Finite Horizon Optimal Control and Communication Co-design for Uncertain Networked Control System with Transmit Power Constraint

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Abstract—**In this paper, finite horizon optimal control and communication co-design problem has been investigated for uncertain networked control system (NCS) with limited transmit power. Firstly, the mathematical relation between network imperfections (e.g. network-induced delays and packet dropouts) and practical wireless communication channel quality has been studied. Then, a novel networked control system model has been represented that includes both physical system dynamics and network dynamics. Next, adopting the emerging neuro dynamics programming (NDP) technique and actor-critic-identifier (ACI) architecture, a novel time-based finite horizon optimal control and communication co-design has been proposed. The proposed scheme cannot only learn the optimal co-design within fixed final time, but also relax the requirement of NCS dynamics and satisfy the practical transmit power constraint. The Lyapunov Theorem is used to validate the effectiveness of proposed scheme. With the proper NN weight update law, proposed scheme can ensure all closed-loop signals and NN weights are uniformly ultimately bounded (*UUB*). Moreover, simulation results are also included to demonstrate the effectiveness of the proposed scheme.**

Keywords—Neuro Dynamics Programming, Optimal Co-design, Networked Control Systems

I. INTRODUCTION

During the past decade, networked control systems (NCS) [1], which has been considered as the next-generation of advanced physical system, attract tremendous interests from both control and communication societies [2], [3]. Using the communication network to connect multiple distributed physical systems, NCS will be able to exchange the real-time local information more efficiently and further construct a more effective global control. However, due to the communication uncertainties, engaging wireless network with control system will also produce several serious challenges such as network-induced delays, packet dropouts and so on. Before truly reaping the benefits from NCS, how to handle the effects from network imperfections is critical.

Therefore, authors in [4] and [5] studied how the network-induced delay and packet dropout affect the control system performance, and further developed a NCS stability region. In addition to the stability analysis, optimal design is much more preferred. Therefore, authors in [6] proposed an infinite horizon stochastic optimal control for NCS under network-induced delays. In [6], authors considered the network-induced delays as a random variable which is independent from the practical communication network protocol. Then, using stochastic optimal theory, an infinite horizon stochastic optimal control be easily generated for NCS. However, these NCS designs [4], [5], [6] have several issues: 1) The practical network imperfections (e.g. delays and packet dropouts) are highly depending on real-time network protocol. Considering network imperfection as random variables is not efficient and practical, 2) Most existing optimal design for NCS need to know the NCS dynamics and network imperfections information that are usually unknown beforehand, 3) Current designs [7] solved the stochastic optimal NCS control backward-in-time which cannot be implemented into practical NCS and 4) Existing NCS designs [4], [5], [6] are usually for infinite horizon scenarios whereas finite horizon optimal design is more preferred.

To overcome these challenges, a novel finite horizon optimal control and communication co-design has been developed for NCS with limited transmit power in this paper. Firstly, adopting Shannon-Hartley Theorem [8] and relevant wireless communication theory [3], network imperfections can be represented by using practical wireless channel quality and relevant limited transmit power mathematically. Then, transmit power constrained NCS can be modeled from both control and communication aspects in the real-time. With the novel NCS model, a novel transmit power constrained optimal control and communication co-design problem has been formulated. However, as shown in [9], [10], solving optimal co-design directly is very difficult and even impossible since the practical NCS dynamic cannot be known beforehand. To circumvent this issue, the neuro dynamics programming techniques proposed by Werbos [11], Bertsekas and Tsitsiklis [12], and actor-critic-identifier architecture [13] have been adopted. To relax the requirement of system dynamics, a neural network (NN) based identifier is designed to learn the NCS model online. Subsequently, a critic NN is developed to estimate the solution of Hamilton-Jacobi-Bellman (HJB) equation (i.e. optimal co-design cost function) within fix final time. In addition, the proposed critic NN can also satisfy the given

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terminal constraint during finite horizon. Then, two actor NNs have been developed to approximate the optimal control signals and transmit power respectively. Eventually, the effectiveness of proposed design has been validated from both theoretical closed-loop stability analysis and numerical simulations.

The main contribution of this paper includes 1) A novel NCS modeling that effectively integrating the physical dynamics and real-time network dynamics; 2) The need for NCS dynamics has been relaxed; 3) The transmit power constrained optimal control and communication co-design can be learnt forward-in-time within fixed final time; and 4) Practical transmit power constraint can be satisfied.

This paper is organized as: Section II presents the novel NCS modeling and optimal co-design problem formulation. The novel finite horizon transmit power constrained optimal co-design is developed in Section III where the closed-loop stability is analyzed by using Lyapunov Theorem. Section IV illustrates the effectiveness of proposed approach while concluding remarks are given in Section V.

II. Networked Control Systems Modeling and Problem Formulation

A. Novel Networked Control Systems (NCS) Modeling

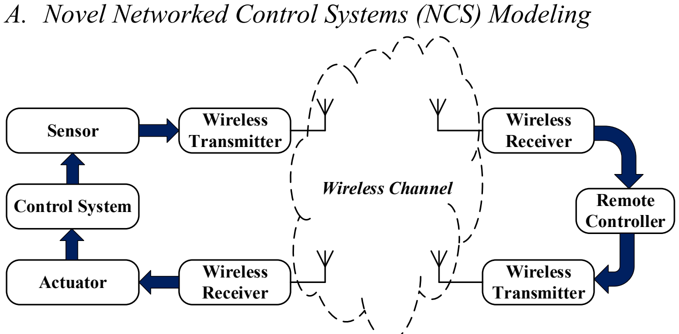

Figure 1. Networked Control Systems

Unlike the most NCS literatures [4], [5], [6], the practical wireless communication channel has been incorporated into the NCS modeling in this paper. Before we model the NCS, the new structure of NCS is shown in Figure 1 where the feedback control loop is closed through wireless communication channel. According to the recent NCS literatures [4], [5], [6] and Shannon-Hartley theorem [8], there are two important facts needed to be considered, i.e. 1) the communication delay and packet dropout are two common factors affecting control systems, and 2) Delays and packet dropout are depending on channel quality and practical transmit power design. Therefore, considering original time-invariant system $\dot{x}(t) = Ax(t) + Bu(t)$, the novel NCS can be modeled from both control and communication aspects as

Control Aspect:

$$x_{k+1} = A_s x_k + B_1(\tau_k, \gamma_{k-1}) u_{c,k-1} + B_0(\tau_k, \gamma_k) u_{c,k} \tag{1}$$

where $A_s = e^{AT_s} \in \Re^n$, $B_1(\tau_k, \gamma_{k-1}) = \gamma_{k-1} \int_0^{\tau_k} e^{A(T_s - s)} B \bullet ds \in \Re^{n \times m}$ and $B_0(\tau_k, \gamma_k) = \gamma_k \int_{\tau_k}^{T_s} e^{A(T_s - s)} B \bullet ds \in \Re^{n \times m}$ are control system dynamics that included delays and packet dropout with T_s being sampling interval. $x_k \in \Re^n, u_{c,k} \in \Re^m$ denote the control system states and control inputs respectively.

Communication Aspect [3]:

$$\tau_{k+1} = \frac{L}{W_B \log_2\left(1 + \dfrac{P_{k+1}}{N_0 W}\right)} = \frac{L}{W_B \log_2\left(1 + \dfrac{P_k + u_{P,k}}{N_0 W_B}\right)} \tag{2}$$
$$\gamma_k = \frac{1}{2} + \frac{1}{2}\operatorname{sgn}\left(\frac{1}{T_s} - \frac{1}{\tau_k}\right)$$

with $\tau_k, \gamma_k \in \Re$ denote the delay and packet dropout, $P_k, u_{P,k} \in \Re$ are transmit power and relevant power control inputs at time kT_s that satisfying the transmit power dynamics as $P_{k+1} = P_k + u_{P,k}$ along with the constraint as $0 \le P_k \le P_M$ and P_M is maximum transmit power. Moreover, $L \in \Re$ is the size of data exchanged through the wireless channel. $W_B \in \Re$ denotes the bandwidth of the channel and N_0 is the average power unit of the noise over bandwidth. $\operatorname{sgn}(\bullet)$ is the sign function [13].

Remark 1: To better understand NCS modeling given in (1) and (2), three things needed to be clarified: 1) When delay is larger than one sampling interval (i.e. $\tau_k > T_s$), the packet is considered to be dropped (i.e. $\gamma_k = \frac{1}{2} + \frac{1}{2}\operatorname{sgn}\left(\frac{1}{T_s} - \frac{1}{\tau_k}\right) = 0$); 2) The packet dropout, γ_k, can be represented by using delay τ_k as equation (2); and 3) The dynamics of delays developed in (2) provides the ability to combine the control and communication aspects into NCS modeling. The details are shown below.

Next, to model NCS more efficiently, a novel augment states has been defined from both control and communication aspects as $z_k = col\{x_k, u_{c,k-1}, \varsigma_k, \varsigma_{k-1}\} \in \Re^{m+n+2}$ with $\varsigma_k = 1/\tau_k$. Then, novel NCS can be represented as

$$z_{k+1} = f_z(z_k) + g_z(z_k) u_{c,k} + g_\tau u_{p,k} \tag{3}$$

with $f_z(z_k) = col\left\{A_z(\varsigma_k, \varsigma_{k-1}), 0, \varsigma_k, \varsigma_k\right\} \in \Re^{n+m+2}$, $g_z(z_k) = col\left\{B_z(\tau_k, \varsigma_k), 0, 0\right\} \in \Re^{(n+m+2)\times m}$ and $g_\tau = col\left\{0, \dfrac{W_B}{L}, 0\right\} \in \Re^{n+m+2}$

wher $A_z(\varsigma_k, \varsigma_{k-1}) = \begin{bmatrix} A_s & B_1(\tau_k, \gamma_{k-1}) \\ 0 & 0 \end{bmatrix}$ and $B_z(\tau_k, \varsigma_k) = \begin{bmatrix} B_0(\tau_k, \gamma_{k-1}) \\ 1 \end{bmatrix}$.

Also, transmit power control needs to satisfy time-varying constraint as $-P_k \le u_{p,k} \le P_M - P_k$.

Remark 2: Compared with most existing NCS literatures [4], [5], [6], there are three things to note: 1) To the best knowledge of author, it is the first time that NCS has been represented in terms of both control and communication aspects. Using this novel representation, most stochastic NCS model becomes real-time deterministic model which is better for practical implementation; 2) Owing to the effects from communication (e.g. delays and packet dropouts), developed NCS (3) becomes nonlinear affine system with two controllers from system and transmit power. 3) It is very difficult to attain the optimal control and communication co-design due to the uncertainties from NCS dynamics and constraint from transmit power.

Next, the transmit power constrained finite horizon optimal co-design problem is formulated.

B. Ideal Transmit Power Constrained Finite Horizon Optimal Co-design

According to [9], the transmit power constrained finite horizon optimal co-design needs to minimize the following cost function

$$V(z_k,k)=\chi(z_N)+\sum_{i=k}^{N-1}\left[Q(z_i,i)+u_{c,i}^T R_c u_{c,i}+\rho(u_{p,i})\right]\ k=0,...,N-1;$$
$$V(z_N,N)=\chi(z_N) \tag{4}$$

where $\chi(z_N)\geq 0$ is the terminate cost constraint [10], $Q(z_k,k)\in\Re$ is positive semidefinite function, and $u_{c,i}^T R_c u_{c,i},\rho(u_{p,i})\in\Re$ are positive definite functions. Inspired from [14], to confront the transmit power constraint, the non-quadratic cost function, $\rho(u_{p,k})$, is defined as

$$\rho(u_{p,k})=2\int_0^{u_{p,k}}\left\{\left[\varphi_k^{-1}(v)\right]\times R_p\right\}dv \tag{5}$$

with $\varphi_k(\bullet)$ is a bounded function satisfying $-P_k\leq\varphi_k(\bullet)\leq P_M-P_k$ and belonging to the continuous function set C_φ. Furthermore, $\varphi_k(\bullet)$ is also a monotonic odd function whose first derivative is bounded by a constant B_φ. A practical example is hyperbolic tangent function (i.e. $\varphi_k(\bullet)=\tanh(\bullet)$). Note that $\varphi_k^{-1}(\bullet)$ is monotonic odd and R_p is positive definite, and $\rho(u_{p,k})$ is positive definite. Recall to Bellman's principle of optimality [9], [11], the optimal cost function needs to satisfy the following Bellman Equation as

$$V^*(z_k,k)=\min_{u_{c,k},u_{p,k}}\left\{Q(z_k,k)+u_{c,k}^T R_c u_{c,k}+2\int_0^{u_{p,k}}\left\{\left[\varphi_k^{-1}(v)\right]\times R_p\right\}dv+V^*(z_{k+1},k+1)\right\} \tag{6}$$

Next, the optimal co-design, $u_{c,k}^*,u_{p,k}^*$, can be derived as

$$\left\{u_{c,k}^*,u_{p,k}^*\right\}=\arg\min\left\{Q(z_k,k)+u_{c,k}^T R_c u_{c,k}+2\int_0^{u_{p,k}}\left\{\left[\varphi_k^{-1}(v)\right]\times R_p\right\}dv+V^*(z_{k+1},k+1)\right\} \tag{7}$$

Using the optimal control theory [9], transmit power constrained optimal co-design can be developed as

$$u_{c,k}^*=-\frac{1}{2}R_c^{-1}g_z^T(z_k)\frac{\partial V^*(z_{k+1},k+1)}{\partial z_{k+1}}$$
$$u_{p,k}^*=-\varphi_k\left[\frac{1}{2}R_p^{-1}g_\tau\frac{\partial V^*(z_{k+1},k+1)}{\partial z_{k+1}}\right] \tag{8}$$

Remark 3: Note that the transmit power constrained optimal co-design is very difficult to obtain due to three challenges, i.e. 1) Owing to the uncertainties, NCS dynamics, $g_z(z_k),g_\tau$, cannot be known completely, 2) Due to the nonlinearity of HJB equation, optimal cost function, $V^*(z_k,k)$, is very hard to solve directly from HJB equation, and 3) The future NCS system state, z_{k+1}, cannot be known beforehand.

To overcome these challenges and further approximate the transmit power constrained finite horizon optimal co-design for NCS, a neuro dynamics programming (NDP) based actor-critic-identifier design is developed next.

III. Finite Horizon Optimal Co-design for NCS with Transmit Power Constraint

In this section, a novel NDP-based actor-critic-identifier (ACI) is utilized to approximate the transmit power constrained finite horizon optimal co-design.

A. Finite Horizon ACI Optimal Co-design Part 1: Identifier

To attain the ideal optimal co-design given in (8), system dynamics are needed. However, due to the effects from unreliable wireless communication, the practical NCS system dynamics (i.e. $f_z(\bullet),g_z(\bullet),g_\tau$) cannot be known accurately. To circumvent this challenge, a novel online NN-based identifier is developed as follows.

Using the universal function approximation theorem of NN [15], the developed NCS dynamics in (3) can be represented as

$$f_z(z_k)=W_{fz}^T\phi_{fz}(z_k)+\varepsilon_{fz}$$
$$g_{aug}(z_k)=\left[g_z(z_k)\quad g_\tau\right]=W_{aug}^T\phi_{aug}(z_k)+\varepsilon_{aug} \tag{9}$$

where $W_{fz}\in\Re^{l_{fz}\times(n+m+2)}$, $W_{aug}\in\Re^{l_{aug}\times(n+m+2)}$ denote the NN target weights, $\phi_{fz}(\bullet)\in\Re^{l_{fz}},\phi_{aug}(\bullet)\in\Re^{l_{aug}}$ are relevant NN activation function, and $\varepsilon_{fz}\in\Re^n,\varepsilon_{aug}\in\Re^{(n+m+2)}$ represent the NN reconstruction errors.

Furthermore, using (3), the NCS dynamics (3) can be expressed as

$$\begin{aligned}z_{k+1}&=f_z(z_k)+g_z(z_k)u_{c,k}+g_\tau u_{p,k}\\&=\begin{bmatrix}W_{fz}\\W_{aug}\end{bmatrix}^T\begin{bmatrix}\phi_{fz}(z_k)&0\\0&\phi_{aug}(z_k)\end{bmatrix}\begin{bmatrix}1\\u_{aug}\end{bmatrix}+\varepsilon_{fz}+\varepsilon_{aug}u_{aug}\\&=W_{I\_NCS}^T\phi_{I\_NCS}(z_k)\bar{u}_{NCS,k}+\varepsilon_{I\_NCS}\\&=W_{I\_NCS}^T\vartheta_{I\_NCS}(z_k,\bar{u}_{NCS,k})+\varepsilon_{I\_NCS}\end{aligned} \tag{10}$$

with the target weights and relevant activation function of NN-based identifier as $W_{I\_NCS}=\left[W_{fz}^T\quad W_{aug}^T\right]^T\in\Re^{(l_{fz}+l_{aug})\times(n+m+2)}$, $\vartheta_{I\_NCS}(z_k,\bar{u}_{NCS,k})=\phi_{I\_NCS}(z_k)\bar{u}_{NCS,k}\in\Re^{(l_{fz}+l_{aug})}$ where $\phi_{I\_NCS}(z_k)=diag\left\{\phi_{fz}(z_k),\phi_{aug}(z_k)\right\}$. Then, $\varepsilon_{I\_NCS}=\varepsilon_{fz}+\varepsilon_{aug}u_{aug}\in\Re^{n+m+2}$ is the reconstruction error for NN-based identifier. Since the activation function of NN-based identifier is known [], the given NCS dynamics (3) can be learnt while the target NN weights have been estimated effectively. Therefore, a proper NN weights update law is needed.

First, adopting the proposed NN-based identifier, we develop that NCS state estimator as

$$\hat{z}_{k+1}=\hat{W}_{I\_NCS}^T(k)\vartheta_{I\_NCS}(z_k,\bar{u}_{NCS,k})-A\tilde{z}_k \tag{11}$$

where $\tilde{z}_k=z_k-\hat{z}_k$ is defined as NCS state estimation error, $\hat{W}_{I\_NCS}(k)$ denotes the estimated NN weights for proposed identifier and A is the design matrix satisfying $\|A\|^2\leq 1/2$.

Next, using the (10) and (11), the dynamics of NCS state estimation error can be represented as

$$\begin{aligned}\tilde{z}_{k+1} &= z_{k+1} - \hat{z}_{k+1} \\ &= \tilde{W}_{I\_NCS}^{T}(k)\vartheta_{I\_NCS}\left(z_k, \bar{u}_{NCS,k}\right) + A\tilde{z}_k + \varepsilon_{I\_NCS}\end{aligned} \tag{12}$$

In order to force the estimated NN weights to converge close to targets, W_{I_NCS}, within fixed final time, the NN-based identifier update law can be designed as

$$\hat{W}_{I\_NCS}(k+1) = \hat{W}_{I\_NCS}(k) + \alpha_{I\_NCS} \frac{\vartheta_{I\_NCS}\left(z_k, \bar{u}_{NCS,k}\right)}{1 + \left\| \vartheta_{I\_NCS}\left(z_k, \bar{u}_{NCS,k}\right) \right\|^2} \tag{13}$$

with α_{I_NCS} being the tuning parameter for proposed NN-based identifier. It is important to note that proposed NN-based identifier (13) updates along with time that is easier for real-time application than traditional iteration-based scheme [14].

Furthermore, according to (13), the dynamics of NN-based identifier weights estimation error can be obtained as

$$\tilde{W}_{I\_NCS}(k+1) = \tilde{W}_{I\_NCS}(k) - \alpha_{I\_NCS} \frac{\vartheta_{I\_NCS}\left(z_k, \bar{u}_{NCS,k}\right)}{1 + \left\| \vartheta_{I\_NCS}\left(z_k, \bar{u}_{NCS,k}\right) \right\|^2} \tag{14}$$

where weights estimation error is defined as $\tilde{W}_{I\_NCS}(k) = W_{I\_NCS} - \hat{W}_{I\_NCS}(k)$. Next, the convergence of the NCS state estimation (12) and NN-based identifier weight estimation errors are analyzed

Theorem 1 (*NN-based Identifier*): Considering that the target weights, W_{I_NCS}, reside in a compact set $\boldsymbol{\Omega}_I$. Let the NCS state estimator is designed as (11) and NN-based identifier is updated as (13). Then, there exists a positive tuning parameter $\alpha_{I\_NCS} > 0$ such that the NCS state estimation error, $\tilde{z}_k$, and NN-based identifier weights estimation errors, $\tilde{W}_{I\_NCS}(k)$, are all uniformly ultimately bounded (*UUB*) within fixed final time.

Proof: Similar to [13] and omitted due to the space limitation.

B. Finite Horizon ACI Optimal Co-design Part 2: Critic NN

According to formulated finite horizon optimal co-design problem in Section II.B and standard optimal theory [9], the optimal co-design cost function can be represented as

$$V^*(z_k, k) = \min_{u_{c,k}, u_{p,k}} \left\{ \chi(z_N) + \sum_{i=k}^{N-1} \left[Q(z_i, i) + u_{c,i}^T R_c u_{c,i} + \rho(u_{p,i}) \right] \right\};$$

$$V^*(z_N, N) = \chi(z_N) \tag{15}$$

Recall to the universal approximation theorem of NN [15] and relevant NN literatures [14], the optimal co-design cost function can be expressed by using a critic NN as

$$V^*(z_k, k) = W_V^T \vartheta_V(z_k, k) + \varepsilon_{V,k}, \forall k = 0,1,...,N \tag{16}$$

with $W_V, \varepsilon_{V,k}$ represent the target weights and reconstruction errors of the critic NN respectively. The critic NN time-dependent activation function is given as $\vartheta_V(z_k, k)$. Compared with infinite horizon scenarios [14], time-dependent critic NN activation function is introduced. Through properly adjusting the activation function along with time, we will be able to guarantee that estimated cost function converges close to the ideal optimal co-design cost function (15) within fixed final time. Furthermore, similar to [13], the target critic NN weights, the reconstruction error and relevant gradient are bounded as $\|W_V\| \le W_{VM}$, $\|\varepsilon_{V,k}\| \le \varepsilon_{VM}$ and $\left\| \partial\varepsilon_{V,k} / \partial z_k \right\| \le \varepsilon_{VM}^{'}$.

Similar to NN-based identifier, the main challenge is to learn the critic NN target weights effectively. To overcome this challenge, a novel update law is developed next.

First, optimal co-design cost function can be estimated as

$$\begin{aligned}\hat{V}(z_k, k) &= \hat{W}_V^T(k)\vartheta_V(z_k, k), \forall k = 0,1,...,N-1 \\ \hat{V}(z_N, N) &= \hat{W}_V^T(k)\vartheta_V(\hat{z}_{N|k}, N)\end{aligned} \tag{17}$$

with $\hat{W}_V(k)$ denotes the estimated weights at time kT_s and $\hat{z}_{N|k}$ is the final NCS state estimated by using current system state information and identified system dynamics.

Next, while estimated cost function (17) is substituting into HJB equation (4), it will not hold due to the optimal co-design cost function estimation error. Therefore, similar to [13], the HJB residual error is introduced and represented as

$$\begin{aligned}e_{HJB,k} &= \left[Q(z_k, k) + u_{c,k}^T R_c u_{c,k} + \rho(u_{p,k}) \right] + \hat{V}(z_{k+1}, k+1) - \hat{V}(z_k, k) \\ &= r(z_k, u_{c,k}, u_{p,k}) + \hat{W}_V^T(k)\Delta\vartheta_V(z_k, k), \ \forall k = 0,1,...,N-1\end{aligned} \tag{18}$$

where $\Delta\vartheta_V(z_k, k) = \vartheta_V(z_{k+1}, k+1) - \vartheta_V(z_k, k)$. Recall to HJB equation and ideal optimal co-design cost function, the cost-to-go function, $r(z_k, u_{c,k}, u_{p,k})$, can be represented as

$$r(z_k, u_{c,k}, u_{p,k}) = -W_V^T \Delta\vartheta_V(z_k, k) - \Delta\varepsilon_{V,k}, \forall k = 0,1,...,N-1 \tag{19}$$

with $\Delta\varepsilon_{V,k} = \varepsilon_{V,k+1} - \varepsilon_{V,k}$.

Then, substituting the (19) into (18), the relationship between HJB residual error and critic NN weight estimation errors can be derived as

$$\begin{aligned}e_{HJB,k} &= \hat{W}_V^T(k)\Delta\vartheta_V(z_k, k) - W_V^T \Delta\vartheta_V(z_k, k) - \Delta\varepsilon_{V,k}, \\ &= -\tilde{W}_V^T(k)\Delta\vartheta_V(z_k, k) - \Delta\varepsilon_{V,k} \ \ \forall k = 0,1,...,N-1\end{aligned} \tag{20}$$

where $\tilde{W}_V(k) = W_V - \hat{W}_V(k)$ is the critic NN weight estimation error. Next, due to the cost function estimation error, the terminal constraint estimation error at time kT_s is given as

$$e_{TR,k} = \chi(z_N) - \hat{W}_V^T(k)\vartheta_V(\hat{z}_{N|k}, N), \ \forall k = 0,1,...,N \tag{21}$$

Recall to (15), the relationship between terminal constraint estimation error and critic NN weight estimation error is

$$\begin{aligned}e_{TR,k} &= W_V^T \vartheta_V(z_N, N) + \varepsilon_{V,0} - \hat{W}_V^T(k)\vartheta_V(\hat{z}_{N|k}, N) \\ &= \tilde{W}_V^T(k)\vartheta_V(\hat{z}_{N|k}, N) + W_V^T \tilde{\vartheta}_V(z_N, \hat{z}_{N|k}) + \varepsilon_{V,0}\end{aligned} \tag{22}$$

with $\tilde{\vartheta}_V(z_N, \hat{z}_{N|k}) = \vartheta_V(z_N, N) - \vartheta_V(\hat{z}_{N|k}, N)$. Due to the boundedness of critical NN activation function, i.e. $\|\vartheta_V\| \le \vartheta_M$ [13], the $\tilde{\vartheta}_V(z_N, \hat{z}_{N|k})$ is also bounded as $\left\| \tilde{\vartheta}_V(z_N, \hat{z}_{N|k}) \right\| \le 2\vartheta_M$.

According to (19) and (22), we will use both HJB residual error and terminal constraint estimation error to force the estimated critic NN weight to converge to targets. The update law is developed as

$$\hat{W}_V(k+1)=\hat{W}_V(k)+\alpha_{V1}\frac{\Delta\vartheta_V(z_k,k)e^T_{HJB,k}}{1+\|\Delta\vartheta_V(z_k,k)\|^2}+\alpha_{V2}\frac{\vartheta_V(\hat{z}_{N|k},N)e^T_{TR,k}}{1+\left\|\vartheta_V(\hat{z}_{N|k},N)\right\|^2} \quad (23)$$

Remark 4: It is important to note that both ideal and estimated optimal co-design cost function (15) and (17) will become zeroes when the NCS system states are zeroes. Therefore, the critic NN will stop updating once NCS states converge to zeroes. According to existing NN literatures [14], this can be considered as a persistently of excitation (PE) requirement for the critic NN inputs. Namely, the NCS system state need to persistently existing long enough in order for the critic NN to effectively learn the optimal co-design cost function within the fixed final time. Similar to [13], the PE requirement can be satisfied by introducing exploration noise.

Next, the stability of proposed critic NN design and effectiveness of proposed update law (23) are demonstrated.

Theorem 2 (Critic NN): With the initial admissible system and transmit power control, $u_{c,0}(z_k),u_{p,0}(z_k)$, and using critic NN update law given in (23), there exists the positive tuning parameters α_{V1},α_{V2} satisfying $0<\alpha_{V1},\alpha_{V2}\le\frac{2-\xi}{7}$ with ξ being $0<\xi=\frac{\|\vartheta_V(z_k,k)\|^2+\|\Delta\vartheta_V(z_k,k)\|^2+2}{(\|\vartheta_V(z_k,k)\|^2+1)(\|\Delta\vartheta_V(z_k,k)\|^2+1)}<2$ such that critic NN weight estimation error, $\tilde{W}_V(k)$, is uniformly ultimately bounded (*UUB*) within the fixed final time.

Proof: Similar to [13] and omitted due to space limitation.

C. Finite Horizon ACI Optimal Co-design Part 3: Actor NNs

According to our previous researches [13] and the universal approximation theorem of NN [15], the ideal optimal co-design (i.e. optimal system control $u^*_{c,k}$, and optimal transmit power control $u^*_{p,k}$) can be also represented by using two actor NNs as

$$\begin{aligned} u^*_{c,k}&=W_c^T\vartheta_c(z_k,k)+\varepsilon_{c,k}\\ u^*_{p,k}&=W_p^T\vartheta_p(z_k,k)+\varepsilon_{p,k}\end{aligned},\ \forall k=0,1,...,N \quad (24)$$

with W_c,W_p are the target weights for two actor NNs, $\vartheta_c(z_k,k)$, $\vartheta_p(z_k,k)$ denote the relevant NN activation functions and $\varepsilon_{c,k}$, $\varepsilon_{p,k}$ represent the reconstruction errors respectively. Similar to [13], without loss of generality, the actor NNs target weights, activation function and reconstruction error are all bounded, i.e. $\|W_c\|\le W_{cM},\|W_p\|\le W_{pM}$, $\|\vartheta_c(z_k,k)\|\le\vartheta_{cM},\|\vartheta_p(z_k,k)\|\le\vartheta_{pM}$, and $\|\varepsilon_{c,k}\|\le\varepsilon_{cM},\|\varepsilon_{p,k}\|\le\varepsilon_{pM}$.

Then, the optimal co-design can be approximated as

$$\begin{aligned} \hat{u}_{c,k}&=\hat{W}_c^T(k)\vartheta_c(z_k,k)\\ \hat{u}_{p,k}&=\hat{W}_p^T(k)\vartheta_p(z_k,k)\end{aligned},\ \forall k=0,1,...,N \quad (25)$$

where $\hat{W}_c(k),\hat{W}_p(k)$ denote the estimated actor NNs weights.

Next, using the (8), identified NCS system dynamics (11) and estimated optimal co-design cost function (17), the two actor NNs estimation errors can be derived as

$$\begin{aligned} e_{uc,k}&=\hat{W}_c^T(k)\vartheta_c(z_k,k)+\frac{1}{2}R_c^{-1}\hat{g}_z^T(z_k)\frac{\partial\hat{V}(z_{k+1},k+1)}{\partial z_{k+1}}\\ e_{up,k}&=\hat{W}_p^T(k)\vartheta_p(z_k,k)+\varphi_k\left[\frac{1}{2}R_p^{-1}\hat{g}_\tau\frac{\partial\hat{V}(z_{k+1},k+1)}{\partial z_{k+1}}\right]\end{aligned} \quad (26)$$

Using (26), the update laws for two actor NNs can be derived as

$$\begin{aligned} \hat{W}_c(k+1)&=\hat{W}_c(k)-\alpha_c\frac{\vartheta_c(z_k,k)e^T_{uc,k}}{\|\vartheta_c(z_k,k)\|^2+1}\\ \hat{W}_p(k+1)&=\hat{W}_p(k)-\alpha_p\frac{\vartheta_p(z_k,k)e^T_{up,k}}{\|\vartheta_p(z_k,k)\|^2+1}\end{aligned},\ \forall k=0,1,...,N \quad (27)$$

with α_c,α_p are two actor NNs tuning parameters.

Furthermore, the ideal actor NNs (24) should be equal to derived transmit power constraint optimal co-design (8), i.e.

$$\begin{aligned} 0&=W_c^T\vartheta_c(z_k,k)+\varepsilon_{c,k}+\frac{1}{2}R_c^{-1}g_z^T(z_k)\frac{\partial V^*(z_{k+1},k+1)}{\partial z_{k+1}}\\ 0&=W_p^T\vartheta_p(z_k,k)+\varepsilon_{p,k}+\varphi_k\left[\frac{1}{2}R_p^{-1}g_\tau\frac{\partial V^*(z_{k+1},k+1)}{\partial z_{k+1}}\right]\end{aligned} \quad (28)$$

According to (28) and (26), the relationship between actor NNs estimation errors and relevant weights estimation errors can be derived as

$$\begin{aligned} e_{uc,k}&=-\tilde{W}_c^T(k)\vartheta_c(z_k,k)-\frac{1}{2}R_c^{-1}\hat{g}_z^T(z_k)\frac{\partial\tilde{V}(z_{k+1},k+1)}{\partial z_{k+1}}\\ &\quad-\frac{1}{2}R_c^{-1}\tilde{g}_z^T(z_k)\frac{\partial V^*(z_{k+1},k+1)}{\partial z_{k+1}}-\varepsilon_{c,k}\\ e_{up,k}&=-\tilde{W}_p^T(k)\vartheta_p(z_k,k)-\varphi_k\left[\frac{1}{2}R_p^{-1}\hat{g}_\tau\frac{\partial\tilde{V}(z_{k+1},k+1)}{\partial z_{k+1}}\right]\\ &\quad-\varphi_k\left[\frac{1}{2}R_p^{-1}\tilde{g}_\tau\frac{\partial V^*(z_{k+1},k+1)}{\partial z_{k+1}}\right]-\varepsilon_{c,k}\end{aligned} \quad (29)$$

with $\tilde{W}_c(k)=W_c-\hat{W}_c(k),\tilde{W}_p(k)=W_p-\hat{W}_p(k)$ being two actor NNs weights estimation errors, and $\tilde{g}_z(k)=g_z(k)-\hat{g}_z(k)$, $\tilde{g}_\tau(k)$ $=g_z(k)-\hat{g}_z(k)$.

D. Finite Horizon ACI Optimal Co-design Part 4: Closed-loop Stability Analysis

In this section, the closed-loop stability of proposed finite horizon ACI optimal co-design is studied. Before showing the main theorem, the flowchart of proposed scheme is demonstrated Figure 2.

Furthermore, similar to [13], [15], there are three things needed to be stated before providing the main theorem, i.e. 1) Using the initial admission system control and transmit power design (i.e. $u_{c,0}(z_k),u_{p,0}(z_k)$), the initial NCS system states remain in a compact set Ω, 2) The proposed ACI activation functions and relevant gradient are all bounded in the compact set Ω, 3) Adding exploration noises will guarantee the PE requirement [15]. Next, the main theorem and a relevant lemma are given.

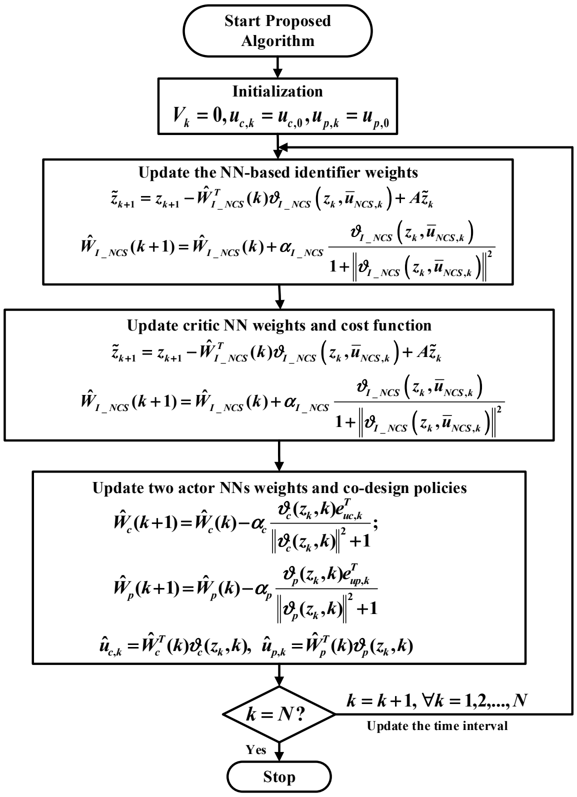

Figure 2. Flowchart of proposed finite horizon ACI optimal co-design

Lemma 1: The ideal optimal co-design, $u_{c,k}^*, u_{p,k}^*$, given in (8) can make the NCS (3) asymptotically stable [13]. Namely, the closed-loop NCS, $z_{k+1} = f_z(z_k) + g_z(z_k)u_{c,k} + g_\tau u_{p,k}$, satisfy

$$\left\| f_z(z_k) + g_z(z_k)u_{c,k}^* + g_\tau u_{p,k}^* \right\| \le l_{NCS}\|z_k\|,\ \forall k = 0,...,N-1 \quad (30)$$

with $0 < l_{NCS} < 1$ being a positive constant.

Proof: Similar to [13] and omitted due to space limitation.

Theorem 3 (Closed-loop Stability): Given $u_{c,0}(z_k), u_{p,0}(z_k)$ as initial admissible co-design for NCS (3), and using the proposed ACI design as (13), (23) and (27), there exists positive NN tuning parameters $\alpha_{I\_NCS}, \alpha_{V1}, \alpha_{V2}, \alpha_c$ and α_p satisfying $0 < \alpha_{I\_NCS} < \min\left\{\frac{1}{2\vartheta_{I\_NCS,M}}, \frac{\vartheta_{I\_NCS,\min}}{\sqrt{2}\vartheta_{I\_NCS,M}}\right\}$, $0 < \alpha_{V1}, \alpha_{V2} \le \frac{2-\xi}{7}$ and $0 < \alpha_c, \alpha_p < 1$ with ξ given in the *Theorem 2* such that NCS state z_k, identification error $\tilde{z}_k$, NN-based identifier weights estimation error $\tilde{W}_{I\_NCS}(k)$, the critic NN weights estimation error $\tilde{W}_V(k)$, and two actor NNs weights estimation errors $\tilde{W}_c(k), \tilde{W}_p(k)$ are all uniformly ultimately bounded (*UUB*) within fixed final time.

Proof: Omitted due to space limitation.

IV. Simulation Results

The performance of proposed finite horizon transmit power constrained optimal control and communication co-design for NCS with uncertain system dynamics has been evaluated. First, the simulation example is introduced before we demonstrate the results.

Example: The batch reactor considered as a benchmark NCS [16] whose system dynamics are shown as

$$\dot{x} = \begin{bmatrix} 1.38 & -0.2077 & 6.715 & -5.676 \\ -0.5814 & -4.29 & 0 & 0.675 \\ 1.067 & 4.273 & -6.654 & 5.893 \\ 0.048 & 4.273 & 1.343 & -2.104 \end{bmatrix} x + \begin{bmatrix} 0 & 0 \\ 5.679 & 0 \\ 1.136 & -3.146 \\ 1.136 & 0 \end{bmatrix} u \quad (31)$$

Then, the co-design parameters are given as: 1) The sampling time $T_s = 0.05\sec$; 2) Wireless channel bandwidth $B = 1000Hz$; 3) The noise spectrum density is $N_0 = 10^{-3}\ Watts/Hz$; 4) The transit power constraint is $P \le 10Watts$; 5) The length of information packet size is $L = 100bits$; and 6) Fixed final time is given as $t_f = NT_s = 10s$ with simulation time steps $N = 200$ and terminal constraint $\chi(z_N) = 0.8$.

First, we will study the stability performance of proposed finite horizon transmit power constrained optimal co-design. The initial admissible policy is given as $u_p = 0.1z_k$ and control

$$u_{c0} = -\begin{bmatrix} 0.87 & 0.85 & -0.1 & 1.24 & 0.03 & 0 & 0.13 & 0.01 \\ -1.51 & 0.09 & -2.55 & 2.47 & 0 & 0.08 & -0.05 & 0.52 \end{bmatrix} z_k .$$

Then, the NN-based identifier activation function is given as $\tanh\{(z_{k,1})^2, z_{k,1}z_{k,2}, ..., (z_{k,6})^2, ..., (z_{k,1})^6, (z_{k,1})^5(z_{k,2}), ..., (z_{k,6})^6\}$, state dependent portion of activation function for critic NN is given as sigmoid of sixth order polynomial (i.e. $\text{sigmoid}\{(z_{k,1})^2, z_{k,1}z_{k,2}, ..., (z_{k,6})^2, ..., (z_{k,1})^6, (z_{k,1})^5(z_{k,2}), ..., (z_{k,6})^6\}$) and time dependent portion of critic NN activation function is defined as saturation polynomial time function (i.e. $sat\{(N-k)^{10}, (N-k)^9, ..., 1; \cdots; 1, (N-k)^{10}, ..., N-k\}$), and activation function of two actor NN are selected as the gradient of critic NN activation function. Please note the saturation for time function is to ensure the magnitude of time function stays instead a reasonable range such that the NN weights are computable. The tuning parameters of NN-based identifier, critic NN and two actor NN are selected as $\alpha_{I\_NCS} = 0.05, \alpha_{V_1} = 0.006, \alpha_{V2} = 0.02$ and $\alpha_c = 0.5, \alpha_p = 0.1$ with the weights of NN-based identifier and critic NN are initialized as zeros and two actor NN weight are set to reflect the initial admissible control at the beginning of simulation.

As shown in Figure 3, 4 and 5, the proposed optimal co-design can force the NCS system state and control inputs converge close to zeroes within finite time horizon even while the system dynamics are unknown and transmit power is limited. It is important to note that 1) The designed transmit power is always less than 10 watts that satisfies the constraint. 2) When control signals become zeroes, transmit power will also be decreased to zero since there is no information needed to be transmitted and 3) Since the proposed design needs some time to tune the NNs at the beginning, there is a slight overshoot initially which dies away fast with time.

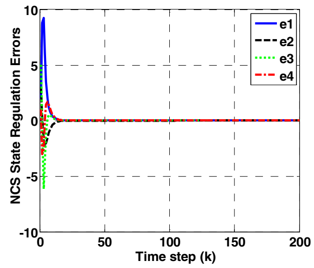

Fig. 3. NCS state regulation errors

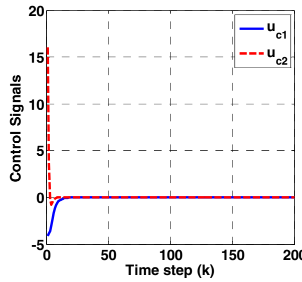

Fig. 4. Estimated NCS control inputs

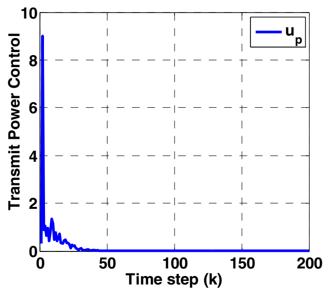

Fig. 5. Estimated transmit power control

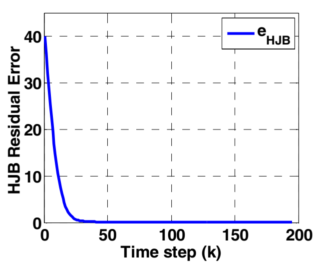

Fig. 6. HJB residual error.

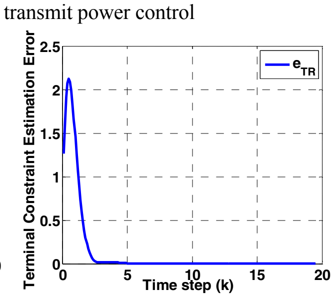

Fig. 7. Terminal constraint estimation error

Next, in order to evaluate the optimality of proposed scheme, the HJB residual error and terminal constraint estimation error are considered. As shown in Figures 6 and 7, both HJB residual error and terminal constraint estimation error converge close to zeroes within fixed final time which indicates that proposed design converges close to finite horizon transmit power constrained optimal co-design while satisfying the terminal constraint.

V. CONCLUSION

In this paper, we investigate the finite horizon optimal control and communication co-design for NCS with transmit power constraint. By adopting the emerging NDP technique and ACI architecture, our proposed design cannot only converge close to the ideal optimal co-design within fixed final time, but also does not require the knowledge of NCS system dynamics. Specifically, using the historical system information along with developed NN-based identifier, the real-time NCS dynamics can be quickly learnt. Meanwhile, the introduced critic NN is effectively estimating the optimal co-design cost function by using HJB residual error and terminal constraint estimation error. Eventually, with the identified NCS dynamics and approximated optimal cost function, two actor NNs are developed to estimate the optimal system control signals and transmit power control inputs efficiently within final horizon. Moreover, the effectiveness of proposed scheme has been validated through theoretical stability analysis and numerical simulations. To sum up, the major three contributions of this paper are: 1) It is the first time to combine the practical communication effects into the real-time NCS dynamics modeling, and 2) Without known the NCS dynamics, the proposed novel NDP-based design can still properly and quickly learn the ideal optimal control and transmit power co-design within the fixed final time and 3) The practical transmit power constraint has been efficiently incorporated into the proposed design.

REFERENCES

[1] Y. Tipsuwan, M. Y. Chow, "Control methodologies in networked control systems," *Contr. Engineering Practice*, vol.11, 2003, pp. 1099–1111.

[2] F. Y. Wang and D. Liu, *Networked control systems: theory and applications*, Springer, London, 2010.

[3] D. Tse and P. Viswanath, *Fundamentals of wireless communication*, Cambridge University Press, Cambridge, UK, 2005.

[4] Y. Halevi and A. Ray. "Integrated communication and control systems: Part I---Analysis," *J. Dynamic Syst., Measure. Contr.*, vol. 110, pp. 367-373, 1988.

[5] W. Zhang, M. S. Branicky, and S. Phillips, "Stability of networked control systems," *IEEE Contr. Syst. Mag.*, vol. 21, pp. 84–99, 2001.

[6] H. Shousong and Z. Qixin, "Stochastic optimal control and analysis of stability of networked control systems with long delay," *Automatica*, vol. 39, 2003, pp. 1877–1884.

[7] L. L. Feng, J. Moyne, and D. Tilbury, "Optimal control design and evaluation for a class of networked control systems with distributed constant delays," in *Proc. IEEE Amer. Contr. Conf.*, 2002, pp. 3009-3014.

[8] T. M. Cover, *Elements of information theory*, wiley, MI, 2007.

[9] F. L. Lewis and V.L. Syrmos. *Optimal Control.* 2nd ed., Wiley, New York, 1995.

[10] F. Y. Wang, N. Jin, D. Liu, and Q. L. Wei, "Adaptive dynamic programming for finite horizon optimal control of discrete-time nonlinear systems with ε-error bound," *IEEE Trans. On Neur. Net.*, vol. 22, 2011, pp. 24-36.

[11] P.J. Werbos, "A menu of designs for reinforcement learning over time," *J. Neural Networks Contr.*, vol. 3, pp. 835-846, 1983.

[12] D. P. Bertsekas and J. Tsitsiklis, *Neuro-dynamics programming*, Athena Scientic, 1996.

[13] H. Xu, and S. Jagannathan, "Stochastic optimal controller design for uncertain nonlinear networked control system via neuro dynamic programming," *IEEE Trans. on Neur. Net. Learn Syst.*, 2012, in press.

[14] A. Heydari and S. N. Balakrishan, "Finite-horizon Input-Constrained Nonlinear Optimal Control Using Single Network Adaptive Critics," in *Proc. American Control Conf.*, San Francisco, USA, 2011, pp. 3047–3052.

[15] S. Jagannathan, *Neural network control of nonlinear discrete-time systems*, CRC Press, 2006.

[16] D. Carnevale, A. R. Teel, and D. Nesic, "A Lyapunov proof of improved maximium allowable transfer interval for networked control systems," *IEEE. Trans. Automa. Contr.*, vol. 52, pp. 892-897, 2007.