. 2(a) is again here used to illustrate how to translate solutions from unified search space to VRP solution space by the proposed *split* based decoding operator. Suppose the unified solution is given as $S = \{1, 2, 3, 4, 5\}$. The decoding operator will evaluate all the feasible routes as shown in Fig. 6(b) and update $\mathbf{F}$ and $\mathbf{P}$ according to the costs of the routes. For instance, for customer

Algorithm 1: Pseudo code of the *split* based decoding process.

Input : A solution S in the unified search space.
Output: A feasible VRP solution R.

Initialize $F[0, \cdots, N]$, $P[0, \cdots, N]$, N is the number of customers;
$F[0] \leftarrow 0$, $F[1, \cdots, N] \leftarrow +\infty$, $P[1, \cdots, N] \leftarrow 0$;
for $i = 1$ ***to*** N **do**
  $Demand \leftarrow 0, Cost \leftarrow 0$;
  $j \leftarrow i$;
  while $j < N$ ***and*** $Demand \leq Q$ **do**
    $Demand \leftarrow Demand + q_{S(j)}$;
    if $i == j$ **then**
      $Cost \leftarrow c_{0,S(j)} + c_{S(j),0}$;
    else
      $Cost \leftarrow Cost + c_{S(j-1),S(j)} - c_{S(j-1),0}$
        $+ c_{S(j),0}$;
    if $Demand \leq Q$ **then**
      if $F[i-1] + Cost < F[j]$ **then**
        a. $F[j] \leftarrow F[i-1] + Cost$;
        b. $P[j] \leftarrow i - 1$
    $j \leftarrow j + 1$
Obtain R by inserting the delimiters '0' into S based on $\mathbf{P}$.

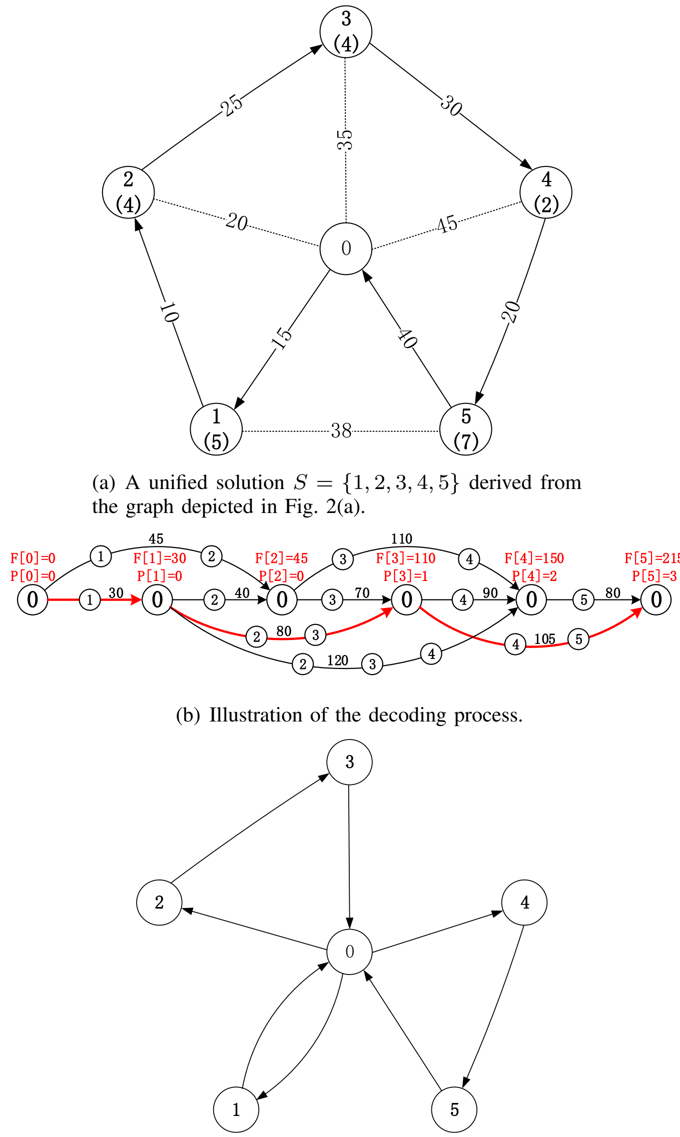
$S_2 = 2$, there are two routes with S_2 as the last customer to be served, which are $R_1 = \{0, 1, 2, 0\}$ and $R_2 = \{0, 1, 0, 2, 0\}$. Since $c(R_1) = 45 < c(R_2) = 70$, F_2 and P_2 are then set as 45 and 0, respectively. In this manner, $\mathbf{F}$ and $\mathbf{P}$ can be updated from dimension 0 to 5. Lastly, the delimiter 0 will be inserted into S based on P (i.e., $P_5 = 3 \rightarrow P_3 = 1 \rightarrow P_1 = 0$) to obtain the VRP solution $R = \{0, 1, 0, 2, 3, 0, 4, 5, 0\}$ with a minimal cost $c(R) = 215$, as depicted in Fig. 6(c).

IV. Empirical Study

To evaluate the efficacy of the proposed *P-MFEA*, empirical study in the domain of CVRP is presented in this section.

A. Experiment Setup

For the setup of multi-tasking environment, as there is no common multi-tasking routing benchmark in the literature, here we build 12 multi-tasking routing problems based on the existing CVRP instances. In particular, the commonly used 14 Christofides CVRP benchmarks [15] with diverse properties (e.g., number of vertices, distribution of vertices, routing time constraints, etc.) are considered in this study. The detailed properties of the CVRP instances are summarized in Table I. "$CMTi$" denotes the number i instance in the Christofides CVRP benchmark. "N", "*Capacity*", "*RT*", "*ST*" and "*BKS*" represent the number of customers, the capacity of vehicle,



(a) A unified solution $S = \{1, 2, 3, 4, 5\}$ derived from the graph depicted in Fig. 2(a).

(b) Illustration of the decoding process.

(c) The decoded VRP solution R.

Fig. 6. A demonstration of the decoding process on the VRP presented in Fig. 2(a) by the proposed *split* based operator.

the routing time constraint, the service time of vehicle and the best known solution[1], respectively.

Further, the multi-tasking routing problem is built by pairing instances with different routing properties, such as different number of customers, different routing time constraints, etc. The built 12 multi-tasking problems for investigation in this study are summarized in Table II.

Next, in this study, the traditional single evolutionary algorithm (label as *SEA* hereafter) is considered as the baseline for comparison. For fair comparison, both the *SEA* and the proposed *P-MFEA* are configured with the same evolutionary operators and parameters, which are detailed as follows:

1) Population size: Popsize = 30.

[1]The best known solution of the CVRPs to date in the literature are drawn from: http://neo.lcc.uma.es/vrp/known-best-results/.

TABLE I
PROPERTIES OF THE CHRISTOFIDES CVRP INSTANCES (n, RT, ST AND BKS DENOTE THE *Customer Number*, *Routing Time Constraint*, *Service Time* AND *Best Known Solution*, RESPECTIVELY).

CVRP Instances	N	$Capacity$	RT	ST	BKS
$CMT1$	50	160	∞	0	524.61
$CMT2$	75	140	∞	0	835.26
$CMT3$	100	200	∞	0	826.14
$CMT4$	150	200	∞	0	1028.42
$CMT5$	199	200	∞	0	1291.29
$CMT6$	50	160	200	10	555.43
$CMT7$	75	140	160	10	909.68
$CMT8$	100	200	230	10	865.94
$CMT9$	150	200	200	10	1162.55
$CMT10$	199	200	200	10	1395.85
$CMT11$	120	200	∞	0	1042.11
$CMT12$	100	200	∞	0	819.56
$CMT13$	120	200	720	50	1541.14
$CMT14$	100	200	1040	90	866.37

2) Max Generation: Gmax $= 1000$.
3) Independent run times: Runs $= 20$.
4) Evolutionary operators:
   - Crossover: OX(Ordered Crossover) [22].
   - Mutation: Swap mutation [23].
5) Local search operators: Exchange [24], Or-opt [24], 2-opt move [24].
6) Probability in assortative mating [1]: $RMP = 0.7$.
7) Local search probability: $PLS = 0.1$.

B. Results and Discussion

Table III tabulates the performance of the single evolutionary algorithm *SEA* and the proposed permutation based multifactorial evolutionary algorithm *P-MFEA* on the 12 multi-tasking CVRPs across 20 independent runs. Note that *SEA* solves the CVRP instances in each multi-tasking problem separately while *P-MFEA* on the other hand, optimizes the two CVRP instances simultaneously. "*B.Cost*" and "*Ave.Cost*", "*Std.Dev*" denote the best cost, averaged cost and standard deviation, respectively. The superior performance in Table III is highlighted in bold.

As can be observed, in Table III, *P-MFEA* achieved superior or competitive performance against *SEA* on 22 out of total 24 CVRP instance, in terms of "*Ave.Cost*". With respect to "*B. Cost*", *P-MFEA* has found better best cost solutions on 6 CVRP instances than *SEA*. Further, it is noted that on instances, such as "$CMT1$", "$CMT6$", and "$CMT8$", which have small number of customers, *P-MFEA* and *SEA* always obtained the same '*B. Cost*" and "*Ave.Cost*". However, when it turns to solve the CVRP instances, such as "$CMT5$", "$CMT10$", and "$CMT13$", which have larger number of customers and contain more complex routing structure, *P-MFEA* always can find better solutions than *SEA* with respect to both '*B. Cost*" and "*Ave.Cost*".

Further, to access the efficiency of the proposed *P-MFEA*, the representative search convergence traces of *P-MFEA* and *SEA* on the multi-tasking CVRPs are presented in Fig. 7. The Y-axis of the figures denote the averaged travel cost obtained, while X-axis gives the respective computational effort incurred in terms of generations made so far. As depicted, generally, *P-MFEA* converges faster than *SEA* on most of the multi-tasking CVRPs. It is worth noting that on the instances with small number of customers, i.e., "$CMT1$", "$CMT6$", and "$CMT8$', although *P-MFEA* and *SEA* obtained the same solution quality (as presented in Table III), *P-MFEA* converges much fast than *SEA*, e.g., "$CMT6$" in Fig. 7(b), "$CMT8$' in Fig. 7(d), etc. Further, on the instance with larger number of instances, i.e., "$CMT5$", "$CMT13$", etc. faster or competitive convergence speed can also be observed by *P-MFEA* against *SEA*, e.g., "$CMT5$" in Fig. 7(c), "$CMT13$" in Fig. 7(e), etc.

Since *P-MFEA* and *SEA* share the same configuration of evolutionary operators and parameters, and the only difference is the *MFO* paradigm incorporated in the former, the superior performance obtained by *P-MFEA* in terms of both solution quality and search efficiency confirmed the efficacy of automated implicit information transfer in *MFO* paradigm as well as the effectiveness of our proposed unified representation and specific decoding operator for solving multi-tasking CVRPs.

V. CONCLUSION

In this paper, a permutation-based *MFEA* has been proposed to improve the multi-tasking performance of *MFEA* in the context of VRPs. Our main contributions can be summarized in two-folds. First of all, we have presented a permutation based unified solution representation, which is more effective than the random-key unified representation in *MFEA*. Secondly, we have proposed a *split* based decoding operator to translate solution from unified space to problem specific space. It takes the VRP constraints into consideration and optimizes the number of vehicle routes simultaneously, during the decoding process. The efficacy of the proposed *P-MFEA* have been confirmed on 12 multi-tasking CVRPs, which are extended from existing commonly used CVRP benchmarks.

As this paper only considered CVRP as the routing problem domain of study, in the future, we would like to further investigate the proposed *P-MFEA* on other variants of VRPs, such as VRP with time window, VRP with stochastic demand, etc, towards more generalized and efficient multi-tasking VRPs.

VI. ACKNOWLEDGEMENTS

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TABLE II
SUMMARY OF THE MULTI-TASKING PROBLEMS INVESTIGATED.

Multi-tasking Problems	$P1$	$P2$	$P3$	$P4$	$P5$	$P6$	$P7$	$P8$	$P9$	$P10$	$P11$	$P12$
CMT Instances	$CMT1 + CMT3$	$CMT1 + CMT6$	$CMT3 + CMT4$	$CMT3 + CMT5$	$CMT3 + CMT12$	$CMT6 + CMT8$	$CMT6 + CMT10$	$CMT8 + CMT13$	$CMT8 + CMT14$	$CMT11 + CMT13$	$CMT12 + CMT14$	$CMT13 + CMT14$

TABLE III
NUMERICAL RESULTS OF *P-MFEA* AND *SEA* ("*B.Cost*", "*Ave.Cost*", "*Std.Dev*" DENOTE THE BEST COST, AVERAGED COST AND STANDARD DEVIATION ACROSS 20 INDEPENDENT RUNS, RESPECTIVELY. THE SUPERIOR PERFORMANCE IS HIGHLIGHTED IN BOLD.).

Multi-tasking Problems	CVRP Instance	Proposed P-MFEA			Single Solver		
		$B.Cost$	$Ave.Cost$	$Std.Dev$	$B.Cost$	$Ave.Cost$	$Std.Dev$
$P1$	$CMT1$	524.61	524.61	0.0	524.61	524.61	0.0
	$CMT3$	826.14	**828.51**	1.3	826.14	829.02	0.9
$P2$	$CMT1$	524.61	524.61	0.0	524.61	524.61	0.0
	$CMT6$	555.43	555.43	0.0	555.43	555.43	0.0
$P3$	$CMT3$	826.14	**828.19**	1.4	826.14	829.02	0.9
	$CMT4$	**1031.96**	1037.19	4.7	1032.18	**1035.90**	3.2
$P4$	$CMT3$	826.14	**828.72**	1.0	826.14	829.02	0.9
	$CMT5$	**1300.76**	**1314.02**	4.5	1306.01	1314.76	3.5
$P5$	$CMT3$	826.14	**828.46**	1.3	826.14	829.02	0.9
	$CMT12$	819.56	819.56	0.0	819.56	819.56	0.0
$P6$	$CMT6$	555.43	555.43	0.0	555.43	555.43	0.0
	$CMT8$	865.94	865.94	0.0	865.94	865.94	0.0
$P7$	$CMT6$	555.43	555.43	0.0	555.43	555.43	0.0
	$CMT10$	**1403.59**	**1410.45**	3.8	1409.08	1410.84	2.7
$P8$	$CMT8$	865.94	865.94	0.0	865.94	865.94	0.0
	$CMT13$	**1545.55**	**1555.21**	6.6	1548.78	1556.83	4.9
$P9$	$CMT8$	865.94	865.94	0.0	865.94	865.94	0.0
	$CMT14$	866.37	866.37	0.0	866.37	866.37	0.0
$P10$	$CMT11$	1042.11	1045.94	6.9	1042.11	**1044.65**	2.1
	$CMT13$	**1545.73**	**1551.17**	3.7	1548.78	1556.83	4.9
$P11$	$CMT12$	819.56	819.56	0.0	819.56	819.56	0.0
	$CMT14$	866.37	866.37	0.0	866.37	866.37	0.0
$P12$	$CMT13$	**1546.36**	**1553.73**	5.1	1548.78	1556.83	4.9
	$CMT14$	866.37	866.37	0.0	866.37	866.37	0.0

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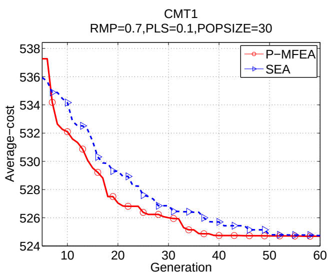
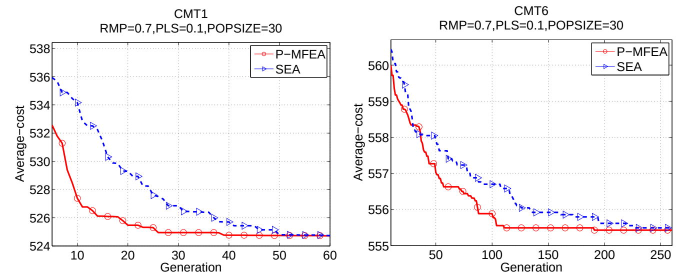
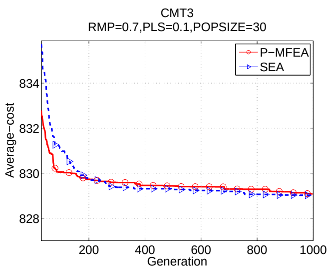


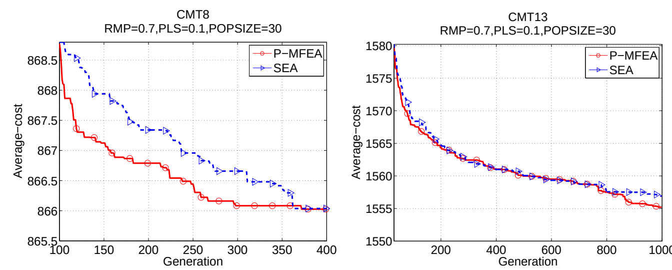
(a) Convergence traces of $P1$



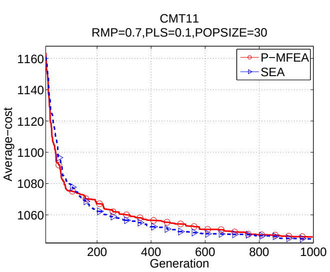
(b) Convergence traces of $P2$



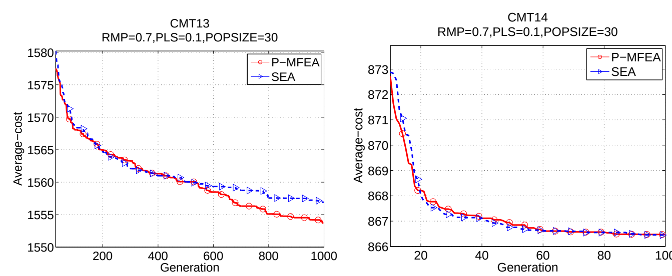
(c) Convergence traces of $P4$



(d) Convergence traces of $P8$



(e) Convergence traces of $P10$



(f) Convergence traces of $P12$

Fig. 7. Convergence traces of *P-MFEA* versus the SEA on representative multi-tasking CVRPs. Y-axis: Averaged travel cost; X-axis: Generation.

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